This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

Sci 1520.203

Harvard College Library

FROM THE

UNITED STATES GOVERNMENT

SCIENCE CENTER LIBRARY

Sci 1520.203

DEPARTMENT OF COMMERCE

BULLETIN
OF THE
BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

VOLUME 12
1915-1916

WASHINGTON
GOVERNMENT PRINTING OFFICE
1916

Harvard College Library
Aug. 22, 1916
From the
United States Government.

214-40
5

CONTENTS OF VOLUME 12

No. 1

[Issued October 28, 1915]

Page

245. TEMPERATURE COEFFICIENT OF MAGNETIC PERMEABILITY WITHIN THE WORKING RANGE .... *Raymond L. Sanford* 1

246. METHODS OF MEASURING THE INDUCTANCES OF LOW-RESISTANCE STANDARDS .... *Frank Wenner*, *Ernest Weibel*, and *F. B. Silsbee* 11

247. AN ANEROID CALORIMETER .... *H. C. Dickinson* and *N. S. Osborne* 23

248. SPECIFIC HEAT AND HEAT OF FUSION OF ICE .... *H. C. Dickinson* and *N. S. Osborne* 49

249. THE EMISSIVITY OF METALS AND OXIDES. IV. IRON OXIDE .... *George K. Burgess* and *Paul D. Foote* 83

250. CHARACTERISTICS OF RADIATION PYROMETERS .... *George K. Burgess* and *Paul D. Foote* 91

No. 2

[Issued November 8, 1915]

251. INTERFERENCE MEASUREMENTS OF WAVE LENGTHS IN THE IRON SPECTRUM (2851–3701) WITH NOTES ON COMPARISONS OF LENGTHS OF LIGHT WAVES BY INTERFERENCE METHODS, AND SOME WAVE LENGTHS IN THE SPECTRUM OF NEON GAS .... *Kelvin Burns* and *W. F. Meggers* 179

252. EFFECTIVE RESISTANCE AND INDUCTANCE OF IRON AND BIMETALLIC WIRES .... *John M. Miller* 207

253. A DIRECT-READING DEVICE FOR USE IN COMPUTING CHARACTERISTICS OF VACUUM TUNGSTEN LAMPS .... *J. F. Skogland* 269

254. A STUDY OF THE QUALITY OF PLATINUM WARE .... *George K. Burgess* and *Paul D. Foote* 289

No. 3

[Issued February 3, 1916]

255. CALCULATION OF THE MAXIMUM FORCE BETWEEN TWO COAXIAL CIRCULAR CURRENTS .... *Frederick W. Grover* 317

256. CONSTRUCTION OF PRIMARY MERCURIAL RESISTANCE STANDARDS .... *F. A. Wolff*, *M. P. Shoemaker*, and *C. A. Briggs* 375

257. NOTE ON THE RESISTANCE OF RADIOTELEGRAPHIC ANTENNAS .... *L. W. Austin* 465

No. 4

[Issued May 25, 1916]

258. A METHOD OF MEASURING EARTH RESISTIVITY .... *Frank Wenner* 469

259. A NEW RELATION DERIVED FROM PLANCK'S LAW .... *Paul D. Foote* 479

260. "CENTER OF GRAVITY" AND "EFFECTIVE WAVE LENGTH" OF TRANSMISSION OF PYROMETER COLOR SCREENS, AND THE EXTRAPOLATION OF THE HIGH TEMPERATURE SCALE .... *Paul D. Foote* 483

261. STUDIES OF INSTRUMENTS FOR MEASURING RADIANT ENERGY IN ABSOLUTE VALUE: AN ABSOLUTE THERMOPILE .... Page
.... *W. W. Coblentz* and *W. B. Emerson* 503

262. PRESENT STATUS OF THE DETERMINATION OF THE CONSTANT OF TOTAL RADIATION FROM A BLACK BODY .... *W. W. Coblentz* 553

263. ILLUMINATION FROM A RADIATING DISK .... *Paul D. Foote* 583

264. PHOTOMETRY OF THE GAS-FILLED LAMP ....
.... *G. W. Middlekauff* and *J. F. Skogland* 589

265. LIFE TESTING OF INCANDESCENT LAMPS AT THE BUREAU OF STANDARDS . .
.... *G. W. Middlekauff, B. Mulligan,* and *J. F. Skogland* 607

Sci 1520.2

DEPARTMENT OF COMMERCE

Vol. 12 No. 1

BULLETIN

OF THE

BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

OCTOBER 28, 1915

CONTENTS

Page

Temperature Coefficient of Magnetic Permeability Within the Working Range . . . . *R. L. Sanford* 1

Methods of Measuring the Inductances of Low-Resistance Standards . . . . *Frank Wenner, Ernest Weibel, and F. B. Silsbee* 11

An Aneroid Calorimeter . . . . *H. C. Dickinson and N. S. Osborne* 23

Specific Heat and Heat of Fusion of Ice . . . *H. C. Dickinson and N. S. Osborne* 49

The Emissivity of Metals and Oxides. IV. Iron Oxide . . . . *George K. Burgess and Paul D. Foote* 83

Characteristics of Radiation Pyrometers . . *George K. Burgess and Paul D. Foote* 91

WASHINGTON
GOVERNMENT PRINTING OFFICE
1915

DEPARTMENT OF COMMERCE

Vol. 12 No. 1

BULLETIN
OF THE
BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

OCTOBER 28, 1915

CONTENTS

Page
Temperature Coefficient of Magnetic Permeability Within the Working Range . . . . R. L. *Sanford* 1
Methods of Measuring the Inductances of Low-Resistance Standards . . . . *Frank Wenner, Ernest Weibel, and F. B. Silsbee* 11
An Aneroid Calorimeter . . . . *H. C. Dickinson and N. S. Osborne* 23
Specific Heat and Heat of Fusion of Ice . . *H. C. Dickinson and N. S. Osborne* 49
The Emissivity of Metals and Oxides. IV. Iron Oxide . . . . *George K. Burgess and Paul D. Foote* 83
Characteristics of Radiation Pyrometers . *George K. Burgess and Paul D. Foote* 91

WASHINGTON
GOVERNMENT PRINTING OFFICE
1915

Harvard College Library
Dec. 15, 1915
From the
United States Government.

ADDITIONAL COPIES
OF THIS PUBLICATION MAY BE PROCURED FROM
THE SUPERINTENDENT OF DOCUMENTS
GOVERNMENT PRINTING OFFICE
WASHINGTON, D. C.
AT
25 CENTS PER COPY
SUBSCRIPTION PRICE, $1 PER VOLUME OF 4 NUMBERS

TEMPERATURE COEFFICIENT OF MAGNETIC PERMEABILITY WITHIN THE WORKING RANGE

By R. L. Sanford

CONTENTS

Page
1. Statement of problem 1
2. Materials and apparatus 2
3. Observations and results 5
4. Summary 10

1. STATEMENT OF PROBLEM

The development of methods and apparatus for magnetic measurements capable of an accuracy of 1 per cent makes it necessary to consider factors which have heretofore been considered negligible. One of these factors, the temperature coefficient of magnetic permeability, is the subject of the present paper.

Many workers have studied the effects of temperature on the magnetic permeability of iron and steel. All of the investigations, however, have been carried on with special reference to temperatures far removed from the atmospheric range. However, Bauer,[1] Hopkinson,[2] Terry,[3] Burrows,[4] and a few others have made some observations at temperatures between 0 and 100° C. which show that induction curves at two different temperatures in this region cross each other. For low inductions the magnetizing force necessary to produce a certain induction decreases with increase in temperature while for high inductions it increases.

The materials examined by these investigators include soft iron, mild steel, hard steel, electrolytic iron, and nickel. Ewing[5] states that "the effects which atmospheric fluctuations of temperature exert upon the magnetic quality are too slight to require to be taken account of in specifying the magnetic properties of a sample, or in stating the results of experiments." He gives curves for annealed iron wire and also for the same wire hardened by stretching beyond the elastic limit. The temperatures were 7 or 8° and 100°. The data show that for a given induction changes as high as 0.14 per cent per degree in permeability may occur.

---

[1] Wied. Ann., **11**, p. 394; 1880.

[2] Phil. Trans., **180**, A, p. 443; 1889.

[3] Phys. Rev., **28**, p. 393; 1909.

[4] This Bulletin, **4**, p. 270; Scientific Paper No. 78.

[5] Magnetic Induction in Iron and Other Metals, p. 178, 3d edition.

In magnetic measurements at the Bureau of Standards it has been found that for magnetizing forces between 100 and 300 gausses the heating due to the current in the magnetizing coils is sufficient to change quite appreciably the induction corresponding to a given magnetizing force. For this reason it has been the practice, when making measurements where an accuracy of 1 per cent is desired, to immerse the magnetizing coils in oil, which is maintained at a standard temperature of 25°. The present work was undertaken to determine what the magnitude of this temperature effect is and whether it is feasible to apply a correction for the reduction to a standard temperature of data taken at other temperatures.

2. MATERIALS AND APPARATUS

The materials and heat treatments indicated in Tables 1 and 2 were chosen for preliminary work as being representative of the range ordinarily met with in magnetic measurements. The results on this relatively small number of samples are such, however, that further experimental work appears to be unnecessary.

TABLE 1.—Chemical Composition

	Carbon	Sulphur	Phosphorus	Manganese	Silicon
Cast-iron	3.65	0.074	0.656	0.52	1.57
Wrought-iron	.053	.028	.155	.15	.18
Low-carbon steel	.073	.036	.100	.33	.10

TABLE 2.—Heat treatment

Designation	Material	Heat treatment
C-1	Cast iron	As received.
C-2	do	Quenched from 1000°.
C-3	do	Annealed from 900°.
W-1	Wrought iron	As received.
W-2	do	Annealed from 900°.
S-1	Low-carbon steel	As received.
S-2	do	Annealed from 900°.

The magnetic measurements were made by the usual ring method. This method presents the fewest experimental difficulties and yields reliable results. In the ring method two assumptions are made: First, that the average value of the magnetizing force is that at the center of the cross section; and, second, that the permeability is constant within the limits of the magnetizing force across the section. The dimensions of the rings are such that the errors due to these assumptions are negligible, but even though they were much greater they enter to the same extent at all temperatures, and therefore would not introduce any error in the temperature coefficients which are obtained by difference. The error due to magnetic viscosity was rendered negligible by making the rings of small cross section and using a long-period galvanometer (30 seconds). The constants of the rings and windings are shown in Table 3.

TABLE 3.—Constants of Rings and Windings

Ring	Density	Mass	Mean diameter	Radial width	Cross section	Mean length	Magnetizing turns	$\frac{H}{I}$	Turns in test coil	Thermometer resistance at 25°
C-1	7.06	615	11.4	1.0	2.43	35.8	312	10.95	100	1.9334
C-2	6.96	653	11.4	1.0	2.62	35.8	305	10.71	100	1.7729
C-3	6.86	648	11.4	1.0	2.64	35.8	295	10.36	100	1.7933
W-1	7.67	380	10.8	.6	1.46	33.9	317	11.75	100	1.6296
W-2	7.55	399	10.8	.6	1.56	33.9	305	11.31	100	2.0231
S-1	7.83	283	8.4	.5	1.38	26.2	222	10.65	100	1.6104
S-2	7.76	285	8.4	.5	1.40	26.2	231	11.08	100	1.7727

The temperature measurements were made by means of a resistance thermometer, consisting of two turns of silk-covered copper wire wound around the outer circumference of the ring. The measuring current of 0.1 ampere produces, therefore, no component of magnetizing force in the direction of the magnetic flux within the iron. The wire was separated from the iron by only a single layer of thin paper, so that its temperature may be assumed as equal to that of the iron itself. Separate potential leads were brought out and the resistance of the part of the wire next to the iron determined by measuring the potential drop across it when 0.1 ampere was flowing. The current and voltage measurements were made with a potentiometer and a standard resistance. The thermometers were calibrated by measuring their resistances when placed in an oil bath at known temperatures. No error is introduced by the heating of the current in the thermometer coils, since the calibrations are made under working conditions. The resistance of the thermometer wires was of the order of 1.5 ohms with a change per degree of about 0.006 ohm, so that measurements could easily be made to 0.1° C, which was ample for the purpose.

The induction was measured in terms of the deflections of a ballistic galvanometer calibrated by means of a mutual inductance. The test coil of 100 turns was wound as closely to the iron as the thermometer wire would permit so as to inclose only the flux in the iron itself.

The magnetizing winding consists of two layers of No. 14 double-cotton covered wire, separated from the other windings by cotton tape. The concentration of the winding is such that the magnetizing force is about ten times the current. The magnetizing current was found by measuring the potential drop across a standard resistance by means of a potentiometer. The magnetizing coils of all of the rings were connected in series. This arrangement allows all of the rings to be demagnetized at one operation and also requires but one adjustment of current for each value of the magnetizing force used.

An oil bath was employed for regulating temperature during the magnetic measurements. The tank was provided with an outer compartment which could be packed with ice for the low temperature. The higher temperatures were secured by means of an electric heating coil placed directly in the oil. The use of oil, which was kept in circulation by a motor-driven stirrer, served to maintain the rings at a unifrom temperature.

3. OBSERVATIONS AND RESULTS

The magnetic measurements were made at different temperatures in the following order: 25°, 88°, 25°, 3°, 25°. The 25° readings were repeated to determine whether any permanent change had taken place in the material. The first set of observations did show a change in some of the rings, and it was found necessary to alternately heat and cool them a number of times in order to bring the material to a stable state. This progressive decrease in permeability due to heating has been observed before [6] and is termed "aging."

Table 4 gives an example of this aging effect, which was most marked in the wrought iron as received and steel as received.

TABLE 4.—Aging Effect in Steel as Received

Induction (gausses)	Magnetizing force (gausses) at 25° C		Induction (gausses)	Magnetizing force (gausses) at 25° C	
	Before heating	After heating to 88°		Before heating	After heating to 88°
2000	2.02	2.31	10 000	7.86	9.02
4000	2.83	3.28	12 000	12.1	13.9
6000	3.89	4.47	14 000	21.1	24.3
8000	5.42	6.28	16 000	47.2	53.0

This shows an average change of about 15 per cent in magnetizing force for a given induction due to aging. The values for wrought iron show the same order of magnitude. It is obvious that temperature coefficients can not be determined until the material has been brought to a stable state. When the stable condition had been reached the oil bath was adjusted successively to temperatures of approximately 3°, 25°, and 88°, and normal induction measurements made. No attempt was made to keep the temperature strictly constant, but small variations from the

---

[6] Mazotto, N. Cimento, 7, pp. 393–421, June, and 8, pp. 5–27, July, 1904; abstracted in Sc. Ab. B 955; 1904.

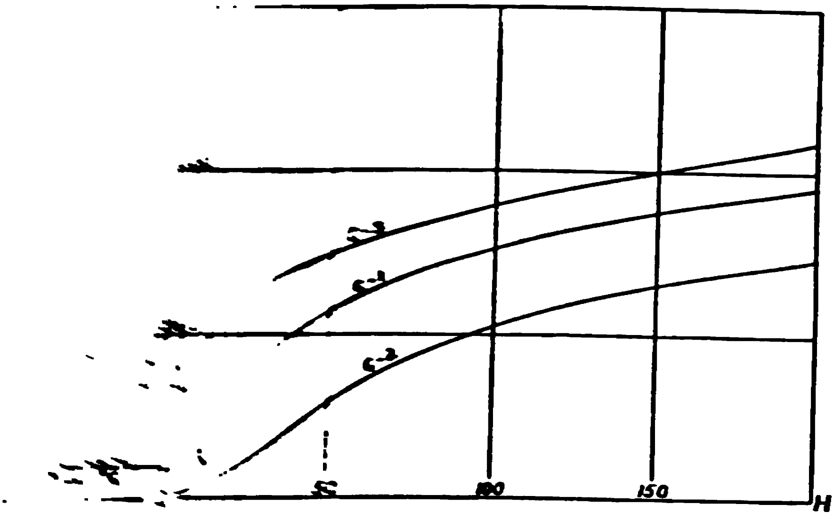


Fig. [illegible].—Magnetic normal induction for cast iron at 25° and percentage change in magnetizing force for a given induction due to 1 degree rise in temperature

C-1, as received   C-2, quenched from [illegible]°   C-3, annealed from 900°

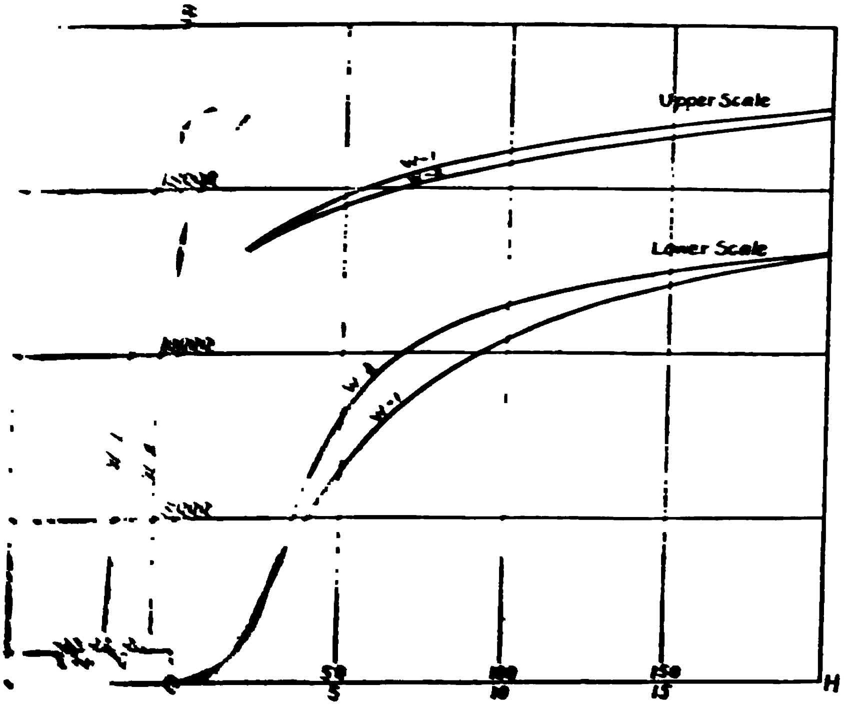


Fig. [illegible].—Magnetic normal induction for wrought iron at 25° and percentage change in magnetizing force for a given induction due to 1 degree rise in temperature

W-1, as received   W-2, annealed from 900°

standard temperature were permitted. As the temperature was actually determined for each point the coefficient of induction for a given magnetizing force could be calculated from the data at two temperatures. In order to draw curves for the standard temperatures 3°, 25°, and 88°, corrections were applied to the observed values by means of these temperature coefficients. From these curves the coefficients of magnetizing force corresponding to a given induction were determined. The values calculated from these three curves indicate that, within the limits of experimental error, the temperature coefficient in this range is independent of temperature.

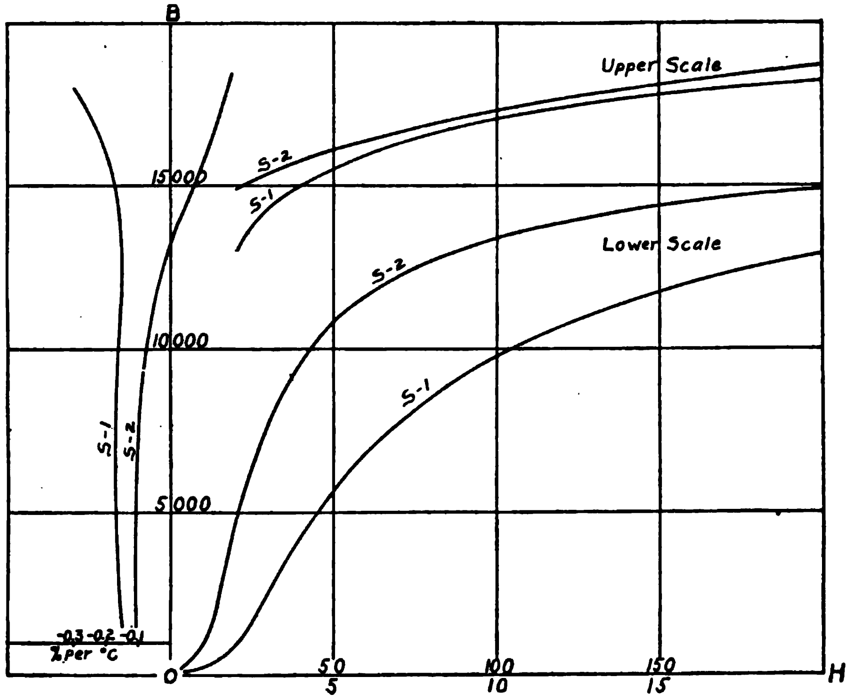


FIG. 3.—*Showing normal induction for low-carbon steel at 25° and percentage change in magnetizing force for a given induction due to 1 degree rise in temperature*

S-1, as received S-2, annealed from 900°

Figs. 1, 2, and 3 give the normal induction curves at 25° and the temperature coefficients in terms of the percentage change per degree in the magnetizing force corresponding to a given induction. It will be noted that the temperature coefficient

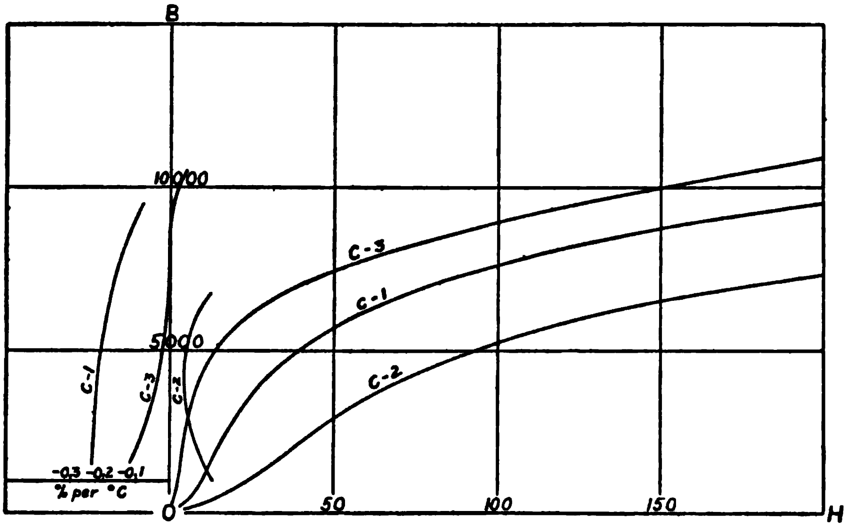


FIG. 1.—*Showing normal induction for cast iron at 25° and percentage change in magnetizing force for a given induction due to 1 degree rise in temperature*

C-1, as received   C-2, quenched from 1000°   C-3, annealed from 900°

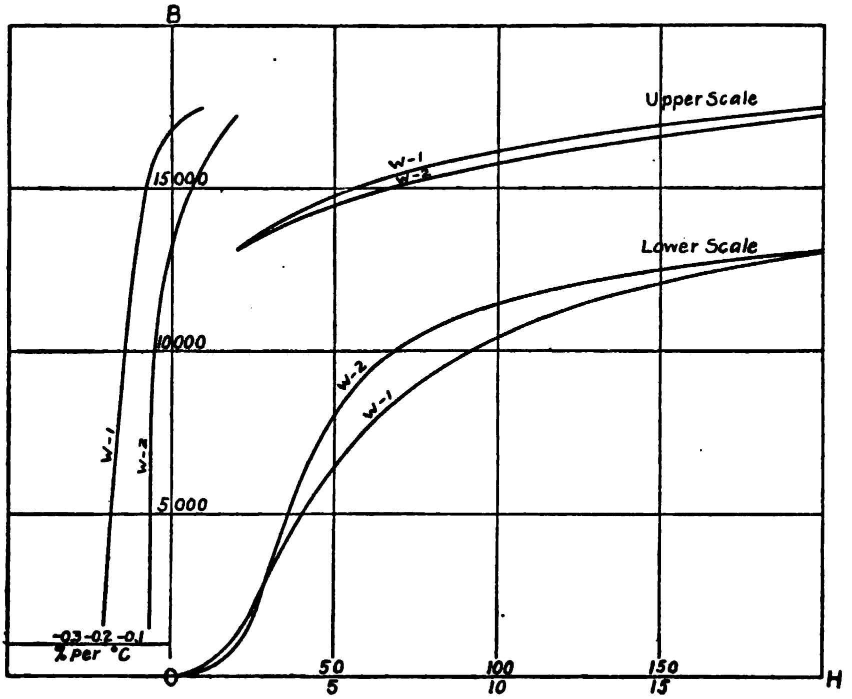


FIG. 2.—*Showing normal induction for wrought iron at 25° and percentage change in magnetizing force for a given induction due to 1 degree rise in temperature*

W-1, as received   W-2, annealed from 900°

standard temperature were permitted. As the temperature was actually determined for each point the coefficient of induction for a given magnetizing force could be calculated from the data at two temperatures. In order to draw curves for the standard temperatures 3°, 25°, and 88°, corrections were applied to the observed values by means of these temperature coefficients. From these curves the coefficients of magnetizing force corresponding to a given induction were determined. The values calculated from these three curves indicate that, within the limits of experimental error, the temperature coefficient in this range is independent of temperature.

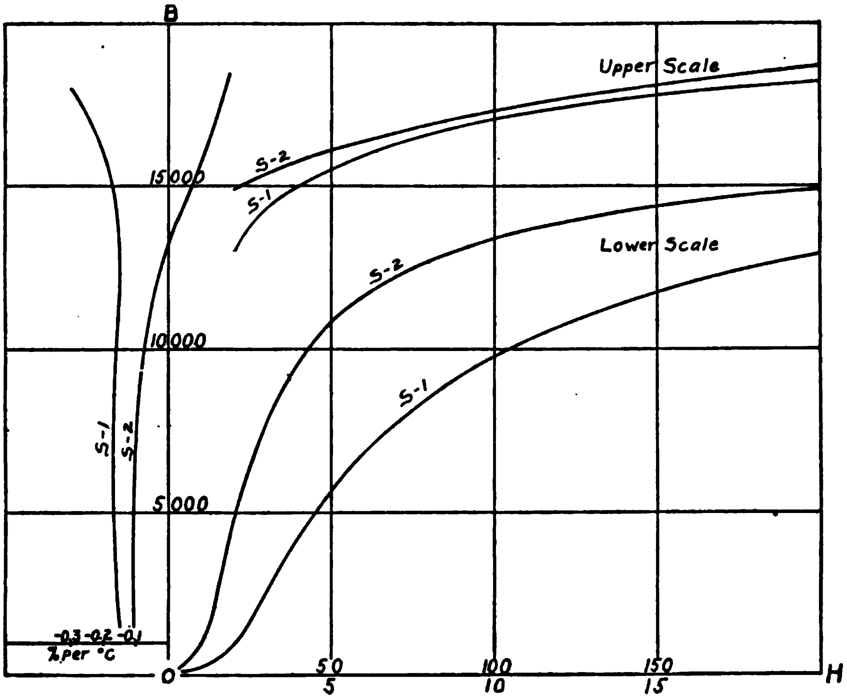


Fig. 3.—*Showing normal induction for low-carbon steel at 25° and percentage change in magnetizing force for a given induction due to 1 degree rise in temperature*

S-1, as received S-2, annealed from 900°

Figs. 1, 2, and 3 give the normal induction curves at 25° and the temperature coefficients in terms of the percentage change per degree in the magnetizing force corresponding to a given induction. It will be noted that the temperature coefficient

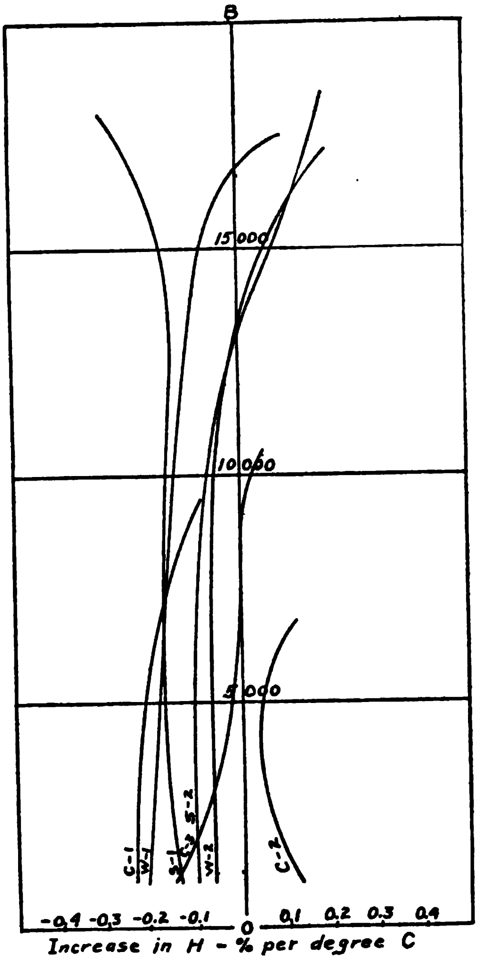


FIG. 4.—*Showing percentage change in magnetizing force for a given induction due to 1 degree rise in temperature*

C-1, cast iron as received
C-2, cast iron quenched from 1000°
C-3, cast iron annealed from 900°
W-1, wrought iron as received
W-2, wrought iron annealed from 900°
S-1, low-carbon steel as received
S-2, low-carbon steel annealed from 900°

changes sign in some cases and not in others. The wrought iron both as received and annealed shows this change, while the steel shows it only in the annealed condition. Cast iron shows quite different characteristics, according to the heat treatment it has received. In the material as received the values of the coefficient are all negative, though they decrease in magnitude with increase in induction. For the annealed material the temperature coefficient is negative at low inductions and positive at high inductions, while for the quenched specimen the values are all positive.

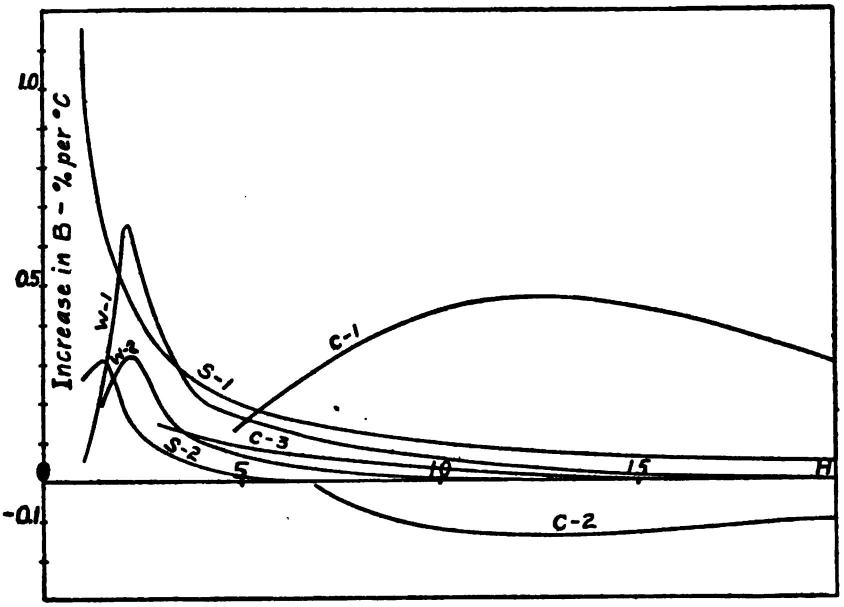


FIG. 5.—*Showing percentage change in induction for a given magnetizing force due to 1 degree rise in temperature*

C-1, cast iron as received
C-2, cast iron quenched from 1000°
C-3, cast iron annealed from 900°
W-1, wrought iron as received
W-2, wrought iron annealed from 900°
S-1, low-carbon steel as received
S-2, low-carbon steel annealed from 900°

Figs. 4 and 5 bring together the coefficients for all of the materials. Fig. 4 shows the percentage change per degree in magnetizing force for a given induction, while Fig. 5 shows the percentage change per degree in induction for a given magnetizing force. The maximum percentage change in induction occurs at comparatively low magnetizing forces, while the percentage change in magnetizing force shows no such well-defined maximum. From these figures it can also be seen how great a variation there is, not only with different kinds of material, but in the same material which has received different heat treatments.

SUMMARY

Magnetic measurements at different temperatures within the atmospheric range have been made on a number of materials with different heat treatments. The results of these measurements are of such a nature that the following conclusions seem to be warranted:

1. The temperature coefficient of magnetic permeability, though small, can not be neglected in magnetic measurements of high accuracy.

2. On account of the wide variation in temperature coefficient, not only for different materials, but also for the same material with different heat treatments, correction can not be made to standard temperature from data obtained from other materials.

3. Unless the temperature coefficient is known for the particular material under test, temperature control offers the only means of avoiding the error due to temperature changes, at least where errors as great as 1 per cent are to be avoided. Conditions often arise in practice where the temperature of a specimen may be raised from 10° to 20° above the temperature of the room, due either to a comparatively heavy current or to the use of coils already heated from a previous test. Since temperature coefficients may be as great as 0.3 per cent per degree, errors amounting to 2 per cent or more may exist.

These conclusions hold in general, even though there may be materials which have very small or even zero temperature coefficients.

The author is indebted to Dr. C. W. Burrows for many valuable suggestions throughout the course of the work, also to the chemistry division of the Bureau for chemical analyses and the division of metallurgy for the heat treatment of the specimens.

WASHINGTON, December 2, 1914.

METHODS OF MEASURING THE INDUCTANCES OF LOW-RESISTANCE STANDARDS

By Frank Wenner, Ernest Weibel, and F. B. Silsbee

I. INTRODUCTION

1. GENERAL USE OF LOW-RESISTANCE STANDARDS

In precise measurements of alternating-current phenomena it frequently becomes necessary to use low-resistance standards of large current capacity. The most common example is in the calibration of instruments for the accurate measurement of power in alternating-current circuits. Power is usually measured by a wattmeter in connection with current and voltage transformers, and these transformers are in turn calibrated by the use of standards of known resistance and inductance. An error in the phase angle of a standard will appear in the apparent phase angle of the transformer, and hence enter directly into the power measurement. For example, if a 0.0001-ohm standard is assumed to be noninductive when its inductance is 10 cm, or 10×10^{-9} henry, the current in the standard at 60 cycles will lag 2.15° behind the electromotive force at its potential terminals, and the error in measuring power at a power factor of 0.71 would be 3.7 percent.

2. DEFINITION OF RESISTANCE AND INDUCTANCE

The low-resistance standards considered in this paper are conductors having two pairs of terminals, namely, current and potential terminals. We shall treat only such conductors in which when a current

$$i = I_m \sin pt \qquad (1)$$

flows through the current terminals, the difference of potential between the potential terminals is

$$e = A \sin pt + B \cos pt \qquad (2)$$

The resistance R is A/I_m, the inductance L is B/pI_m, the time constant T is $B/pA = L/R$, and the phase angle θ is $\tan^{-1}B/A = \tan^{-1} pL/R$ where $p = 2\pi$ times the frequency.

In order that the resistance and inductance of a low-resistance standard be definite,[1] it is necessary to have the potential and current leads brought out in a definite manner. In a well-designed standard for general use in alternating-current measurements the resistance and inductance are practically independent of the manner of making the connections to the terminals of the standard.

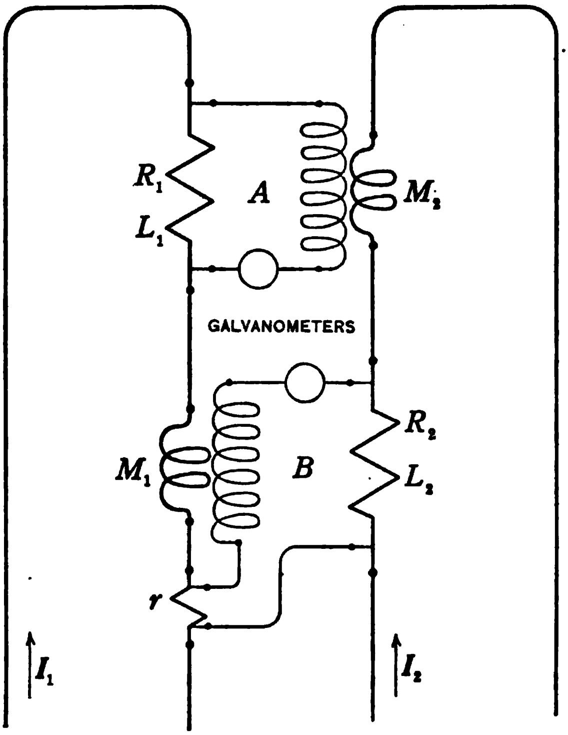


Fig. 1.—*Simple diagram of connections for measuring sum of phase angles of two resistance standards*

---

[1] Wenner, this Bulletin, 8, p. 560; 1912; Scientific Paper No. 181.

3. PREVIOUS MEASUREMENTS

All values heretofore obtained for the residual inductance of "noninductive" standards of low resistance have been based upon the calculated inductance of some conductor. Such a calculation can only be performed for simple shapes and for ideal conditions as regards current distribution, current and potential leads, end effects, etc., so that the construction of a standard of which the inductance can be calculated with sufficient certainty offers some difficulties.

The inductances of standards of odd shapes have then been obtained by measuring the differences between their phase angles and that of a calculated standard. These difference measurements have been made, using a Rosa curve tracer,[2] an electrometer,[3] an electrodynamometer,[4] a Thomson bridge,[5] and a current transformer.[6]

The two methods described in this paper for determining the phase angles of low-resistance standards without recourse to computed values are believed to be the only ones now available.

II. THEORY OF METHODS

1. FIRST METHOD

This method gives primarily the sum of the phase angles of two resistance standards. By using three resistance standards three sums[7] can be measured from which each phase angle can be obtained.

The resistance standards are each connected in series with a mutual inductance, as indicated in Fig. 1, and excited by alternating currents substantially in quadrature. The constants of the circuits and the relative phase and magnitude of the two currents are then adjusted so that no current flows in either galvanometer.

---

[2] By plotting curves of the voltages across the potential terminals of 2 resistance standards carrying the same current, Dr. Rosa obtained directly the difference of the phase angles.

[3] Orlich, Zs. für Instk., 29, p. 241; 1909.

[4] Agnew and Fitch, this Bulletin, 6, p. 281; 1909; Scientific Paper No. 130.

[5] Sharp & Crawford, Trans. Amer. Inst. E. E., 29, p. 1540; 1910. Wenner, this Bulletin, 8, p. 603; 1912.

[6] Agnew and Silsbee, Trans. A. I. E. E., 31, p. 1635; 1912.

[7] By measuring the sum of the phase angles of two resistance standards by this method and the difference by any of those mentioned above, one can determine both phase angles.

If then we let the real parts of $I_1\epsilon^{jpt}$ and $I_2\epsilon^{jpt}$ (where I_1 and I_2 are complex quantities or vectors and j represents the $\sqrt{-1}$) represent the instantaneous values of the currents, and R_1, L_1, M_1, r, R_2, L_2, and M_2 represent the constants of the circuits, we have

$$(R_1+jpL_1)I_1=jpM_2I_2 \tag{3}$$

and

$$(R_2+jpL_2)I_2=(r-jpM_1)I_1. \tag{4}$$

Eliminating I_1/I_2, we get

$$\frac{jpM_2}{R_1+jpL_1}=\frac{R_2+jpL_2}{r-jpM_1} \tag{5}$$

or

$$jrpM_2+p^2M_1M_2=R_1R_2+j(pL_1R_2+pL_2R_1)-p^2L_1L_2.$$

Therefore

$$R_1R_2=p^2(M_1M_2+L_1L_2) \tag{6}$$

and

$$\frac{pL_1}{R_1}+\frac{pL_2}{R_2}=\frac{rpM_2}{R_1R_2} \text{ or } \frac{r}{pM_1} \tag{7}$$

very closely. The first of these equations, (6), can be used in the absolute measurement of resistance and the second gives a measure of the sum of the phase angles of the two resistances.

The above relations can be shown very simply by the vector diagram shown in Fig. 2. If Z_1 and Z_2 are the impedances of the two resistances, then Z_1I_1 leads I_1 by $\theta_1=\tan^{-1}pL_1/R_1$, and when the galvanometer A carries no current this equals $-jpM_2I_2$, so that I_2 must lead Z_1I_1 by 90°. Now, Z_2I_2 leads I_2 by $\theta_2=\tan^{-1} pL_2/R_2$, as shown. Also $-jp(-M_1)I_1$ or jpM_1I_1 leads I_1 by 90°, as shown. From this we subtract rI_1 parallel to I_1, and when the galvanometer B carries no current we know that $jpM_1I_1-rI_1=Z_2I_2$. By inspection of the diagram we see that

$$\tan\,(\theta_1+\theta_2)=\frac{r}{pM_1} \tag{8}$$

and if θ_1 and θ_2 are small we have

$$\frac{pL_1}{R_1}+\frac{pL_2}{R_2}=\frac{r}{pM_1} \tag{9}$$

very closely. A modification of this method of measuring the sum is to use, instead of the small resistance r, a small mutual inductance m, whose primary is in series with R_2 and whose secondary is in series with galvanometer B. The sum of the phase angles would then be pm/R_2 instead of r/pM_1.

In the above proofs it is assumed that the mutual inductances are pure; i. e., give electromotive forces in exact quadrature with the primary current. Any deviation from quadrature represents an equal error in the measurement of the sum of the phase angles.

It is also assumed that the currents are sinusoidal. If this is not the case, there will be unbalanced harmonic electromotive forces in the galvanometer circuits. The detectors must therefore

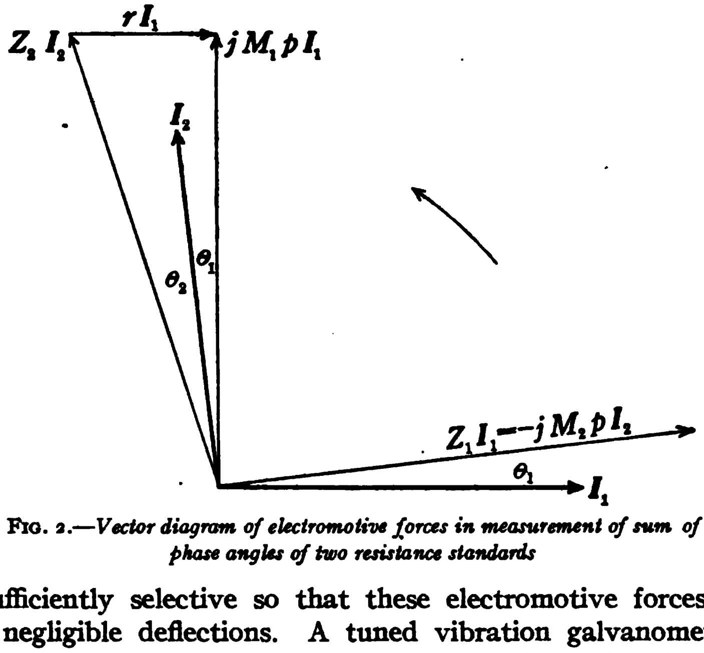


FIG. 2.—*Vector diagram of electromotive forces in measurement of sum of phase angles of two resistance standards*

be sufficiently selective so that these electromotive forces give only negligible deflections. A tuned vibration galvanometer is very selective. If it is not sufficiently so, it is possible by using an electrical resonating system in series with it to increase greatly the selectiveness of the arrangement. The use of such a system, however, necessarily introduces resistance into the galvanometer circuit, and thus reduces somewhat the sensitivity.

The sensitivity of the arrangement is easily calculated, since we know that for a lack of balance in phase of one minute there is 0.0003 R_1I_1 volt in circuit A and 0.0003 R_2I_2 volt in circuit B.

The deflections produced by these voltages can be calculated from the constants of the galvanometers and galvanometer circuits.

In order to obtain as large a voltage as possible for a given difference in phase, the mutual inductances should be of such magnitudes and of such construction as will allow both resistances to be used at full current capacity. The resistances of the secondaries of the mutual inductances should be kept low in order not to diminish unduly the deflection produced by this voltage.

Since the balance depends directly upon the frequency, it is necessary that the speed of the generators be kept very constant. For example, if the measurement is to be made to one minute of phase angle, the speed must be kept constant to about 0.03 per cent.

In making the measurement one observer who reads galvanometer A controls the ratio and phase difference of the two currents so as to keep his galvanometer undeflected, while the other observer reading galvanometer B varies M_1 and r (or m) so as to bring the deflection to zero. With a simultaneous balance of the two galvanometer circuits the relations stated above are satisfied.

2. SECOND METHOD

This method makes use of a standard whose resistance can be changed without changing its inductance. This can be done, for example, by using a copper standard whose temperature can be varied. The measurement consists in determining the change in phase angle or in time constant when the resistance is changed a known amount. Then, if the observed decrease in phase angle is $\Delta\theta$, or in time constant is ΔT, and the corresponding proportional increase in resistance be a, we have

$$\frac{pL}{R}=\frac{1+a}{a}\Delta\theta \tag{10}$$

and

$$\frac{L}{R}=\frac{1+a}{a}\Delta T \tag{11}$$

very closely. To show this, we make use of the known relations (See Fig. 3.)

$$\tan\theta=\frac{pL}{R}\text{ and }\tan(\theta-\Delta\theta)=\frac{pL}{(1+a)R} \tag{12}$$

Since the angles are small, these give

$$\Delta\theta = \frac{pL}{R} - \frac{pL}{(1+a)R} = \frac{pL}{R}\,\frac{a}{1+a} \tag{13}$$

This method requires that there be no change in form or in current distribution sufficient to cause an appreciable change in inductance. Systematic errors due to this cause can be eliminated by the use of well-constructed standards of different designs.

The change in phase angle can be measured by any of the methods used for measuring differences of phase angles referred to on page 13. The precision attainable by the use of this method for measuring the phase angle of a standard is determined by the precision of measurement of the change in phase angle and by the magnitude of the proportional change in resistance. Thus, if $a = 0.5$, which with a copper resistance corresponds to a change from 0 to 117° C, and it is desired to measure pL/R to 1 minute, it is necessary to measure $\Delta\theta$ to $\frac{0.5}{1+0.5}$, or about 0.3 minute.

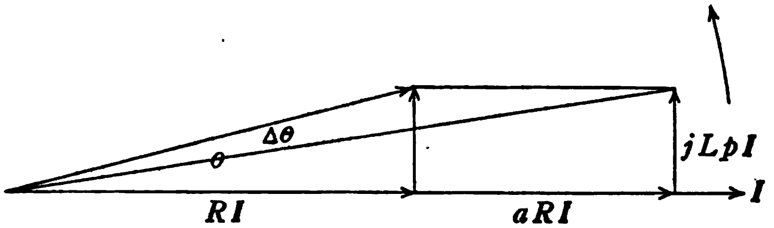


FIG. 3.—*Vector diagram showing change in phase angle with change in resistance*

III. EXPERIMENTAL WORK

1. STANDARDS

In order to test the practicability of these methods, measurements have been made on four low resistances. These were all made of flat-strip resistance material, and the current and potential leads were brought out in a definite manner. The values of the resistances and the current capacities in oil are given in Table 1. R and S were of manganin, K of kulmitz metal, and C of copper. The standards C and K were such that the inductance could be calculated from the dimensions with some certainty.

2. FIRST METHOD

All of the six possible sums of the phase angles of the four standards were measured by the first method. To do this, the apparatus was connected as shown in Fig. 4. R_1 and R_2 are the resistances to be measured; r_1, r_2, and r_3 are resistances for obtaining the adjustable low resistance r in Fig. 1. For example, with $r_1=0.001$ ohm, $r_2=0.1$ ohm, and $r_3=100$ ohms, $r=10^{-6}$ ohm. r_3 was variable so that r could be easily varied over a considerable range. The two circuits were excited through transformers from two generators directly connected to a motor. By manipulation of the resistance r_5 and r_6 and the field rheostats of the generators the currents were easily adjusted. The phase difference of the currents could be varied by rotating the stationary armature of one generator and by changing the resistance r_7. It was necessary to use the double transformation and ground connection as shown to avoid leakage from the high-voltage circuits through the measuring circuits and galvanometers to ground. The vibration galvanometers were of the moving coil type having a resistance of about 1.5 ohm, a voltage sensitivity of 0.16 mm per microvolt, and an amplitude time constant of 2 seconds. The frequency was 60 cycles per second. A resonating system consisting of an inductance of 0.14 henry in series with a capacity of 50 microfarads was used in series with each galvanometer. These were found to be very effective. The impedance at the fundamental frequency was about 1 ohm, while to the third harmonic it was about 140 ohms.

A Maxwell bridge with a charge and discharge commutator on the shaft of the generators was used to indicate changes in speed. The motor was run from a storage battery, so that the speed was very constant.

The procedure followed in making the measurements was practically as described above. In addition reversals of current and potential connections of the resistances and reversals of the primary and secondary connections of the mutual inductances were made in various combinations in order to eliminate errors due to stray alternating magnetic fields not considered in the theory above.

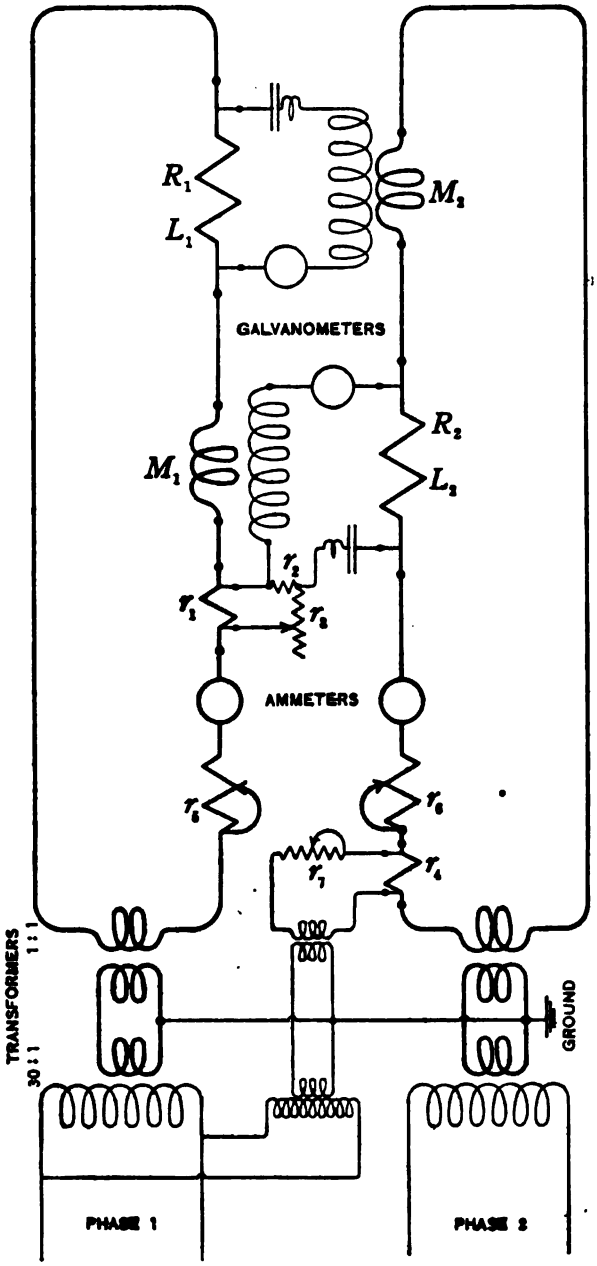


FIG. 4.—*Diagram of connections as used in measuring sum of phase angle of two resistance standards*

The phase angles at 60 cycles determined from the observations on the six sums are given in Table 1. These values satisfy all of the observations within two minutes.

TABLE 1

Designation	Resistance	Current capacity	Phase angle at 60 cycles	Time constant $\times 10^6$	Inductance
	ohm	amp.	min.	sec.	cm.
C	0.000337	300	29.0	22.4	7.6
K	0.01	75	2.5	1.9	19.0
R	0.001	200	19.0	14.7	14.7
S	0.001	500	16.5	12.7	12.7

The phase angles of *C* and *K* at 60 cycles calculated from dimensions are 24.1 minutes and 0.8 minutes, respectively. The values determined experimentally are therefore higher, showing that the ideal conditions assumed in the calculations were not fulfilled.

3. SECOND METHOD

Measurements were made by the second method on standard *C*. The standard was connected in the primary circuit of a current transformer and the apparent phase angle of the transformer measured by the usual method,[8] with the shunt at various temperatures from 20° C to 80° C. The phase angle at 60 cycles computed, assuming the change in the apparent phase angle of the transformer to be equal to the change in phase angle of the standard, was 30.5 minutes.

IV. CONCLUSIONS

The experimental work described in this paper was undertaken to determine the practicability of the methods suggested above, with a view to their use in a more complete investigation of the inductance of four-terminal conductors by one of the authors. It served also to show some of the precautions which must be taken in case the first method is used in the absolute measurement of resistance.

---

[8] Agnew and Silsbee, Trans. A. I. E. E., 31, p. 1635; 1912.

The first method is rather complicated and requires much special apparatus, but affords a valuable check on results obtained by other methods. In the work described here the precision was limited by fluctuations in the speed of the generators, but it is believed that the precision attained can be improved by further attention to this and other limiting factors. As has been pointed out above (p. 15), the results depend directly upon the magnitude and purity of the mutual inductances used, and these are to be investigated further in connection with the work on the inductance of four-terminal conductors.

The second method was found to be much simpler, and with the current-transformer method of measuring the change in phase angle and resistance is easily carried out. A precision of 0.1 minute in the measurement of the change in phase angle, at 60 cycles, should be attainable; so with an increase of 100 per cent in the resistance the phase angle could be measured to 0.2 minute. The accuracy would probably be slightly lower than this on account of systematic errors.

WASHINGTON, August 5, 1914.

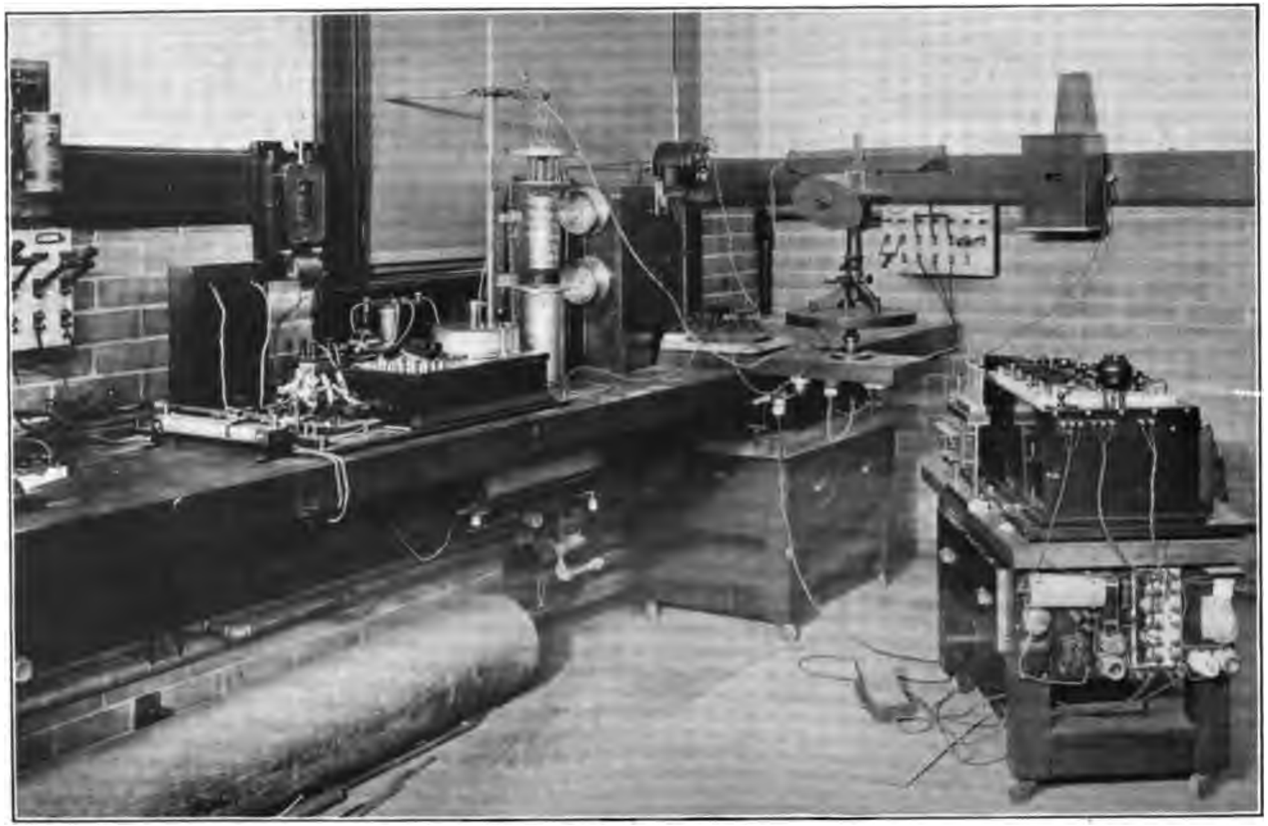


FIG. 1.—*General view of calorimeter and accessories*

AN ANEROID CALORIMETER

By H. C. Dickinson and N. S. Osborne

CONTENTS

Page
INTRODUCTION 23
  Characteristics 24
GENERAL PRINCIPLES OF DESIGN 25
  Equalization of temperature 25
  Control of jacket temperature 25
DETAILED DESCRIPTION 25
  Calorimeter 25
  Thermometer 27
  Heating element 27
  Thermocouples 27
  Jacket 29
  Lead wires 29
  Thermostat bath 29
    Cooling system 30
    Heating element 31
    Thermoregulator 31
CALIBRATION 32
  Thermometer calibration 32
  Heat-capacity determinations 38
  Calibration of thermocouples 39
  Sample heat-capacity determination 39
  Method of computation of results 41
  Results of heat-capacity determinations 42
SPECIFIC HEAT OF WATER 45
SUMMARY 47

INTRODUCTION

Several years ago the feasibility of making use of a calorimeter consisting entirely of metal, depending upon metallic conduction instead of the stirring of a liquid, was considered at the Bureau. The high thermal conductivity of copper seemed to offer a prospect of success, and with a view to testing its performance such a calorimeter was built at about the time that the work of Eucken [1] and of Nernst, Koref, and Lindemann,[2] utilizing the same principle, became known. This instrument was described by D. R. Harper 3d,[3] in a paper before the American Physical Society in April, 1913. Meanwhile this principle has also been applied by Féry.[4] The original calorimeter built at the Bureau, however, differed materially from these, in that both a thermometric element and a heating element were imbedded within the mass of the metal itself. This earlier calorimeter was in the form of a hollow cylinder of cast copper, 10 cm in diameter, 10 cm high outside, and having a wall thickness of 1 cm. The heating element and the resistance thermometer element were laid in grooves, insulated with mica, and covered by copper strips in the manner described later, except that the two coils were laid in the same deep groove with a copper strip between them, instead of in separate grooves as in the later instrument. Although the cast copper was quite impure, thus having a much lower thermal conductivity than pure copper, it became evident at once that the method was not only capable of considerable precision, but was applicable to a wide range of temperature and to a wide variety of problems which could hardly be undertaken with calorimeters of any other single type.

---

[1] Eucken, Physikalische Zeitschrift, 10, p. 586; 1909.

Characteristics.—Upon undertaking a number of calorimetric problems in connection with an investigation of refrigerants, it became necessary to adopt a calorimeter suitable for use under unusual conditions. The "aneroid calorimeter" combines the following characteristics: (1) It is applicable to both solids and liquids even under high pressure; (2) the same instrument, without any changes, can be used over a very wide range of temperature, and by suitably adapting the jacket and the liquid used therein, this range can be extended to the lowest temperatures obtainable and up to +200° C or higher; (3) the heat capacity of the calorimeter and contents can be measured over small temperature intervals, thus approximating the heat capacity at a definite temperature; (4) the troublesome correction due to evaporation of a calorimetric liquid and to the energy supplied by the stirring device are eliminated.

---

[2] Nernst, Koref, and Lindemann, Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften, 1910, p. 247.

[3] Harper, Physical Review (2), 1, p. 469; 1913.

[4] Féry, Comptes Rendus, 154, p. 691; 1912.

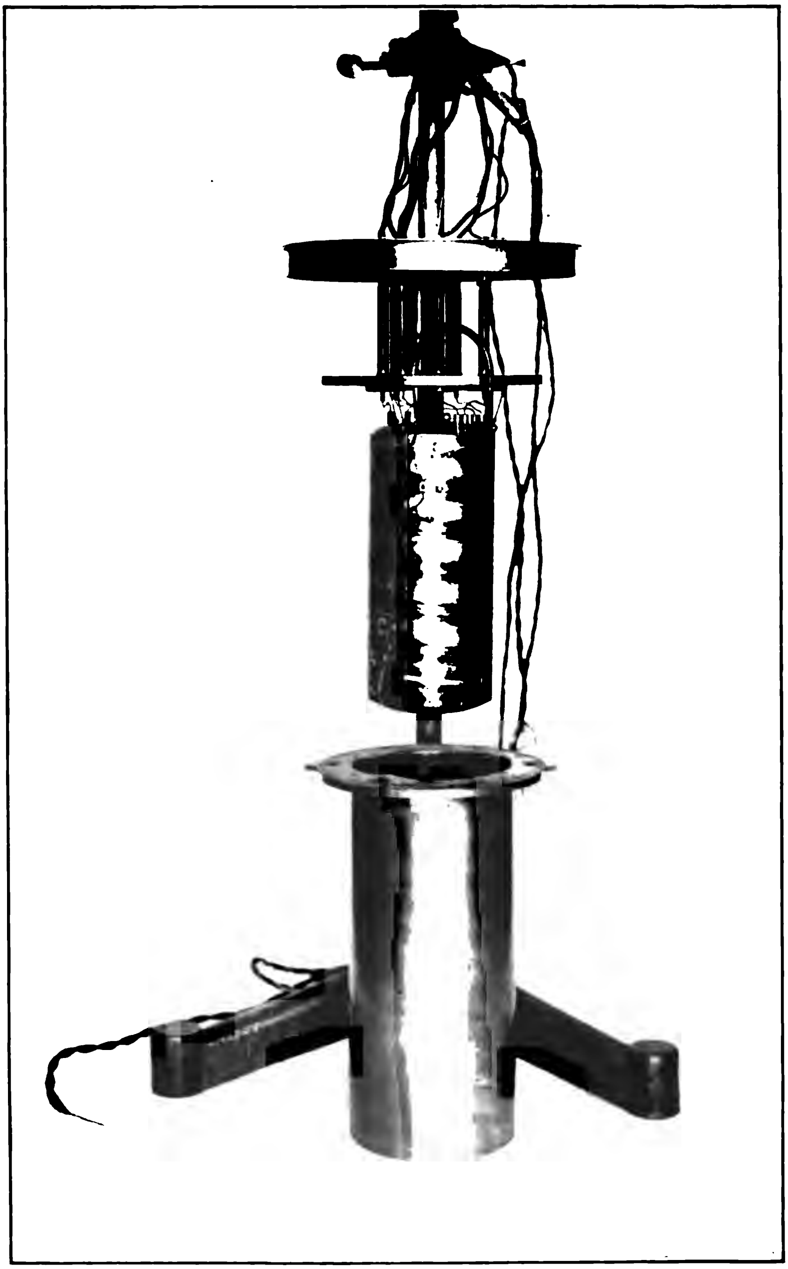


FIG. 2.—*Calorimeter and jacket*

GENERAL PRINCIPLES OF DESIGN

Equalization of Temperature.—The factor of prime importance in a calorimeter of this type is the rapid equalization of temperature over the surface of the calorimeter and throughout the entire mass of material. To provide for this, the material of the calorimeter should have as high thermal conductivity as possible, and the design should be such as to facilitate equalization of temperature.

A cylinder of copper of about 8 cm diameter, 18 cm length outside, and 5 mm wall thickness forms a highly conducting container within which can be inclosed either solids or liquids. Determination of the mean temperature of this cylinder is effected by measuring the electrical resistance of a platinum wire insulated with mica and wound at the bottom of a helical groove, which is then filled in with a copper strip. The energy required for heating is supplied electrically by means of a constantan wire similarly mounted in a second helical groove.

Control of Jacket Temperature.—The calorimeter is suspended by fine steel wires within a thin walled brass vessel or jacket which can be so closed as to permit total immersion in liquid. In order to control its temperature, the jacket containing the calorimeter is immersed in a stirred bath containing a carbon dioxide expansion cooling coil, an electric heating coil, and a coil of copper tubing forming the bulb of a thermostat. The stirrer is so placed as to give rapid circulation of the liquid, particularly around the heating and cooling coils and thermostat bulb.

The whole bath is surrounded by a thick layer of insulating material to adapt it for use at low temperatures.

DETAILED DESCRIPTION

Calorimeter.—The copper calorimeter, as shown in Figs. 2 and 3, was turned from a solid block of electrolytic copper. The helical grooves, four in number, each making about four turns around the cylinder, were milled with a concave bottom. The platinum wire and the flat constantan wire were wound in the bottom of these grooves and insulated with overlapping mica strips. A rectangular copper strip, concave on one side, was driven in place so as to compress the edges of the mica strips but leave the

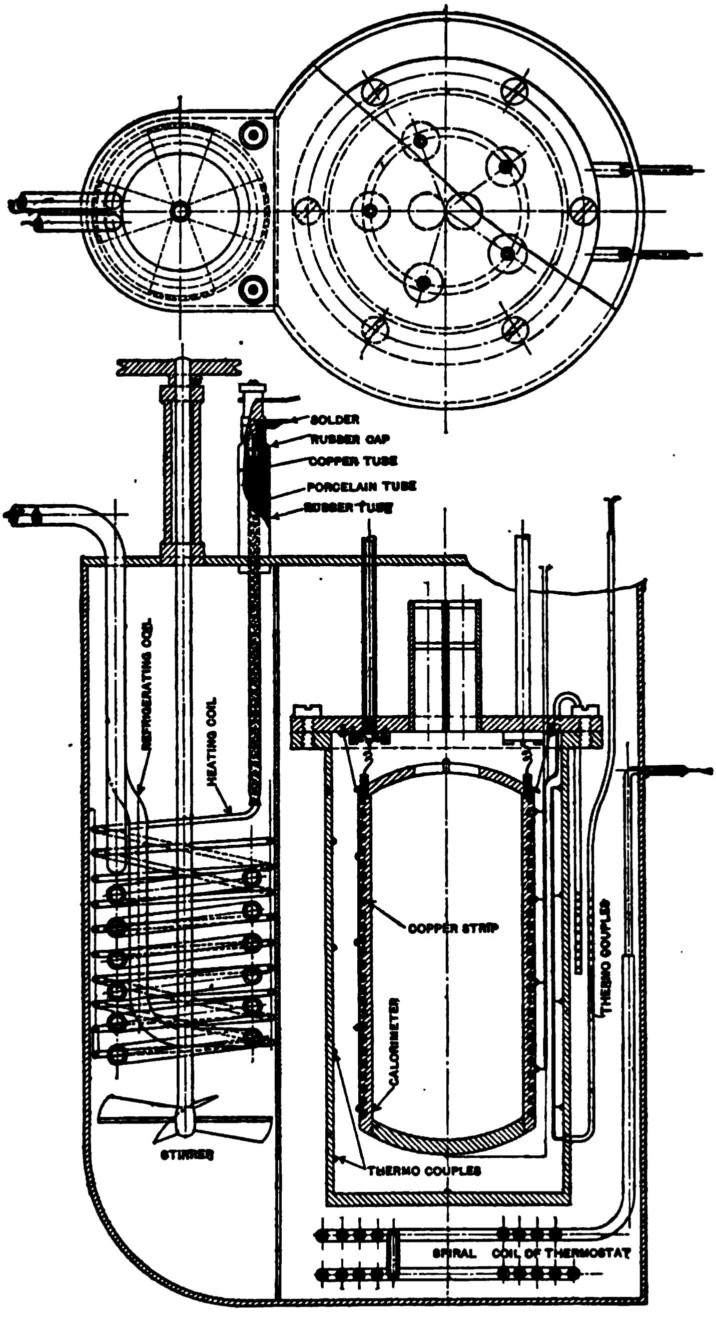


FIG. 3.—*Assembly drawing of calorimeter and jacket*

wire free, as shown in Fig. 4. Each of the two wires occupies two grooves, going down in one and back in the adjacent one, so that both the thermometer and the heating element are distributed quite uniformly through the entire cylindrical portion of the calorimeter. After the wires and copper strip were in place the latter was sweated in with pure tin and the surface turned true and nickel plated. Unfortunately, after this was done a defect developed in the thermometer coil, making it necessary to remove a part of this coil, thus removing a portion of the nickel plating.

Thermometer.—The resistance thermometer consists of a 0.1 mm platinum wire about 3 meters long, having a resistance at 0° C of 31.01 ohms. The method of winding, while avoiding direct pressure of the copper covering, does leave the wire subject to some irregularities of mechanical strain due to elasticity of the mica strips, as well as to differential thermal expansion. It was found that the resistance of the thermometer at a given temperature (e. g., ice point) changed considerably during the initial calibration but later became more nearly constant.

Heating Element.—The heating element consists of a constantan wire of the same length as that of the thermometer wire and having a resistance of about 9.2 ohms. This wire was rolled flat before winding and was wound in the same manner as the thermometer coil. The connections to both heater and thermometer were brought out from the calorimeter through short porcelain tubes within copper tubes.

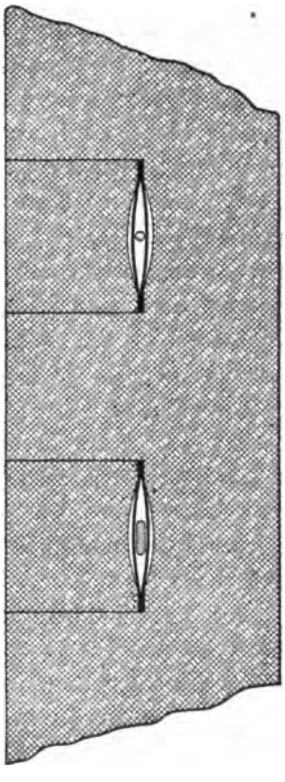


Fig. 4.—*Detail of thermometer and heating coil construction*

Thermocouples.—In order to make corrections for thermal leakage between calorimeter and jacket it is necessary to know the difference in temperature between the surface of the calorimeter and that of the jacket. A careful survey of the distribution of temperature on both of these surfaces, when the temperatures were changing, disclosed the fact that the distribution was not uniform over the surface of either calorimeter or jacket. In order conveniently to measure, at any given instant, the difference between the mean temperatures of the surfaces, two sets of copper constantan thermocouples, 10 in each set connected in series, were used. The junctions applied to the surfaces were soldered to thin copper plates each about 4 mm square, while the two sets of complementary junctions were imbedded in paraffin in copper tubes. These junctions were mounted as shown in Fig. 2 and diagrammatically in Fig. 3. Ten junctions of one set were distributed over the surface of the calorimeter and held in place by small phosphor bronze clips and insulated with mica, and the corresponding 10 junctions of the other set were similarly mounted on the inner surface of the jacket. The copper tubes containing the two complementary sets were attached one to the cover and the other to the side of the jacket and extended into the surrounding liquid, where they were clamped side by side in a split-copper bar. The calorimeter and the jacket cover can thus be removed together without disturbing any of the junctions, except the single one on the cover.

One set of thermocouples indicates the temperature difference between a point in the liquid and the surface of the calorimeter while the other set indicates the temperature difference between the same point in the liquid and the surface of the jacket. By suitably connecting the terminals of the two sets, since the junctions in the liquid are at the same temperature, the temperature difference between the calorimeter surface and the jacket surface can be measured. The two sets of couples can be read either independently or combined, direct or reversed. For the purpose of setting its zero the galvanometer can be connected to a coil free from thermoelectromotive forces and having the same resistance as the couples.

The sensibility of the arrangement could be varied in four steps by means of suitable series and shunt coils so chosen as to preserve critical damping of the galvanometer. All the operations of changing connections could be quickly performed by means of a switchboard, the arrangement of which is shown in Fig. 5.

Jacket.—The construction of the jacket of the calorimeter is shown in Figs. 2 and 3.

Electrical connections and inlet tubes, when required, are brought out through the cover from which the calorimeter is suspended so that when the body of the jacket is removed the calorimeter is accessible for adjustment or changing its contents.

The material to be used for a leak-proof gasket between the jacket and its cover depends upon the nature of the liquid used in the jacket. When gasoline is used, a satisfactory gasket is one made by impregnating cloth with a compound of gelatin and glycerin.

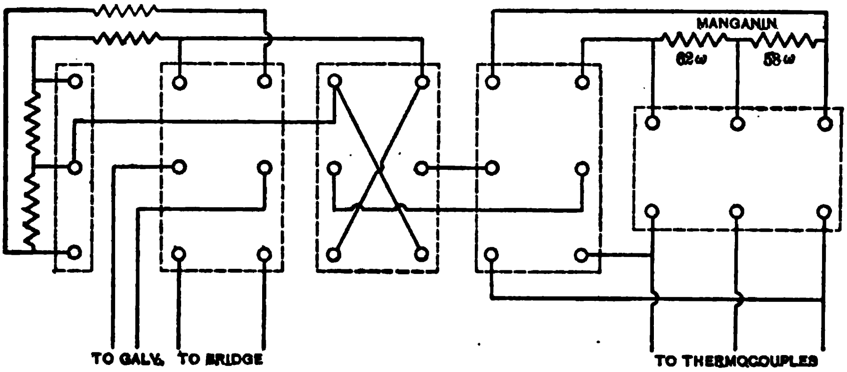


FIG. 5.—*Diagrammatic representation of circuits*

Lead Wires.—The method used for the insulation of the leads to the thermometer and the heating coil is shown in Fig. 3. The leads to the thermocouple were drawn through copper tubes which were then filled with wax to close the opening and exclude condensed moisture.

Thermostat Bath.—The essential details of construction of the thermostatically controlled bath are shown in Fig. 3. The brass vessel serving as container for the liquid is supported within a cork-filled box and is supplied with a cover of 2-inch cork board to minimize the transfer of heat between the liquid and the surroundings. The offset contains the stirrer, cooling coil, and heating coil.

Cooling system.—The cooling system consists of a copper expansion coil containing about 2 meters of tubing, 8 mm in diameter, and connected to a carbon dioxide compression system. It was found that great care was necessary in drying the carbon dioxide and freeing it from oil in order that it should not clog in the expansion coil or at the very small throttle valve which was used. A drier in the high-pressure line, containing fused calcium chloride, together with a filter of glass wool and an oil trap, served to eliminate moisture and oil from the liquid carbon dioxide. Just outside the thermostat bath the incoming and outgoing currents of carbon dioxide pass in opposite directions through a heat exchanger shown in Fig. 6. This exchanger consists simply of two copper pipes in the shape of adjacent helical spirals soldered together throughout their length and covered with cork board for thermal insulation. With the present arrangement it is possible to maintain constant to within a few thousandths of a degree any desired temperature between −55° and +40° C.

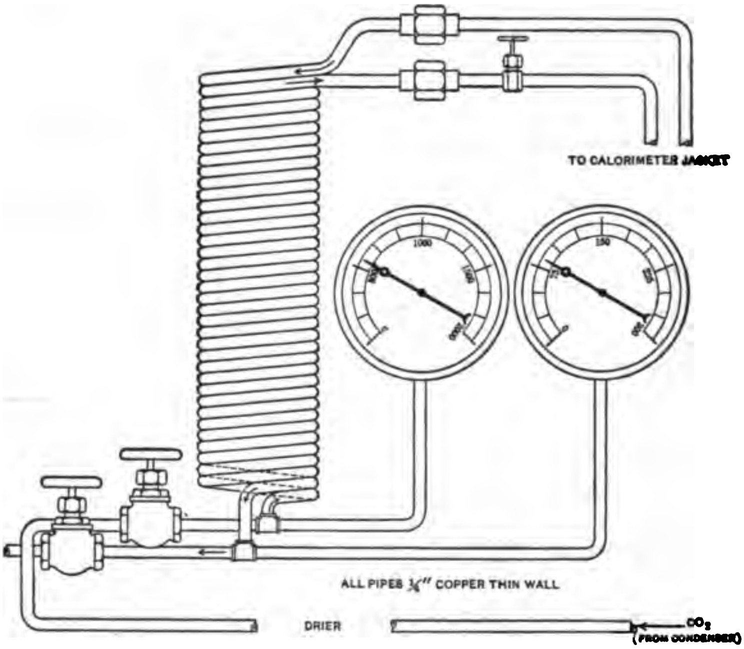


Fig. 6.—*Heat exchanger*

Heating Element.—The heating coil, shown in Fig. 3, consists of a length of asbestos covered "advance" resistance wire, which, for protection from the liquid, is inclosed in a copper tube of about 0.3 mm wall thickness. By alternate drawing, annealing, and stretching, a copper tube is reduced to the desired thickness and to as small a size as will readily permit introduction of the insulated wire. Then the wire, which has received several coats of thin shellac baked on over the asbestos covering, is threaded into the tube, which is then passed through a draw plate bringing it down into better contact with the insulated wire.

This type of coil has proven very satisfactory, as it has a small heat capacity and will dissipate a very large amount of energy without excessive heating, and is also easy of construction. Its use has greatly improved the behavior of the thermostat on account of the small amount of heat remaining in the coil when the heating current is cut off. The lead wires to the heating coil, as well as all other electric leads, are carefully covered with waterproof material (Ozokerite) for some distance from the calorimeter to prevent serious electric leakage through the moisture condensed on the cold portion of the wires.

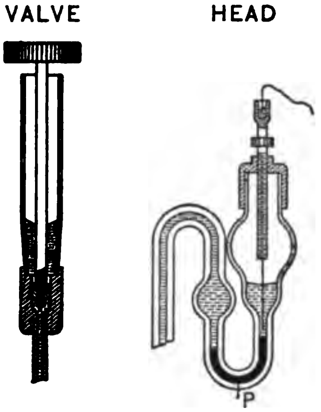


Fig. 7.—*Detail of thermoregulator*

Thermoregulator.—The temperature control is secured by means of a thermoregulator, consisting of a coil of about 2 meters of 6-mm copper tubing wound in a double spiral and placed near the bottom of the container. From each end of this copper tube a smaller copper tube extends up and out through the wall of the containing vessel, as shown in Fig. 3. One end opens through a small needle valve into a glass tube, and the other into a glass thermostat head, shown in Fig. 7. The thermoregulator is filled with toluol, and the bottom of the glass U tube with a small amount of mercury serving to make and break an electric contact operating, through a relay, a shunt in the main electric-heating circuit.

A regulator of this type is convenient for the present purpose, since whenever a desired temperature is reached the regulator may be set to maintain this temperature by opening and then closing the needle valve.

CALIBRATION

Thermometer Calibration.—The thermometer built into the copper calorimeter was found by preliminary calibration not to have the characteristics of the ordinary platinum resistance thermometer. Hence, a rather extended study of its behavior was necessary. The finished calorimeter was inclosed in a close-fitting copper container which could be immersed in steam, water, and other comparison media. A series of measurements of the resistance of the thermometer were then made in ice and steam, and a few in a mixture of solid carbon dioxide and gasoline. This latter test point was not satisfactory for the comparison of so bulky a thermometer because of the variations in temperature of the bath itself due to hydrostatic pressure. This difference was nearly 0°1 C between the top and bottom of the thermometer. The main object of this latter test was to determine the constancy of the ice and steam points of the thermometer after exposure to quite low temperatures.

The observations made in this way indicated that while the fundamental interval $(R_{100} - R_0)$ remained apparently constant to a few parts in 10 000, the calibration constants of the thermometer differed from those of the ordinary platinum resistance thermometer so greatly as to necessitate comparison with a standard platinum thermometer at shorter temperature intervals. The most convenient method for doing this seemed to be by using a standardized platinum thermometer immersed in the thermostat bath, comparing this with the calorimeter thermometer by means of the thermocouples when the calorimeter was mounted in the jacket as described above. This thermometer was held in contact with the face of the split-copper bar which incloses the thermocouple junctions, and for protection from sudden local fluctuations in the temperature of the liquid these were all inclosed in a thick walled copper tube surrounded by a glass tube. By adjusting the temperatures of jacket and calorimeter very nearly to equality and using the thermoelements to read the small difference, usually only one or two hundredths of a degree, comparisons could be made to an accuracy of about 0°001.

From the observations was obtained a series of resistance differences on the calorimeter thermometer corresponding to a series of temperature differences as determined by the standard resistance thermometer. Taking the ratio of each temperature difference to the corresponding difference in resistance, a series of difference factors varying slightly with temperature was obtained, which may be used to convert observed resistance differences into corresponding temperature differences.

A large number of calibrations were made in this way with somewhat disappointing results. A comparison of the results obtained at different times during several months disclosed disagreements of as much as 1 part in 2000 from the mean without apparent regularity. Although these results were sufficiently accurate for some of the uses to which the calorimeter was to be put, the irregularities were too great to be attributed to accidental experimental errors,[5] and finally a more systematic examination of the behavior of the thermometer disclosed a marked dependence upon the previous temperature to which the calorimeter had been subjected. If cooled to −40° or so, a consistent series of difference factors was found, while if cooled to only −15°, a different series of consistent factors was found. For still higher initial temperatures still different series were found. The results of this examination are given in Table 1,[6] and shown graphically in Fig. 8.

The peculiarity in the behavior of this thermometer can be explained on the ground of mechanical strain in the platinum wire. A method of construction which would obviate irregular strains would doubtless eliminate the peculiarity.

---

[5] The particular standard resistance thermometer used in most of the experimental work with this calorimeter is B. S. No. 4725. This thermometer is one of the four described in this Bulletin, 9, pp. 483–492 (B. S. Scientific Paper No. 200), and has the following constants, ice point resistance $(R_0) = 25.4750$, fundamental interval $(R_{100} - R_0) = 9.9650$, δ (as used in the Calendar equation) $= 1.48$.

[6] $K = \frac{\Delta\theta}{\Delta R}$ = the factor for a resistance thermometer by which an observed change in resistance must be multiplied to find the corresponding change in temperature. This factor varies with the temperature and is stated in such a way that, for the difference of any two observed resistances, the factor to be used corresponds to the mean of these resistances. The computation of this factor from the observed constants of platinum resistance thermometers is treated in a paper by Dickinson and Mueller, this Bulletin, 9, p. 483, 1913 (Scientific Paper No. 200). For determining actual temperatures by means of the built-in thermometer, the values of K given in the table were used, taking the resistance of this thermometer as 31.0135 ohms at the temperature of 0°.

TABLE 1

Calibration of Thermometer in Calorimeter No. 11510

Date	Resistance of platinum thermometer No. 4725	ΔR_{4725}	K_{4725}	$\Delta\theta$	Resistance of thermometer No. 11510	ΔR_{11510}	K_{11510}	Mean resistance of thermometer 11510
1914	*Ohms*	*Ohms*	*Deg/ohm*	*Degrees*	*Ohms*	*Ohms*	*Deg/ohm*	*Ohms*
Sept. 19...	21.84861				26.57352			
		1.07595	9.8017	10.546		1.31749	8.0046	27.23226
	22.92456				27.89101			
		.95097	9.8301	9.3481		1.16402	8.0309	28.47302
	23.87553				29.05503			
		.97617	9.8572	9.6223		1.19504	8.0519	29.65255
	24.85170				30.25007			
		1.06027	9.8860	10.482		1.29815	8.0746	30.89914
	25.91197				31.54822			
		.95501	9.9149	9.4688		1.16945	8.0968	32.13294
	26.86698				32.71767			
Oct. 1.....	21.39239				26.01606			
		1.07458	9.7890	10.519		1.31587	7.9939	26.67399
	22.46697				27.33193			
		1.07749	9.8190	10.580		1.31862	8.0235	27.99124
	23.54446				28.65055			
		1.06584	9.8492	10.498		1.30461	8.0468	29.30285
	24.61030				29.95516			
		1.05427	9.8791	10.415		1.29030	8.0718	30.60131
	25.66457				31.24546			
Oct. 2.....	24.18314				29.42907			
		.99283	9.8660	9.7953		1.21694	8.0491	30.03754
	25.17597				30.64601			
		1.06658	9.8954	10.554		1.30695	8.0753	31.29948
	26.24256				31.95296			
		1.05698	9.9260	10.492		1.29461	8.1044	32.60026
	27.29954				33.24757			
		1.04696	9.9565	10.424		1.28190	8.1317	33.88852
	28.34650				34.52947			
Oct. 3.....	28.53673				34.76197			
		.99532	9.9629	9.9163		1.21886	8.1357	34.15254
	27.54141				33.54311			
		1.00349	9.9338	9.9685		1.22861	8.1136	32.92881
	26.53792				32.31451			
		2.03494	9.8899	20.125		2.49103	8.0790	31.06899
	24.50298				29.82348			
		1.03282	9.8467	10.170		1.26428	8.0441	29.19134
	23.47016				28.55920			
		1.04418	9.8175	10.251		1.27794	8.0215	27.92023
	22.42598				27.28126			
		1.05479	9.7607	10.324		1.29103	7.9967	26.63539
	21.37093				25.98987			

TABLE 1—Continued.

Calibration of Thermometer in Calorimeter No. 11510—Continued

Date	Resistance of platinum thermometer No. 4725	ΔR_{4725}	K_{4725}	$\Delta\theta$	Resistance of thermometer No. 11510	ΔR_{11510}	K_{11510}	Mean resistance of thermometer 11510
1914	*Ohms*	*Ohms*	*Deg/ohm*	*Degrees*	*Ohms*	*Ohms*	*Deg/ohm*	*Ohms*
Oct. 6.....	23.53755				28.63794			
		1.03043	9.8485	10.1482		1.26267	8.0371	29.26927
	24.56798				29.90061			
		1.03623	9.8777	10.2356		1.26937	8.0635	30.53529
	25.60421				31.16998			
		1.00528	9.9068	9.9591		1.23148	8.0871	31.78572
	26.60949				32.40146			
		.99493	9.9357	9.8853		1.21868	8.1115	33.01080
	27.60442				33.62014			
Oct. 12....	21.89236				26.62643			
		1.04517	9.8025	10.2453		1.27986	8.0050	27.26636
	22.93753				27.90629			
		1.03177	9.8316	10.1439		1.26314	8.0307	28.53786
	23.96930				29.16943			
		1.01922	9.8604	10.0568		1.24866	8.0541	29.79376
	24.98922				30.41809			
		1.00879	9.8891	9.9760		1.23525	8.0761	31.03573
	25.99801				31.65334			
		.99869	9.9181	9.9051		1.22273	8.1008	32.26470
	26.99670				32.87607			
		.98886	9.9469	9.8361		1.21104	8.1220	33.48159
	27.98556				34.08711			
		.98305	9.9756	9.8065		1.20403	8.1447	34.68913
	28.96861				35.29114			
Oct. 13....	20.74645				25.22703			
		1.06179	9.7710	10.3747		1.29953	7.9834	25.87679
	21.80824				26.52656			
		1.04837	9.8002	10.2742		1.28277	8.0094	27.16794
	22.85661				27.80933			
		1.03676	9.8294	10.1907		1.26887	8.0313	28.44377
	23.89337				29.07820			
		1.02604	9.8584	10.1151		1.25572	8.0552	29.70606
	24.91941				30.33392			
		1.01559	9.8873	10.0414		1.24316	8.0773	30.95550
	25.93500				31.57708			
		1.00579	9.9163	9.9737		1.23126	8.1004	32.19271
	26.94079				32.80834			
		.99674	9.9453	9.9129		1.22046	8.1223	33.41857
	27.93753				34.02880			

TABLE 1—Continued

Calibration of Thermometer in Calorimeter No. 11510—Continued

Date	Resistance of platinum thermometer No. 4725	ΔR_{4725}	K_{4725}	$\Delta\theta$	Resistance of thermometer No. 11510	ΔR_{11510}	K_{11510}	Mean resistance of thermometer 11510
1914	*Ohms*	*Ohms*	*Deg/ohm*	*Degrees*	*Ohms*	*Ohms*	*Deg/ohm*	*Ohms*
Oct. 15....	25.69525				31.27997			
		0.50958	9.9023	5.0460		0.62477	8.0766	31.59435
	26.20483				31.90474			
		.50843	9.9169	5.0420		.62306	8.0923	32.21627
	26.71326				32.52780			
		.50731	9.9316	5.0384		.62174	8.1037	32.83867
	27.22057				33.14954			
		.50610	9.9463	5.0338		.62003	8.1186	33.45956
	27.72667				33.76957			
		.50536	9.9611	5.0339		.61886	8.1341	34.07900
	28.23203				34.38843			
		.50440	9.9757	5.0317		.61774	8.1453	34.69680
	28.73643				35.00617			
Oct. 17....	25.76816				31.36791			
		.50757	9.9042	5.0271		.62274	8.0726	31.67928
	26.27573				31.99065			
		.50708	9.9189	5.0297		.62159	8.0917	32.30145
	26.78281				32.61224			
	26.78275				32.61211			
		.50652	9.9336	5.0316		.62070	8.1063	32.92246
	27.28927				33.23281			
		.50538	9.9483	5.0277		.61942	8.1168	33.54252
	27.79465				33.85223			
		.50434	9.9631	5.0248		.61803	8.1303	34.16125
	28.29899				34.47026			
		.50331	9.9776	5.0218		.61668	8.1433	34.77860
	28.80230				35.08694			
	29.05588				35.39775			
		.50106	9.9998	5.0105		.61347	8.1675	35.70449
	29.55694				36.01122			

It was found that with suitable regulation of the jacket temperature, calorimetric determinations could be very conveniently made, dispensing with the thermometer in the calorimeter shell, and referring calorimeter temperatures to the standard platinum thermometer in the outside-stirred bath by means of the thermocouples, as described above. This method of observation has therefore been followed in much of the later work.

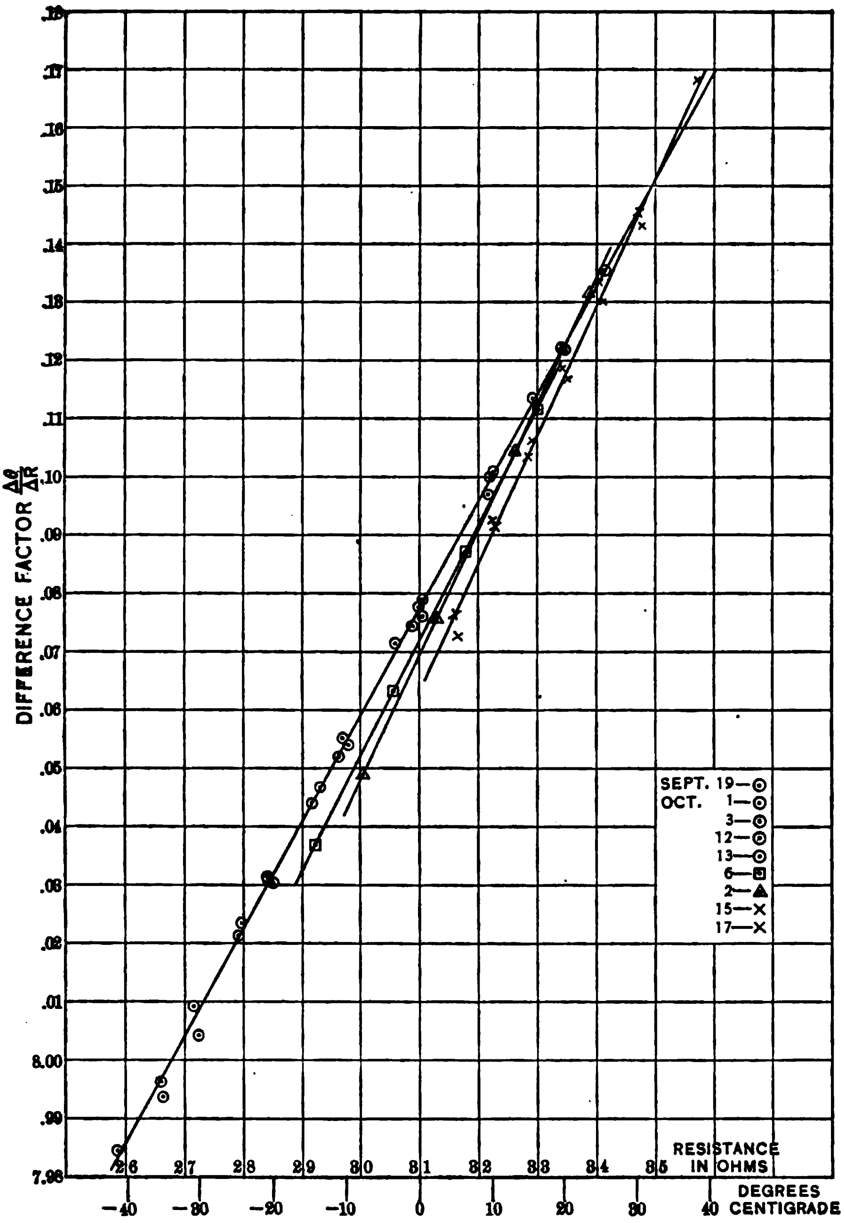


Fig. 8.—*Calibration of resistance thermometer in calorimeter*

Heat-Capacity Determinations.—With the calorimeter empty or containing a specimen, in place in the jacket, the latter is cooled by means of the refrigerating coil to the lowest temperature attainable in the apparatus (about −55°C) and is kept at this temperature until the calorimeter is cooled to the desired temperature for the first experiment of the day. By means of the heating coil the temperature of the jacket is then brought to equality with that of the calorimeter. The setting valve of the thermoregulator is opened to permit the mercury column to assume a position of equilibrium at this temperature, after which the valve is closed. The regulating current is set to the proper amount, and the thermoregulator begins to operate. Final adjustment of the temperature is made by the slow-motion screw controlling the position of the upper contact.

Before beginning the determination sufficient time is allowed for the calorimeter to assume thermal equilibrium. This time rarely exceeds 15 minutes, depending somewhat upon the contents. Meanwhile the current to be used in the calorimeter heating coil is set to the proper value in an auxiliary coil so connected to a quick-throw switch that the current may be instantaneously thrown from the auxiliary coil to the calorimeter. The potentiometer used to measure the current and potential drop in the calorimeter heater is balanced against the standard cell. The initial temperature of the calorimeter is observed by measuring the resistance of the platinum thermometer with the Wheatstone bridge. By means of the special switchboard the galvanometer connections are changed from the bridge to the thermocouple and the zero adjusted to a chosen point on the scale.

After the temperature observation the observer begins to take thermocouple readings at intervals of 30 seconds. At the beginning of the next minute the current is thrown on the calorimeter heater by a second observer, the instant of time being recorded automatically on a chronograph. The current in the jacket heater is increased to the proper amount, and thereafter from time to time adjusted by the observer so as to keep the temperature of the jacket as nearly as practicable equal to that of the calorimeter, as indicated by the thermocouple reading. Potentiometer readings of current and of potential drop in the calorimeter heating coil are taken alternately at equal intervals of time throughout the experiment. At the end of a predetermined time interval the current is thrown off and the time again automatically recorded. The jacket is brought under control of the thermostat at the temperature of the calorimeter and after sufficient time has elapsed for the calorimeter to assume equilibrium, its final temperature is observed. There are thus obtained the following data:

(*a*) The initial and final resistance of the calorimeter thermometer, giving its temperature change.

(*b*) The series of readings of current and of potential drop in the calorimeter coil during the experiment, giving the rate of energy supplied electrically.

(*c*) A thermocouple record of temperature difference between calorimeter surface and the surrounding jacket surface, from which is computed the correction for thermal leakage between calorimeter and its surroundings.

(*d*) A record of the duration of the energy supply to the calorimeter.

Calibration of Thermocouples.—A supplementary experiment is made to determine the relation between the rate of thermal leakage and the difference in temperature between the surfaces of calorimeter and jacket. This is done only at occasional intervals. To make this determination the temperature of the jacket is altered from that of the calorimeter and at definite intervals of time, alternate readings are taken of the resistance thermometer in the calorimeter, and of the thermocouple between calorimeter and jacket. An approximate knowledge of the heat capacity of the calorimeter permits the computation, from this data, of the rate of thermal leakage in terms of the thermocouple readings.

Sample Heat-Capacity Determination.—An example of the record of a single experiment in determining the heat capacity of the calorimeter is given in Fig. 9. In columns 4 and 5 are recorded the readings of the thermocouple, and of the thermometer resistances corresponding to times recorded in column 3. At the left of the page are recorded the potentiometer readings for determining the potential drop and the current in the calorimeter heating coil. The observations for the duration of the calorimeter heating current are recorded under that heading.

CALORIMETER RECORD

Observers N. S. O. and M. S. V. D. *Date Oct. 13, 1914, Experiment No. 7*

Time	Potentiometer readings across volt box
m s	
14 30	0.126051
16 30	57
18 30	65
20 30	74
22 30	81
Mean	0.126065
Corr. for potentiometer	0
Corr. for volt box	+4
Corrected reading	0.126069
Potential drop (E)=	**12.6069 volts**

Time	Potentiometer readings across std. resistance
m s	
15 30	0.138886
17 30	75
19 30	64
21 30	56
23 30	45
Mean	0.138864
Corr. for potentiometer	0
Corr. for std. resistance	−106
Corr. for volt box current	−126
Corrected reading	0.138632
Current (I)=	**1.88632 amperes**

Duration of calorimeter current

	m	s
On	14	0.40
Off	24	0.39

Total time (T)=599.99 seconds

Power	17.4772 watts
Total energy	10486.1 joules
Corr. for thermal leakage	−4.6 joules
Corrected energy	**10481.5 joules**

	Initial	Final
Obs. R.	32.80980	34.03025
Bridge corr.	−144	−144
Corr. R_1	32.80836	R_2=34.02881
ΔR	1.22045	R_m 33.42
K_m	8.1224	R_1-R_0 1.795
$\Delta\theta$	**9°.9130**	K 8.0948
		θ_1 14.5
Heat capacity	**1057.3 J/deg.**	θ_m 19.5

Time m	Time s	Thermocouple readings	Resistance thermometer readings	Notes
m	s			
13			32.80980	
	30	200.0		Bridge B. S. no. 7481 at 30° C
14		190.7		
	30	194.0		Bridge calibration of Aug. 29, 1914
15		198.0		
	30	201.3		Ratio 100 R
16		201.7		Thermometer no. 11510
	30	202.6		
17		203.5		Calorimeter no. 11510 with empty water container
	30	203.2		
18		202.2		
	30	200.0		
19		199.4		
	30	199.8		
20		200.2		
	30	200.7		
21		201.0		Thermocouple zero=200
	30	201.5		
22		201.7		
	30	201.3		When jacket is colder than calorimeter the thermocouple reading is above 200
23		200.8		
	30	200.3		
24		207.5		
	30	206.5		
25		207.5		
	30	206.3		Rate of thermal leakage to calorimeter= −0.2 joules per second per millimeter deflection above 200.
26		204.0		
	30	203.3		
27		203.3		
	30	202.0		
28		201.0		
	30	200.7		
29		200.0		
	30	200.2		Correction for thermal leakage to calorimeter $=-0.2\times1.1\times21=4.6$ J
30		200.0		
	30	200.0		
31		200.0		
	30	200.0		
32		200.0		
	30	200.0		
33		200.0		
	30	200.0		
34	00		34.03025	
37	00		34.03022	
Mean		201.1		

FIG. 9.—*Sample heat-capacity determination*

The initial and final readings of thermometer resistance corrected for bridge errors give the initial and final thermometer resistances R_1 and R_2. The difference between the initial and final resistances, ΔR, multiplied by the difference factor, K_m, gives the change in temperature of the calorimeter, $\Delta\theta$. In obtaining K_m from the chart, Fig. 8, the mean resistance R_m is used.

The average thermocouple deflection multiplied by the rate and by the time between initial and final thermometer readings gives the correction for thermal leakage.

The mean potentiometer readings for potential drop and for current, corrected for instrumental errors, and multiplied by the proper reduction factors, give the potential drop, E, and current, I. The product of the current and potential drop is the power, and this multiplied by the time is the total energy electrically supplied to the calorimeter. This total energy corrected for thermal leakage gives the corrected energy, i. e., the amount of energy received by the calorimeter and contents during the experiment.

The corrected energy divided by the change in temperature, $\Delta\theta$, gives the heat capacity in joules per degree.

To obtain the mean temperature the initial temperature, θ_1, is first computed from R_1, using R_0 for the thermometer resistance at 0° and the appropriate value of K. The initial temperature plus one-half the temperature change gives the mean temperature θ_m.

Method of Computation of Results.—The method of calculation may be summarized as follows:

Supposing the observed data to be corrected for all instrumental errors, the heat capacity of the calorimeter is given by the formula

$$C = \frac{IET + Btd}{\Delta R K_m}$$

where

C = mean heat capacity of calorimeter in joules per degree,

I = current in amperes (mean value),

E = potential drop in volts (mean value),

T = duration of energy supply to calorimeter in seconds,

d = mean thermocouple deflection in millimeters during experiment,

t = time in minutes between initial and final readings of calorimeter resistance thermometer,

B = rate of thermal leakage to calorimeter from surroundings in joules per minute per mm thermocouple deflection,

R_1 = initial resistance of thermometer in ohms,

R_2 = final resistance of thermometer in ohms,

ΔR = difference between the initial and final resistances of thermometer in ohms,

K_m = difference factor for the resistance thermometer, i. e.,

$\frac{\Delta\theta}{\Delta R}$ in degrees per ohm.

The mean temperature of the determination is given by the formula

$$\theta_m = (R_1 - R_0)\,K + \frac{\Delta\theta}{2}$$

where

θ_m = mean temperature,

K = difference factor corresponding to the mean resistance, $\frac{1}{2}(R_1 + R_0)$.

R_0 = resistance of thermometer at 0°C

The current and the potential drop were always so nearly constant that the approximation in taking the product of their mean values multiplied by the time as the total energy is well within the limit of allowable error.

The values of the factor K were taken from the results of calibrations summarized in Table 1, page 34, and plotted in Fig. 8, the difference factor being taken from the proper curve at the point corresponding to the mean thermometer resistance for the observed interval.

Since the performance of the thermometer depended upon its previous thermal treatment, in the use of the chart the appropriate calibration line was chosen to correspond with the initial temperature of the experiments on the particular day.

Results of Heat-Capacity Determinations.—The results of the determinations of the heat capacity of the calorimeter, including the container used for the ice and water samples, are given in

Table 2 and shown graphically in Fig. 10. Two distinct curves are shown, of which the points on the lower curve are more consistent. The upper curve represents the results obtained before the addition of the second set of couples (p. 28), which indicate the difference between the mean temperature of the inner surface of the jacket and that of the point in the liquid at which the other set of junctions is placed. Subsequent experiments made after the second set of couples had been arranged upon the interior surface of the jacket showed that in the earlier arrangement the true difference of temperature between the surfaces of the calorimeter and of the jacket had not been indicated, owing to the effect of lag in the liquid and jacket.

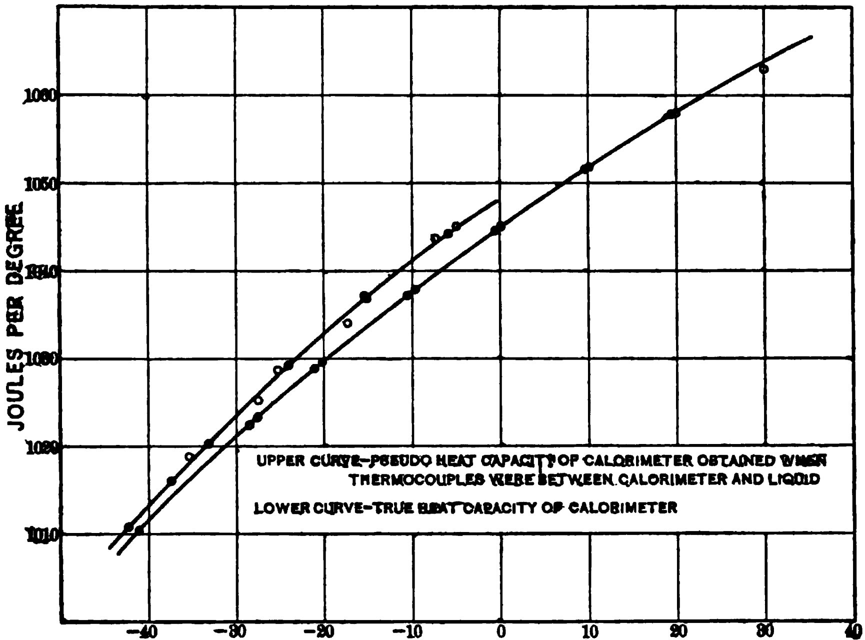


FIG. 10.—*Results of heat capacity determinations*

Unfortunately, some determinations of specific heat had been made when the significance of this effect became apparent, but

TABLE 2

Résumé of Determinations of Heat Capacity of Calorimeter No. 11510

THERMOCOUPLES—CALORIMETER TO LIQUID

Date	Exp.	Total energy	Correction for thermal leakage	Corrected total energy	Temperature difference	Mean heat capacity	Mean temperature
1914		*Joules*[7]	*Joules*	*Joules*	*Deg C*	*Joules/deg C*	*Deg C*
May 20	1	9467.3	+10.2	9477.5	9.3754	1010.9	−42.30
	2	9469.1	− 2.2	9466.9	9.2776	1020.4	−32.98
	3	9463.2	− 5.8	9457.4	9.1883	1029.3	−23.74
	4	9466.1	− 3.8	9462.3	9.1227	1037.2	−14.58
	5	9465.8	− 3.4	9462.4	9.0619	1044.2	− 5.48
	6	9479.7	− 0.9	9478.8	9.0334	1049.3	+ 3.57
July 1	1	10556.3	− 7.5	10548.8	10.3509	1019.1	−35.16
	2	10555.3	− 6.8	10548.5	10.2514	1028.9	−24.86
	3	10555.0	− 4.0	10551.0	10.1731	1037.0	−14.65
	4	10552.9	− 3.1	10549.8	10.0938	1045.2	− 4.50
July 7	1	10480.9	+ 1.5	10482.4	10.3164	1016.1	−37.36
	2	10483.6	− 5.2	10478.4	10.2195	1025.4	−27.08
	3	10485.3	− 2.1	10483.2	10.1365	1034.2	−16.91
	4	10487.6	− 1.5	10486.1	10.0473	1043.7	− 6.81

THERMOCOUPLES—CALORIMETER TO JACKET

Date	Exp.	Total energy	Correction for thermal leakage	Corrected total energy	Temperature difference	Mean heat capacity	Mean temperature
Oct. 12	1	10462.1	0.0	10462.1	10.2453	1021.2	−30.14
	2	10445.9	− 0.5	10445.4	10.1439	1029.7	−19.94
	3	10435.3	− 1.4	10433.9	10.0568	1037.5	− 9.84
	4	10425.3	− 1.0	10424.3	9.9760	1044.9	+ 0.18
	5	10416.5	− 3.4	10413.1	9.9051	1051.3	+10.12
	6	10405.0	− 2.5	10402.5	9.8361	1057.6	+19.99
	7	10423.2	− 1.8	10421.4	9.8065	1062.7	+29.80
Oct. 13	1	10483.0	+ 0.8	10483.8	10.3747	1010.5	−41.25
	2	10483.4	− 1.3	10482.1	10.2742	1020.2	−30.93
	3	10488.1	− 1.8	10486.3	10.1907	1029.0	−20.70
	4	10489.4	− 2.5	10486.9	10.1151	1036.8	−10.55
	5	10487.2	− 0.4	10486.8	10.0414	1044.4	− 0.47
	6	10488.0	− 5.6	10482.4	9.9737	1051.0	+ 9.54
	7	10486.1	− 4.6	10481.5	9.9129	1057.4	+19.48

all of this earlier work could be satisfactorily computed as explained below. The value of the observed total heat capacity of the empty calorimeter and of the calorimeter containing a specimen would be affected to the same extent by the improper placing of the thermocouple, provided that, in the two experiments, the manipulation of the jacket was similar. If, therefore, in computing the result of a specific heat determination, a value of heat capacity of the calorimeter be used, determined under experimental conditions similar to those in the specific heat determination, no error is introduced into the resulting value of specific heat. It was necessary, therefore, to employ the false values of heat capacity shown by the upper curve of Fig. 10 in computing the earlier results.

---

[7] The joule used in this paper is defined by the relation $\frac{e^2}{R}$, where R is in international ohms and e is in international volts, determined by the relation emf of mean Weston normal cell at 20° C equals 1.01830 volts: the international ohm is the resistance offered to an unvarying current by a column of pure mercury at 0° C of length 106.300 cm and uniform cross section such that the mass is 14.4521 grams.

It was ascertained that the variations in manipulation which did occur, such as the use of different amounts of refrigeration and compensating heating, etc., were not sufficient to cause any large systematic error.

SPECIFIC HEAT OF WATER

In Table 3 are given the results of a series of experiments on the specific heat of water in the range 0° to 40° C made with a view to testing the performance of this calorimeter. Fig. 11 shows the same results graphically. These determinations were not undertaken with the idea of establishing authoritative values of this important constant. A great amount of experimental data on this constant, obtained by this and other methods, has been accumulated in the course of the calorimetric investigations carried out at the Bureau during the past three years, but in view of the discrepancies between the most carefully conducted researches published to date it seems that further publications are hardly warranted until the investigations can be regarded as exhaustive with the experimental methods at command.

The experiments were made in connection with observations on the specific heat and latent heat of ice in the same calorimeter,[8] so that the specific heat of water at temperatures near the freezing point, where the published results are very discordant, was of some immediate interest. Observations were made, except in one instance, using the same samples of water which had previously been experimented on as ice.

---

[8] Dickinson and Osborne, this Bulletin, 12, p. 49; 1915.

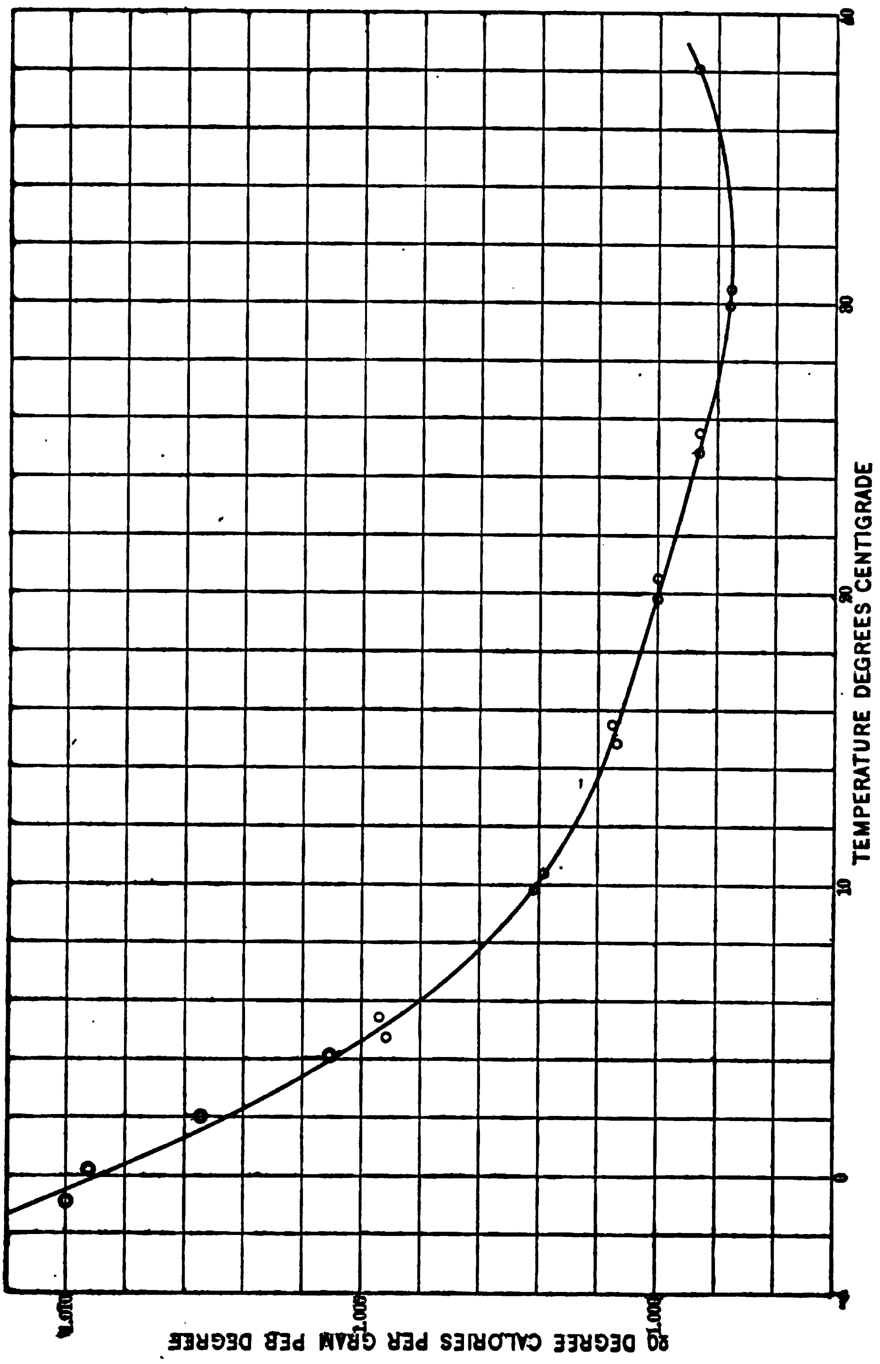


FIG. 11.—*Results of determinations of specific heat of water*

TABLE 3

Specific Heat of Water

Date	Exp.	Energy	Corr.	Corrected energy	Temp. diff.	Heat capacity cal. and contents	Heat capacity calorimeter	Heat capacity contents	$\frac{J}{g \cdot deg.}$	Specific heat	Mean temperature
1914		*Joules*		*Joules*	*Deg. C*	*J/deg.*	*J/deg.*	*J/deg.*		*Calories* 20°	°*C*
Oct. 15	1	16733.6	+30.2	16763.8	5.0460	3322.2	1047.9	2274.3	4.2020	1.0046	+4.70
	2	16729.1	+ 9.7	16738.8	5.0420	3319.9	1051.2	2268.7	4.1916	1.0021	9.74
	3	16719.8	+ 6.5	16726.3	5.0384	3319.8	1054.3	2265.5	4.1857	1.0007	14.78
	4	16716.2	+ 2.2	16718.4	5.0338	3321.2	1057.3	2263.9	4.1827	1.0000	19.82
	5	16714.5	+10.4	16724.9	5.0339	3322.5	1060.3	2262.2	4.1796	.9993	24.78
	6	16713.7	+11.4	16725.1	5.0317	3323.9	1062.8	2261.1	4.1776	.9988	29.90
Oct. 17	1	16701.8	+ 2.8	16704.6	5.0271	3322.9	1048.3	2274.6	4.2025	1.0047	+5.40
	2	16698.9	− 1.0	16697.9	5.0297	3319.9	1051.7	2268.2	4.1907	1.0019	10.45
	3	16702.1	+ 3.6	16705.7	5.0316	3320.1	1054.3	2265.8	4.1862	1.0008	15.47
	4	16698.5	+ 2.6	16701.1	5.0277	3321.8	1057.8	2264.0	4.1829	1.0000	20.51
	5	16699.4	− 2.2	16697.2	5.0248	3323.0	1060.7	2262.3	4.1798	.9993	25.53
	6	16696.0	− 3.2	16692.8	5.0218	3324.1	1063.2	2260.9	4.1776	.9988	30.54
	7	16683.7	− 0.5	16683.2	5.0105	3329.6	1067.1	2262.5	4.1801	.9993$_5$	38.10
Sept. 9	1	15040.2	+ 7.2	15047.4	5.0318	2990.5	1044.1	1946.4	4.2248	1.0100	−0.925
Sept. 8	1	14675.1	+17.0	14692.0	4.9130	2990.5	1044.9	1945.6	4.2229	1.0096	+0.237
Aug. 28	1	30146.3	+34.1	30180.4	10.100	2988.1	1046.1	1942.0	4.2153	1.0078	+2.026
Sept. 9	2	15012.0	+ 9.6	15021.6	5.0316	2985.3	1047.5	1937.8	4.2062	1.0056	+4.110

A comparison of the results given above with the unpublished results of a long series of experiments, made at the Bureau, using a stirred water calorimeter in the temperature range from 10° to 40°, shows an agreement at all points within about 1 part in 2000. The results of October 15 and 17 (Table 3) in the interval 5° to 38° were obtained using the standard platinum resistance thermometer. Those of August 28 to September 9 were obtained using only the thermometer in the walls of the calorimeter. The latter results appear somewhat less consistent than the former, yet serve adequately to extend the curve beyond the freezing point and to indicate that no peculiarity exists in the specific heat of water at zero degrees.

SUMMARY

A calorimeter consisting of a cylindrical copper vessel in the walls of which are embedded a coil of resistance wire to supply heat electrically, and a platinum resistance coil for use as a thermometer, has been found useful over a wide range of temperatures and is applicable to a variety of problems.

For use at low temperatures the calorimeter is mounted in a jacket surrounded by a bath of gasoline the temperature of which can be controlled thermostatically to within a few thousandths of a degree at any temperature between −55° and +40° C, or can be changed rapidly in order to keep it the same as that of the calorimeter when heat is being supplied to the latter.

Differences in temperature between the surface of the calorimeter and that of the jacket are measured by means of multiple thermocouples which have 10 junctions distributed over the surface of each.

The platinum resistance coil embedded in the calorimeter shows slight irregularities in its behavior, probably due to the difference in expansion between the platinum and the copper which surrounds it. Uncertainties on this account, while in general negligible, can be avoided by measuring the temperature of the outer bath with a standard resistance thermometer, using the thermocouples to measure the small difference, usually not more than a few thousandths of a degree, between the calorimeter and the jacket. The thermometer could probably be improved by changing the construction.

Results of a series of experiments give the constants of the resistance thermometer and the heat capacity of the calorimeter, including a tin-lined cell for use in determining the specific heat of ice and water and the latent heat of fusion of ice.

A series of check experiments on the specific heat of water show the order of reproducibility of results which can be obtained with this calorimeter to be 1 part in 2000. Measurements made at temperatures between 0° and 40° C gave results which agree to within the limits of experimental accuracy with the unpublished results of a long series of experiments made in the usual form of stirred water calorimeter. The results are also in satisfactory agreement with the most probable values deducible from the data of the most careful investigations published by other observers.

WASHINGTON, D. C., February 25, 1915.

SPECIFIC HEAT AND HEAT OF FUSION OF ICE

By H. C. Dickinson and N. S. Osborne

CONTENTS

	Page
INTRODUCTION	49
PREVIOUS WORK	50
DESCRIPTION OF CALORIMETRIC METHOD	52
MATERIAL AND PREPARATION OF SAMPLES	53
EXPERIMENTAL PROCEDURE	55
SPECIFIC HEAT OF ICE	63
Experimental results	63
Relation between apparent specific heat of ice and dissolved impurities	69
Discussion of results	73
Conclusion	76
HEAT OF FUSION OF ICE	77
SUMMARY	78
TABLE OF TOTAL HEAT OF ICE AND WATER	79

INTRODUCTION

The present investigation is one of a series undertaken, at the request of the refrigeration industries, for the determination of constants which are of fundamental importance in the design and operation of refrigeration machinery.

A determination made at the Bureau[1] of the heat of fusion of ice was published in 1913. In this publication is given a review of previous work on this subject. As stated there the results presented are subject to a slight uncertainty on account of the lack of adequate knowledge of the specific heat of ice near the melting point. For this reason and also on account of the direct technical significance of the specific heat of ice, it has been made the subject of the work here presented.

---

[1] Dickinson, Harper, and Osborne; this Bulletin, 10, p. 235, 1913, Scientific Paper No. 209.

PREVIOUS WORK

A review of previous determinations of the heat of fusion of ice is given in the publication cited above.

The results of previous experimental determinations of the specific heat of ice are summarized in Table 1.

TABLE 1

Specific Heat of Ice—Results of Previous Observers

Date	Observer	Temperature range	Mean temperature	Heat capacity cal/gram degree
1910	Nernst [2]		− 7.0	0.57
			− 9.5	.550
			− 73	.376
			− 83	.355
			−189.3	.186
1910	Koref [3]	− 2.9 to − 76.9	− 39.9	.435
		− 15.3 to − 75.9	− 45.6	.417
		− 81.7 to −189.5	−135.6	.266
1904	Bogojawlenski [4]		− 15	.501
			− 29.2	.442
			− 48.3	.398
1849	Regnault [5]	0 to − 78	− 39	[6].453
1907	Nordmeyer and Bernoulli [7]	0 to −185	− 92.5	.345
1905	Dewar [8]	− 18 to − 78	− 48	.463
		− 78 to −188	−133	.286
		−188 to −252.5	−220.2	.146
1847	Person [9]		− 10	.502
			− 15	.505
	Gadelin [10]		− 8.9	.524

[2] Nernst, K. Ak. d. Wiss. Sitzb., 1910, p. 262.

[3] Koref, K. Ak. d. Wiss. Sitzb., 1910, p. 253.

[4] Bogojawlenski, Schr. der Dorpater Naturf. Ges., **18**, p. 1.

[5] Regnault, Ann. d. Chim. (3), **26**, p. 261, 1849.

[6] Value given by Nernst (loc. cit.), recalculated from original on basis of specific heat of lead.

[7] Nordmeyer and Bernoulli, Verh. d. Deutch. Phys. Ges.. **9**, p. 175; 1907.

[8] Dewar, Proc. Roy. Soc. Lond., **76**, p. 330; 1905.

[9] Person, Ann. Chim. et Phys. (3), **21**, p. 295; 1847.

[10] Review by Ångstrom, Ann. d. Phys., **90**, p. 509; 1853. Original, Nov. Act. Reg. Soc. Upsala, **5**.

Nernst and his associates have deduced an empirical equation expressing their results on the specific heat of ice relative to its temperature. The graphical representation of this equation, together with the observed results of different experimenters, is shown in Fig. 1. The mean temperatures of the determinations

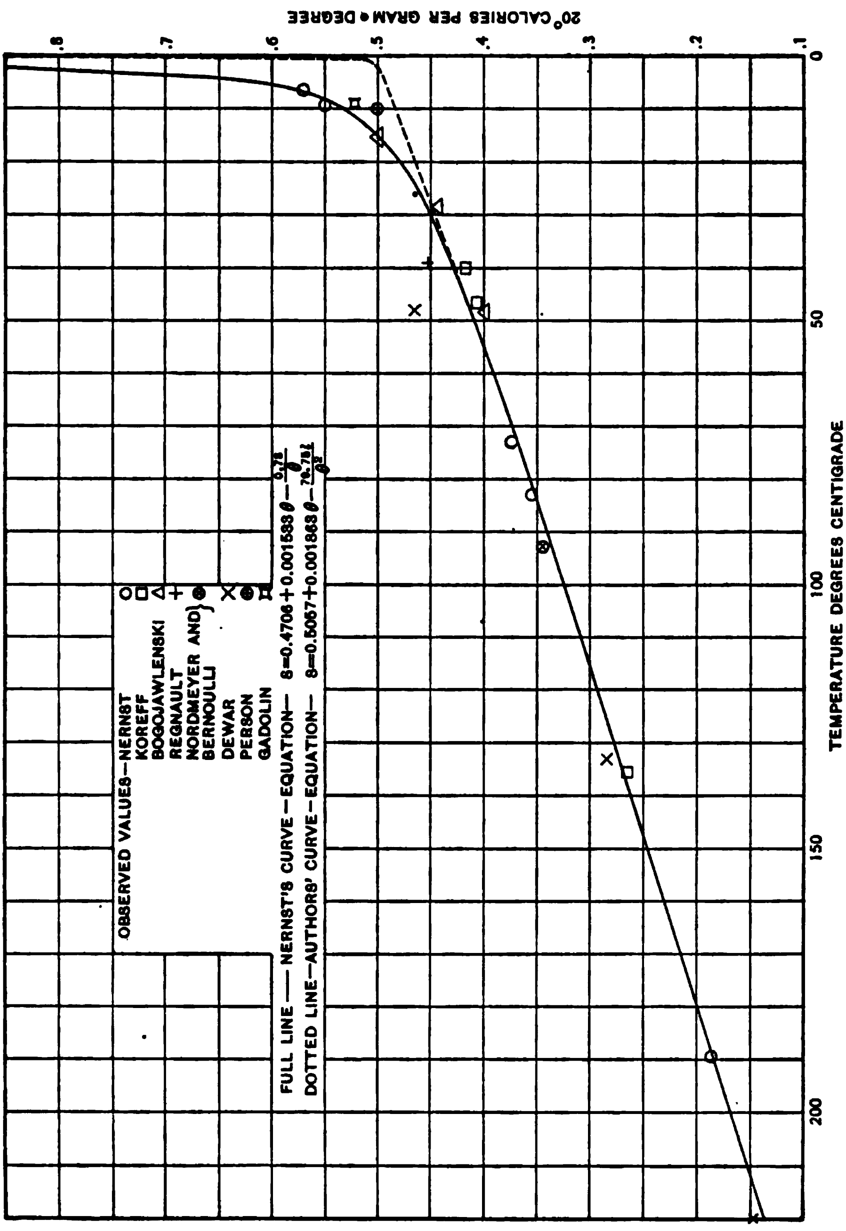


Fig. 1.—*Specific heat of ice*

are all below $-7°$ C. From the trend of these results in the upper part of the range a very significant increase in the specific heat on approaching the melting point is suggested. This might not appear remarkable except for the fact that A. W. Smith[11] has observed that while with impure ice an apparent increase in specific heat is obtained on approaching the melting point, ice of a high degree of purity shows no such abnormal change in specific heat up to a temperature very close to zero.

The presence of certain dissolved impurities lowers the freezing point of water. At any given temperature not too far below zero a certain proportion of an ice sample containing impurity is unfrozen, due to this lowering. A portion of the heat of fusion is thus made to appear as sensible heat, and the observed apparent heat capacity of such ice, especially near zero, is larger than for ice containing no such impurity. Smith's conclusion that the specific heat of pure ice does not change appreciably on approaching zero is therefore plausible, notwithstanding the fact that it appears to be in contradiction to the results of others.

DESCRIPTION OF CALORIMETRIC METHOD

The experiments here described were planned with the object of attaining a high precision in the thermal measurements upon ice of a high degree of purity, and especially of extending the experiments to temperatures near zero, where the specific heat is more important technically and is most in doubt.

For making the measurements a calorimeter was adopted which is described in detail elsewhere.[12] An important feature of this calorimeter is the employment of a shell of copper inclosing the specimen under investigation, the copper acting as the calorimetric medium for the transmission and distribution of heat developed in an electric heating coil which is built into the shell. Temperature changes in calorimeter and contents are measured by means of an electric resistance thermometer likewise built into the shell. The calorimeter is suspended in an air space within an inclosing metal jacket. Multiple thermocouples distributed about the surfaces of the calorimeter and the jacket serve to indicate at any instant the difference between the average temperatures of the surfaces. This enables the corrections for thermal leakage between calorimeter and its surroundings to be controlled and measured.

---

[11] Smith, Physical Review, 17, p. 193; 1903.

[12] Dickinson and Osborne, this Bulletin, 12, p. 23, 1915.

The jacket containing the calorimeter is immersed in a stirred liquid bath which, by means of a refrigerating coil using liquid carbon dioxide, an electric heating coil, and a thermoregulator, is maintained at any temperature between −55° C and +40° C to within a few thousandths of a degree. Using current furnished by a storage battery, the heat supplied to the calorimeter and the contained specimen is developed at a nearly constant rate, which is determined by potentiometer measurements of current and of potential drop.

By this method the heat involved in the temperature changes of the material is measured directly in terms of electrical units, from which it may be reduced to customary heat units by using the proper constants.

The ice specimens were contained in a metal cell which fits inside the calorimeter shell. To promote the rapid equalization of temperature, the cell is provided with radial copper vanes attached to the interior surface. The details of construction of the container are shown in Fig. 2.

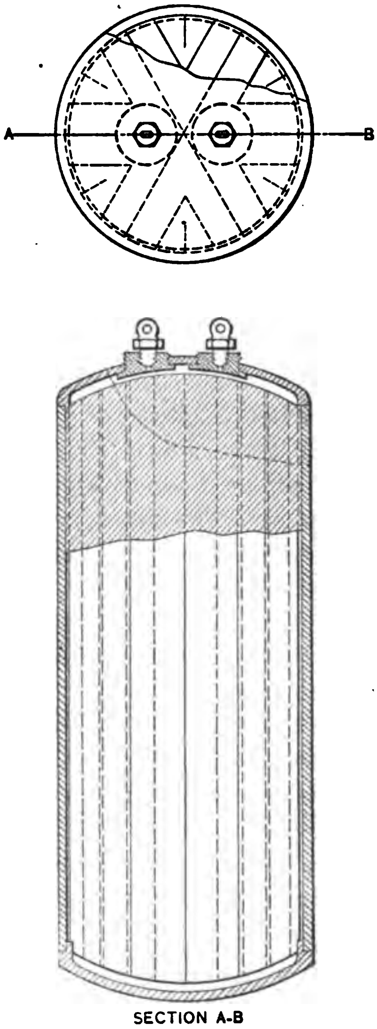


Fig. 2.—*Specimen container*

MATERIAL AND PREPARATION OF SAMPLES

Since even small amounts of impurity would cause considerable increase in the apparent heat capacity, due to incipient melting of portions of the ice, the material used for the determinations of the specific heat and of the heat of fusion of ice was prepared with the object of obtaining specimens of ice as pure as possible. The entire interior surface of the ice container was carefully coated with pure tin to avoid contamination of the sample. The water used was specially prepared by repeated distillations to remove impurities. Four samples of ice were prepared. The water used in samples 1, 2, and 3 was poured into the container through a tin-lined funnel. The water was boiled in the container to expel dissolved gases. Sample No. 1 was frozen in the container directly within the calorimeter by lowering the temperature of the jacket. After the determinations on this sample were completed it was found that this method of freezing had developed sufficient pressure to slightly open the joint at the bottom of the container. The subsequent samples were frozen, beginning at the bottom, by immersing the container into a bath of stirred gasoline cooled to about −10° C, and gradually increasing the depth of immersion as the sample froze.

In the preparation of samples 2 and 3 the residual water was poured from the container after freezing sufficient ice for the sample. The interior cellular construction of the container, however, interfered somewhat with the success of this procedure, and the results on samples 2 and 3 indicated only slightly different purity from sample No. 1.

The results of experiments upon samples 1, 2, and 3 showed a greater increase in the apparent specific heat on approaching zero than would be expected in consideration of Smith's experiments, if the samples possessed the degree of purity sought. On the contrary, they showed less increase than the results of other observers. The results upon sample 3 showed somewhat less increase than samples 1 and 2, which can be accounted for by supposing that sample 3 contained less impurity than the others. To learn, if possible, more in regard to the effect of impurities, an attempt was made to prepare a sample of still greater purity.

Sample No. 4 was distilled directly into the container, the condensed water touching no surface which was not tinned. In doing this the container was first inverted so that the condensed water would drain out, thus washing the entire interior surface with steam and hot distilled water. After about 800 cc had been thus passed through, the container was placed erect and surrounded with ice. Water was condensed until the container was nearly full, leaving about 700 cc in the distilling flask. Thus, only the middle fraction was retained for the sample. The water used in the distilling flask was specially prepared double distilled water. Before sealing the sample in the container, the contained water was boiled to expel any air which might have been absorbed after condensation.

Measurements of the electrical conductivity of the samples, made after the various determinations were completed, failed to show any considerable difference in the observed conductivities of the different samples. These were all of the order of 3×10^{-6} $\text{ohm}^{-1}\ \text{cm}^{-1}$.

While the specimens were being frozen the calorimeter was cooled to a temperature slightly below zero. The object of this was to avoid melting any considerable portion of the ice when the container was introduced into the calorimeter. The operation of inserting the specimen into the calorimeter at a temperature of zero was accompanied by the unavoidable condensation of moisture on the calorimeter, as the work was done when the dew point was above zero. To absorb this moisture and to maintain the dryness of the air within, a small dish of calcium chloride was placed at the bottom of the jacket.

EXPERIMENTAL PROCEDURE

A detailed description of the various operations involved in determination of heat capacity is given in the previous paper referred to above. Briefly stated, the sequence as to manipulations and observations is as follows:

The calorimeter containing the specimen is cooled to the initial temperature of the experiment. The jacket is brought under control of the thermoregulator at the temperature of the calorimeter. The resistance of the built-in platinum thermometer is observed to determine the initial temperature[13] of the calorimeter. Electric current is passed through the heating coil of the calorimeter for a measured interval of time. During this time alternate readings of current[14] and potential drop are made at equal intervals of time to determine the rate of energy supply to the calorimeter. Meanwhile by hand control of the jacket heating current the temperature of the jacket is kept as nearly as practicable equal to the rising temperature of the calorimeter. The readings of the thermocouples, which are recorded at intervals of 30 seconds throughout the entire experiment, indicate the temperature difference between the calorimeter and jacket, and serve the double purpose of guiding the jacket control and giving the data for determining the thermal leakage. After the interruption of the calorimeter heating current, the jacket is again brought under control of the thermoregulator, and when the calorimeter attains thermal equilibrium the thermometer resistance is again observed to determine the final temperature.

The rate of thermal leakage is determined by a separate experiment, in which, with the calorimeter and jacket at different temperatures, alternate readings are made of the resistance thermometer and of the thermocouple between calorimeter and jacket. An approximate value of the heat capacity of the calorimeter at the time of this supplementary experiment is adequate to enable the computation of the rate to be made.

The mass of the sample is determined from weighings in air against brass weights of the empty container, and of the container with specimen included. The difference between these weighings corrected for air buoyancy gives the mass of the specimen.

An example of the record of a single experiment in determining the specific heat of an ice specimen is given in Fig. 3.

---

[13] The temperatures θ employed in this paper are expressed in degrees of the centigrade scale determined by a resistance thermometer of the Heraeus purest platinum according to the equations

$$\theta = pt + \delta\left(\frac{\theta}{100} - 1\right)\frac{\theta}{100}, \quad pt = \frac{R_\theta - R_0}{R_{100} - R_0} 100, \quad \delta = 1.48$$

The value of δ here taken, viz, 1.48, was obtained by a direct comparison of platinum resistance thermometers of Heraeus purest platinum, in the interval 0° to 100° with verre dur thermometers standardized at the Bureau International des Poids et Mesures, and thus serves to reproduce the hydrogen scale of that bureau.

The difference between the scale above defined and the thermodynamic scale as reproduced in the interval −50° to +500° by means of the platinum resistance thermometer standardized at the temperatures of melting ice, steam, and sulphur vapor, consists in the use of $\delta = 1.48$ instead of 1.49+, which would be the value for platinum of the purity here employed when the sulphur boiling point is taken as 444°6. At +50° the difference between the two scales is less than °003, which is within the limits of accuracy of reproduction of the hydrogen scale.

[14] All electrical quantities are expressed in terms of the units adopted by the Bureau of Standards, as given in Bureau Circular No. 29, 1st ed.

CALORIMETER RECORD

Observers, N. S. O., C. H. M.

Date, Aug. 6, 1914
Experiment No. 5

Time	Potentiometer readings across volt box
m	
1	0.117683
3	695
5	708
7	712
9	717
11	723
Mean	.117706
Corr. for potentiometer	0
Corr. for volt box	+5
Corrected reading	.117711
Potential drop (E)	11.7711 volts

Time	Potentiometer readings across std. resistance
m	
2	0.129893
4	903
6	899
8	898
10	892
Mean	.129897
Corr. for potentiometer	0
Corr. for std. resistance	− 99
Corr. for volt-box current	−118
Corrected reading	.129680
Current (I)	1.29680 amp.

Duration of calorimeter current

	m	s
On	1	0.70
Off	11	.71
Total time		600.01 seconds

Power E×I	15.2648	watts
Total energy	9159.1	joules
Corr. for thermal leakage	−3.1	j.
Corrected energy	9156.0	joules

	Initial	Final
Obs. R	30.10950	30.67708
Bridge.corr.	−105	−103
Corr. R_1	$=30.10845$	$R_2=30.67605$
ΔR	0.56760	$R_m=30.39225$
K_m	8.0667	$\theta_1=-7\overset{\circ}{.}304$
$\Delta\theta$	4.5787	$\theta_2=-2\overset{\circ}{.}725$

Total heat capacity	1999.7 j/deg
Heat capacity of calorimeter	1041.1
Heat capacity of ice	958.6
Mass of ice specimen	460.7 g
Heat capacity of ice per g	2.0807 j/deg·g
Mean specific heat θ_1 to θ_2	$0.4974 \frac{\text{cal}_{20}}{\text{g}\cdot\text{deg}}$

Time		Thermocouple readings	Resistance thermometer readings	Notes
m	s	mm	Ohms	
0			30.10950	
	30	200.0		Bridge B. S. No. 7481 at 33° C
1		196.0		
	30	199.9		Bridge calibration of Aug. 29, 1914
2		202.2		
	30	203.0		Ratio 100 R
3		202.3		
	30	201.0		Thermometer No. 11510
4		199.5		
	30	197.8		
5		196.5		
	30	196.3		Calorimeter No. 11510, with ice specimen No. 4
6		197.2		
	30	198.6		
7		200.2		
	30	202.0		
8		203.8		
	30	205.3		Mass of specimen No. 4, 460.7 g
9		205.8		
	30	205.7		
10		205.3		
	30	205.7		
11		212.0		
	30	202.5		Thermocouple zero = 200
12		197.2		
	30	195.2		
13		194.8		
	30	195.3		When jacket is colder than calorimeter, the thermocouple reading is above 200
14		196.3		
	30	197.6		
15		199.2		
	30	201.0		
16		202.8		
	30	204.7		
17		205.0		
	30	202.3		
18		200.3		
	30	199.8		Rate of thermal leakage from jacket to calorimeter = −0.2 joules per minute per millimeter deflection above 200
19		199.7		
	30	200.0		
20		200.0		
	30	200.0		
21		200.0		
	30	200.0		
22			30.67709	
	30	200.0		
23			30.67708	
	30	200.0		
24			30.67708	Correction for thermal leakage to calorimeter = −0.2×6.5×24 = −3.1 j
Mean		200.65		
				Potentiometer No. 9143 Std. cell No. 2682
				Volt box No. 5245 Temp. volt box, 26°8 Std. res. No. 7354
				Temp. std. res., 26°9

FIG. 3.—*Sample record of experiment*

The initial and final readings of thermometer resistance corrected for bridge errors give the initial and final thermometer resistances R_1 and R_2. The difference between the initial and final resistances, ΔR, multiplied by the difference factor, K_m, gives the change in temperature of the calorimeter, $\Delta\theta$. In obtaining K_m from the chart Fig. 4 the mean resistance R_m is used.

The average thermocouple deflection multiplied by the rate and by the time between initial and final thermometer readings gives the correction for thermal leakage.

The mean potentiometer readings for potential drop and for current, corrected for instrumental errors, and multiplied by the proper reduction factors, give the potential drop, E, and current I. The product of the current and potential drop is the power, and this multiplied by the time is the total energy electrically supplied to the calorimeter. This total energy corrected for thermal leakage gives the corrected energy, i. e., the amount of energy received by the calorimeter and contents during the experiment.

The corrected energy divided by the change in temperature, $\Delta\theta$, gives the heat capacity of calorimeter and specimen in joules per degree.

The heat capacity of the calorimeter is obtained from the curve, Fig. 5, at a point corresponding to the mean temperature, $\frac{1}{2}(\theta_1+\theta_2)$.

Deducting from the total mean heat capacity the heat capacity of the calorimeter, the mean heat capacity of the specimen is obtained over the temperature interval of the experiment.

The mean heat capacity of the specimen divided by the mass and by the number of joules[15] in one 20° calorie,[16] gives the mean specific heat of the ice specimen in 20° calories per gram per degree.

---

[15] The relation between the 20° calorie and the joule (international watt second) is taken as represented by the equation

$$1 \text{ calorie}_{20} = 4.183 \text{ joules}$$

[16] The 20° calorie used in this paper is taken as the quantity of heat per gram (mass) per degree centigrade required to raise the temperature of water at 20° C.

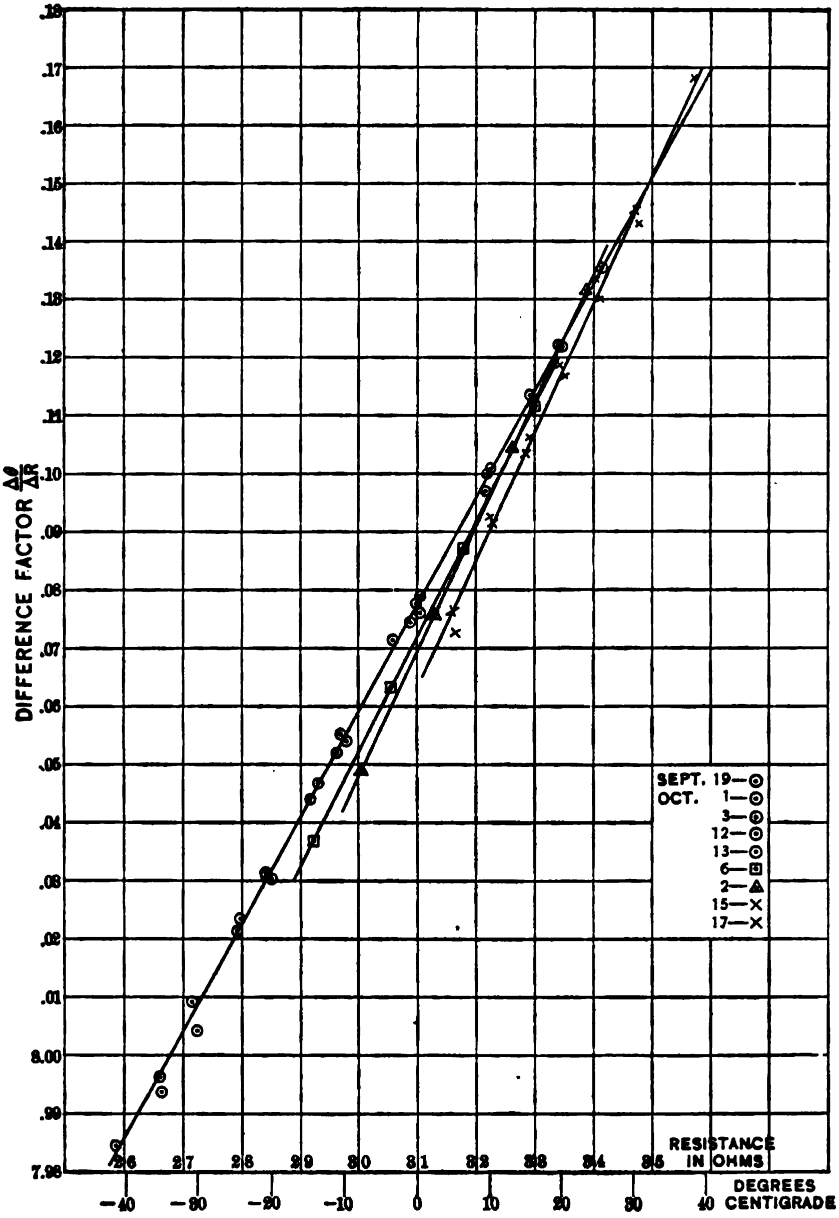


Fig. 4.—*Calibration of resistance thermometer in calorimeter*

The initial temperature, θ_1, may be computed from R_1, using the thermometer resistance at 0° (R_0), determined at the time of calibration, and the value K corresponding to a resistance which is the mean of R_0 and R_1. The final temperature θ_2 may be computed similarly from R_2.

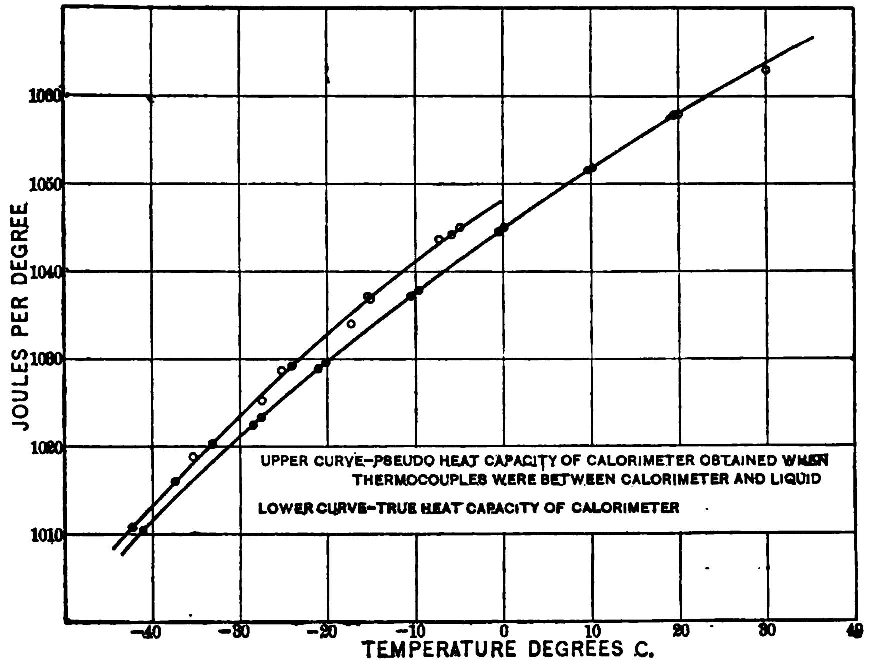


Fig. 5.—*Heat capacity of calorimeter*

The method of calculation may be summarized as follows:

Supposing the observed data to be corrected for all instrumental errors, the specific heat is given by the formula

$$S=\frac{\dfrac{IET+Btd}{\Delta RK_m}-C}{MJ}$$

where

S = mean specific heat of specimen in 20° calories per gram per degree over the interval of temperature employed.

C = mean heat capacity of calorimeter in joules per degree.

I = current in amperes (mean value).

E = potential drop in volts (mean value).

T = duration of energy supply to calorimeter in seconds.

d = average thermocouple deflection in millimeters during experiment.

t = time in minutes between initial and final readings of calorimeter resistance thermometer.

B = rate of thermal leakage to calorimeter from surroundings in joules per minute per millimeter thermocouple deflection.

R_1 = initial resistance of thermometer in ohms.

R_2 = final resistance of thermometer in ohms.

ΔR = difference between initial and final resistance of the thermometer in ohms.

K_m = the difference factor for the resistance thermometer, i. e., $\frac{\Delta\theta}{\Delta R}$ in degrees per ohm.

M = mass of specimen in grams.

The current and potential drop were always so nearly constant that the approximation in taking the product of their mean values multiplied by the time as the total energy is well within the limit of allowable error.

A complete description of the method and results of the calibration of the calorimeter resistance thermometer and of the determinations of the heat capacity of the calorimeter are given in a preceding paper.[17]

The values of the difference factor K as there determined are given graphically in Fig. 4. This chart was used in making calculations of specific heat determinations, the difference factor for any observed interval being taken from the curve at the point corresponding to the mean between the initial and final thermometer resistances. Since the performance of the thermometer was found to depend upon the previous thermal treatment, in the use of the chart the appropriate calibration line was chosen to correspond with the initial temperature of the experiments on the particular day.

---

[17] Dickinson and Osborne, this Bulletin, **12**, p. 23, 1915, Scientific Paper No. 247.

The values of the heat capacity of the calorimeter are given graphically in Fig. 5. The two curves there shown represent the values obtained with the two arrangements of the thermocouples which were used in the determinations. When the earlier arrangement was used, consisting of a single set of thermocouples, the junctions of which were so placed as to indicate the difference in temperature between the surface of the calorimeter and a point in the liquid near the jacket, the upper curve was obtained. The lower curve was obtained with the later arrangement, making use of an additional set of couples the junctions of which were so placed that when joined in series the two sets of couples indicated the difference in temperature between the calorimeter and the jacket surfaces.

These later results showed that in the original arrangement the true temperature difference between calorimeter and jacket had not been indicated owing to the effect of lag in liquid and jacket. The significance of this fault in the apparatus did not appear until after the experimental work on samples 1, 2, and 3 had been completed, but errors from this cause in the final results for these samples could be avoided in the manner explained below.

The value of the observed total heat capacity of the empty calorimeter and of the calorimeter containing a specimen would be affected to the same extent by the improper placing of the thermocouple, provided that in the two experiments the manipulation of the jacket was similar. If, therefore, in computing the result of a specific heat determination a value of heat capacity of the calorimeter be used, determined under experimental conditions similar to those in the specific heat determination, no error is introduced into the resulting value of specific heat. It was necessary therefore to employ the false values of heat capacity shown by the upper curve in Fig. 5 in computing the earlier results.

It was ascertained that the variations in manipulation which did occur, such as the use of different amounts of refrigeration and compensating heating, etc., were not sufficient to cause any large systematic error.

SPECIFIC HEAT OF ICE

Experimental Results.—The results of experiments to determine the specific heat of ice are given in Tables 2, 4, 6, and 8, in which the observed mean specific heats of the several samples are expressed with reference to the initial and final temperatures θ_1, θ_2 of the respective experiments ($S_m = \frac{H_2 - H_1}{\theta_2 - \theta_1}$, where $H_2 - H_1$ represents the total heat per gram over the interval $\theta_2 - \theta_1$).

TABLE 2

Determinations of Specific Heat of Ice

Sample No. 1, Experimental Results

[Mass, 468.7 grams]

Date	Exp	Total energy	Correction for thermal leakage	Corrected total energy	Initial temperature θ_1	Final temperature θ_2	Temperature difference θ_2 to θ_1	Total mean heat capacity θ_1 to θ_2	Mean heat capacity of calorimeter θ_1 to θ_2	Mean heat capacity of ice θ_1 to θ_2 j/deg	Mean specific heat θ_1 to θ_2 S_m
1914		*joules*	*joules*	*joules*	*deg C*	*deg C*	*deg C*	*j/deg*	*j/deg*	*j/deg*	$\frac{cal_{20}}{g \cdot deg}$
June 2	1	19245.7	+22.3	19268	−41.840	−31.559	10.281	1874.1	1016.5	857.6	0.4374
	2	19212.9	− 3.5	19209	31.559	21.553	10.006	1919.7	1026.7	893.0	.4555
	3	19162.3	+ 0.8	19163	21.548	11.799	9.7495	1965.5	1035.4	930.1	.4744
	4	9574.3	− 1.0	9573	11.799	7.014	4.7848	2000.7	1041.2	960.5	.4899
	5	5740.7	0.0	5740.7	7.016	4.182	2.8339	2025.7	1044.2	981.5	.5006
	6	3816.7	0.0	3816.7	4.183	2.327	1.8564	2056.0	1045.7	1010.3	.5153
	7	1909.3	+ 3.2	1912.5	2.327	1.419	.9079	2106.5	1046.5	1060.0	.5407
	8	1909.0	+ 6.1	1915.1	1.419	.583	.8362	2290.2	1047.0	1243.2	.6341
	9	953.9	+12.2	966.1	.584	.284	.3001	3219.3	1047.4	2171.9	1.1078
June 3	1	19106.4	+34.3	19141	−41.853	−31.648	10.205	1875.6	1016.5	859.1	.4382
	2	19055.0	+ 0.6	19056	31.648	21.729	9.9188	1921.2	1026.7	894.5	.4563
	3	19084.8	− 3.9	19081	21.726	12.016	9.7100	1965.1	1035.2	929.9	.4743
	4	9483.5	− 3.4	9480	12.017	7.279	4.7378	2000.9	1041.1	959.8	.4896
	5	5763.7	0.0	5763.7	7.279	4.430	2.8488	2023.2	1043.8	979.4	.4996
	6	3801.6	+ 3.1	3804.7	4.430	2.570	1.8608	2044.7	1045.6	999.1	.5096
	7	1988.4	+ 2.8	1991.2	2.570	1.614	.9558	2083.3	1046.5	1036.8	.5289
	8	1988.1	− 1.6	1986.5	1.615	.714	.9012	2204.3	1047.0	1157.3	.5903
	9	993.2	+ 1.8	995.0	.714	.356	.3581	2778.6	1047.4	1731.2	.8830

TABLE 3

Specific Heat of Ice

Sample No. 1, Reduction of Results

Initial temperature θ_1	Final temperature θ_2	$\sqrt{\theta_1\theta_2}-\theta$	$\frac{\theta_1+\theta_2}{2}-\theta'$	$\theta-\theta'$	$A(\theta-\theta')$	Mean specific heat θ_1 to θ_2 = S_m	Specific heat corrected to θ $S_{\theta obs}$	$S_{\theta calc.}$	$S_{\theta obs}-S_{\theta calc.}$
deg C	*deg C*	*deg C*	*deg C*	*deg C*		$\frac{cal_{20}}{g \cdot deg}$			
−41.840	−31.559	−36.338	−36.700	0.362	0.0007	0.4374	0.4381	0.4381	0.0000
41.853	31.648	36.395	36.750	.355	.0007	.4382	.4389	.4380	+ .0009
31.559	21.553	26.080	26.556	.476	.0009	.4555	.4564	.4572	− .0008
31.648	21.729	26.224	26.688	.464	.0009	.4563	.4572	.4569	+ .0003
21.548	11.799	15.945	16.674	.729	.0014	.4744	.4758	.4764	− .0006
21.726	12.016	16.157	16.871	.714	.0013	.4743	.4756	.4760	− .0004
11.799	7.014	9.097	9.406	.309	.0006	.4899	.4905	.4900	+ .0005
12.017	7.279	9.353	9.648	.295	.0006	.4896	.4902	.4894	+ .0008
7.016	4.182	5.417	5.599	.182	.0003	.5006	.5009	.4990	+ .0019
7.279	4.430	5.679	5.854	.176	.0003	.4996	.4999	.4982	+ .0017
4.183	2.327	3.120	3.255	.135	.0003	.5153	.5156	.5101	+ .0055
4.430	2.570	3.374	3.500	.126	.0002	.5096	.5098	.5081	+ .0017
2.327	1.419	1.817	1.873	.056	.0001	.5407	.5408	.5325	+ .0083
2.570	1.614	2.037	2.092	.055	.0001	.5289	.5290	.5261	+ .0029
1.419	.583	$.909_6$	1.001	.092	.0002	.6341	.6343	.6244	+ .0099
1.615	.714	1.074	1.164	.090	.0002	.5903	.5905	.5901	+ .0004
.584	.284	$.407_3$	.434	.027	.0001	1.1078	1.1079	1.1059	+ .0020
.714	.356	$.504_2$	.535	.031	.0001	.8830	.8831	.8966	− .0135

TABLE 4

Determinations of Specific Heat of Ice

Sample No. 2, Experimental Results

[Mass, 399.8 grams]

Date	Exp	Total energy	Correction for thermal leakage	Corrected total energy	Initial temperature θ_1	Final temperature θ_2	Temperature difference $\theta_2-\theta_1$	Total mean heat capacity θ_1 to θ_2	Mean heat capacity of calorimeter θ_1 to θ_2	Mean heat capacity of ice θ_1 to θ_2	Mean specific heat θ_1 to θ_2 S_m
1914		*joules*	*joules*	*joules*	*deg C*	*deg C*	*deg C*	*j/deg*	*j/deg*	*j/deg*	$\frac{cal_{15}}{g \cdot deg}$
June 12	1	18068.7	+ 0.5	18069	−45.509	−35.094	10.416	1734.8	1012.8	722.0	0.4317
	2	18060.5	+ 4.8	18065	35.094	24.915	10.178	1774.9	1023.5	751.4	.4493
	3	18042.8	− 4.2	18039	24.915	14.974	9.9412	1814.6	1032.7	781.9	.4676
	4	18035.3	−11.8	18023	14.974	5.262	9.7118	1855.8	1040.7	815.1	.4874
	6	3604.1	− 1.1	3603.0	3.344	1.459	1.8853	1911.1	1046.2	864.9	.5172
	7	1914.6	− 0.3	1914.3	1.459	.530	.9289	2060.8	1047.1	1013.7	.6062
June 13	1	18352.7	− 9.1	18344	−26.561	−16.409	10.152	1807.0	1031.3	775.7	.4639
	2	18311.8	0.0	18312	16.371	6.470	9.9013	1849.5	1039.6	809.9	.4843
	3	5487.6	+ 2.6	5490.2	6.470	3.551	2.9189	1880.9	1044.7	836.2	.5000
	4	5484.9	− 2.2	5482.7	3.551	.722	2.8286	1938.3	1046.5	891.8	.5333
	5	914.7	− 4.2	910.5	.722	.352	.3702	2459.5	1047.3	1412.2	.8445
	6	456.3	+ 1.0	457.3	.352	.238	.1137	4022.0	1047.5	2974.5	1.779

TABLE 5

Specific Heat of Ice

Sample No. 2, Reduction of Results

Initial temperature θ_1	Final temperature θ_2	$\sqrt{\theta_1\theta_2}=\theta$	$\frac{\theta_1+\theta_2}{2}=\theta'$	$\theta-\theta'$	$A(\theta-\theta')$	Mean specific heat θ_1 to $\theta_2=S_m$	Specific heat corrected to θ $S_{\theta obs}$	$S_{\theta calc.}$	$S_{\theta obs}-S_{\theta calc.}$
deg C	*deg C*	*deg C*	*deg C*	*deg C*		$\frac{cal_{15}}{g \cdot deg}$			
−45.509	−35.094	−39.964	−40.302	0.338	0.0006	0.4317	0.4323	0.4313	+0.0010
35.094	24.915	29.570	30.004	.434	.0008	.4493	.4501	.4507	− .0006
26.561	16.409	20.877	21.485	.608	.0011	.4639	.4650	.4670	− .0020
24.915	14.974	19.315	19.944	.629	.0012	.4676	.4688	.4700	− .0012
16.371	6.470	10.292	11.420	1.128	.0021	.4843	.4864	.4874	− .0010
14.974	5.262	8.877	10.118	1.241	.0023	.4874	.4897	.4904	− .0007
6.470	3.551	4.793	5.010	.217	.0004	.5000	.5004	.5007	− .0003
3.551	.722	1.601	2.136	.535	.0010	.5333	.5343	.5399	− .0056
3.344	1.459	2.209	2.402	.193	.0004	.5172	.5176	.5212	− .0136
1.459	.530	.879	.994	.115	.0002	.6062	.6064	.6282	− .0218
.722	.352	$.504_1$	.537	.033	.0001	.8445	.8446	.8812	− .0366
.352	.238	$.289_4$	.295	.006	.0000	1.779	1.779	1.6522	+ .1268

TABLE 6

Determinations of Specific Heat on Ice

Sample No. 3, Experimental Results

[Mass, 415.8 grams]

Date	Exp	Total energy	Correction for thermal leakage	Corrected total energy	Initial temperature θ_1	Final temperature θ_2	Temperature difference θ_2 to θ_1	Total mean heat capacity θ_1 to θ_2	Mean heat capacity of calorimeter θ_1 to θ_2	Mean heat capacity of ice θ_1 to θ_2	Mean specific heat θ_1 to θ_2 S_m
1914		*joules*	*joules*	*joules*	*deg C*	*deg C*	*deg C*	*j/deg*	*j/deg*	*j/deg*	$\frac{cal_{15}}{g\cdot deg}$
June 17	1	20120.	−6.0	20114.	−43.284	−31.964	11.320	1776.8	1015.7	761.1	0.4376
	2	18685.8	−9.2	18677.	31.956	21.701	10.2546	1821.3	1026.5	794.8	.4570
	3	18662.4	+0.6	18663.	21.701	11.689	10.0121	1864.0	1035.6	828.4	.4763
	4	9325.1	+0.8	9326.	11.689	6.769	4.9202	1895.5	1041.6	853.9	.4910
	5	5593.2	−0.9	5592.3	6.769	3.848	2.9213	1914.3	1044.5	869.8	.5001
	6	3726.8	−1.1	3725.7	3.848	1.924	1.9240	1936.4	1046.0	890.4	.5120
	7	1931.6	−2.4	1929.2	1.924	.954	.9690	1990.9	1046.7	944.2	.5429
June 18	1	18620.4	−1.2	18619.	−16.720	− 6.800	9.9206	1876.8	1039.9	836.9	.4812
	2	5664.6	0.0	5664.6	6.809	3.847	2.9619	1912.5	1044.4	868.1	.4991
	3	1912.4	−1.9	1910.5	3.847	2.857	.9902	1929.4	1045.6	883.8	.5082
	4	1913.1	+ .9	1914.0	2.857	1.873	.9844	1944.3	1046.1	898.2	.5164
	5	1899.5	−1.4	1898.1	1.877	.925	.9521	1993.6	1046.7	946.9	.5444
	6	952.2	−1.5	951.7	.925	.494	.4309	2208.6	1047.1	1161.5	.6678
	7	951.3	0.0	951.3	.495	.211	.2837	3353.2	1047.2	2306.0	1.3259

TABLE 7

Specific Heat of Ice

Sample No. 3, Reduction of Results

Initial temperature θ_1	Final temperature θ_2	$\sqrt{\theta_1\theta_2}=\theta$	$\frac{\theta_1+\theta_2}{2}=\theta'$	$\theta-\theta'$	$A(\theta-\theta')$	Mean specific heat θ_1 to $\theta_2=S_m$	Specific heat corrected to θ $S_{\theta obs}$	$S_{\theta calc.}$	$S_{\theta obs}-S_{\theta calc.}$
deg C	*deg C*	*deg C*	*deg C*	*deg C*		$\frac{cal_{15}}{g\cdot deg}$			
−43.284	−31.964	37.196	37.624	0.428	0.0008	0.4376	0.4384	0.4366	+0.0018
31.956	21.701	26.334	26.828	.494	.0009	.4570	.4579	.4566	+ .0013
21.701	11.689	15.927	16.695	.768	.0014	.4763	.4777	.4760	+ .0017
16.720	6.800	10.663	11.760	1.097	.0020	.4812	.4832	.4859	− .0027
11.689	6.769	8.895	9.229	.334	.0006	.4910	.4916	.4892	+ .0024
6.769	3.848	5.104	5.308	.204	.0004	.5001	.5005	.4965	+ .0040
6.809	3.847	5.118	5.328	.210	.0004	.4991	.4995	.4965	+ .0030
3.848	1.924	2.721	2.886	.165	.0003	.5120	.5123	.5108	+ .0015
3.847	2.857	3.315	3.352	.037	.0001	.5082	.5083	.5064	+ .0019
2.857	1.873	2.313	2.365	.052	.0001	.5164	.5165	.5155	+ .0010
1.924	.954	1.355	1.439	.084	.0002	.5429	.5431	.5444	− .0013
1.877	.925	1.318	1.401	.083	.0002	.5444	.5446	.5468	− .0022
.925	.494	$.676_0$	.710	.034	.0001	.6678	.6679	.6701	− .0022
.495	.211	$.323_2$	.353	.030	.0001	1.3259	1.3260	1.2311	+ .0949

TABLE 8

Determinations of Specific Heat of Ice

Sample No. 4, Experimental Results

[Mass, 460.7 grams]

Date	Exp	Total energy	Correction for thermal leakage	Corrected total energy	Initial temperature θ_1	Final temperature θ_2	Temperature difference θ_2 to θ_1	Total mean heat capacity θ_1 to θ_2	Mean heat capacity of calorimeter θ_1 to θ_2	Mean heat capacity of ice θ_1 to θ_2	Mean specific heat θ_1 to θ_2
1914		*joules*	*joules*	*joules*	*deg C* (−)	*deg C* (−)	*deg C*	*j/deg*	*j/deg*	*j/deg*	$\frac{cal_{20}}{g\cdot deg}$
Aug. 5	1	15167.4	+6.2	15174	36.0626	27.9733	8.0893	1875.8	1018.5	857.3	0.4449
	2	15156.1	+2.6	15159	27.9733	20.0447	7.9286	1911.9	1026.8	885.1	.4593
	3	17942.7	−2.2	17941	19.8574	10.6655	9.1919	1951.8	1033.3	918.5	.4766
	4	8959.0	−1.2	8957.8	10.6655	6.1438	4.5217	1981.1	1038.7	942.4	.4890
	5	4468.6	+2.9	4471.5	6.1438	3.8982	2.2456	1991.2	1041.2	950.0	.4930
	6	4478.1	−7.2	4470.9	3.8982	1.6695	2.2287	2006.1	1042.8	963.3	.4999
	7	1789.7	0.0	1789.7	1.6695	.7827	.8868	2018.1	1043.9	974.2	.5055
Aug. 6	1	9154.1	0.0	9154	40.8110	35.8648	4.9462	1850.7	1013.4	837.3	.4345
	2	18356.7	+3.1	18360	35.8648	26.1189	9.7459	1883.8	1020.2	863.6	.4481
	3	18325.1	−1.4	18324	26.1189	16.6050	9.5139	1926.0	1028.4	897.6	.4658
	4	18313.1	+1.2	18314	16.6045	7.3042	9.3003	1969.2	1035.9	933.3	.4843
	5	9159.1	−3.1	9156	7.3037	2.7250	4.5787	1999.7	1041.1	958.6	.4974
	6	1836.4	−3.4	1833.0	2.7236	1.8116	.9119	2010.0	1043.1	966.9	.5017
	7	1835.8	−2.5	1833.3	1.8116	.9028	.9088	2017.2	1043.8	973.4	.5051
	8	1146.6	+1.2	1147.8	.9028	.3401	.5627	2039.8	1044.3	995.5	.5166
Aug. 7	1	18288.0	−3.3	18285	39.5670	29.7674	9.7996	1865.9	1016.9	849.0	.4405
	2	18297.4	−5.3	18292	29.7674	20.1960	9.5714	1911.0	1025.4	885.6	.4596
	3	18276.0	−8.5	18267	20.1960	10.8384	9.3576	1952.2	1033.2	919.0	.4769
	4	18280.0	−4.6	18275	10.8360	1.6742	9.1618	1994.7	1040.3	954.4	.4952
	5	2287.6	−1.4	2286.2	1.9562	.8220	1.1342	2015.7	1042.3	973.4	.5051
	6	1141.9	0.0	1141.9	.8220	.2656	.55635	2052.5	1044.4	1008.1	.5231
	7	454.2	−1.4	452.8	.2656	.0727	.19290	2347.3	1044.7	1302.6	.6759
	8	122.6	−0.2	122.4	.0733	.0453	.02795	4379.3	1044.7	3334.6	1.730

TABLE 9

Specific Heat of Ice

Sample No. 4, Reduction of Results

Initial temperature θ_1	Final temperature θ_2	$\sqrt{\theta_1\theta_2}=\theta$	$\frac{\theta_1+\theta_2}{2}=\theta'$	$\theta-\theta'$	$A(\theta-\theta')$	Mean specific heat θ_1 to $\theta_2=S_m$	Specific heat corrected to θ $S_{\theta obs}$	$S_{\theta calc.}$	$S_{\theta obs}-S_{\theta calc.}$
deg C	*deg C*	*deg C*	*deg C*	*deg C*		$\frac{cal_{20}}{g\cdot deg}$			
−40.811	−35.865	−38.258	−38.338	0.080	0.0001	0.4345	0.4346	0.43.44	+0.0002
39.567	29.767	34.319	34.667	.348	.0006	.4405	.4411	.4418	− .0007
35.865	26.119	30.607	30.992	.385	.0007	.4481	.4488	.4487	+ .0001
36.063	27.973	31.761	32.018	.257	.0005	.4449	.4454	.4465	− .0011
27.973	20.045	23.680	24.009	.329	.0006	.4593	.4599	.4616	− .0017
29.767	20.196	24.519	24.982	.463	.0009	.4596	.4605	.4600	+ .0005
26.119	16.605	20.826	21.362	.536	.0010	.4658	.4668	.4669	− .0001
20.196	10.834	14.792	15.515	.723	.0013	.4769	.4782	.4781	+ .0001
19.857	10.665	14.552	15.261	.709	.0013	.4766	.4779	.4786	− .0007
16.604	7.304	11.012	11.954	.942	.0018	.4843	.4861	.4852	+ .0009
10.665	6.144	8.095	8.404	.309	.0006	.4890	.4896	.4907	− .0011
10.836	1.674	4.259	6.255	1.996	.0037	.4952	.4989	.4980	+ .0009
7.304	2.725	4.461	5.014	.553	.0010	.4974	.4984	.4976	+ .0008
6.144	3.898	4.894	5.021	.127	.0002	.4930	.4932	.4968	− .0036
3.898	1.670	2.552	2.784	.232	.0004	.4999	.5003	.5015	− .0012
2.724	1.812	2.222	2.268	.046	.0001	.5017	.5018	.5024	− .0006
1.956	.822	1.268	1.389	.121	.0002	.5051	.5053	.5058	− .0005
1.812	.903	1.279	1.358	.079	.0001	.5051	.5052	.5057	− .0005
1.669	.783	1.143	1.226	.083	.0002	.5055	.5057	.5066	− .0009
.903	,340	.554	.622	.068	.0001	.5166	.5167	.5177	− .0010
.822	.266	.468	.544	.076	.0001	.5231	.5232	.5234	− .0002
.266	.073	.139	.170	.031	.0001	.6759	.6760	.7116	− .0356
$.073_8$	$.045_8$	$.057_6$	$.059_8$	$.001_7$	.0000	1.730	1.730	1.7326	− .0026

A preliminary examination of these results was made by plotting the observed mean specific heats with reference to the average temperatures, $\frac{1}{2}(\theta_1+\theta_2)$ taken as abscissas. This examination tion indicated that the curves of specific heat were asymptotic to a straight line, the departure from which was apparent only above −8° for samples 1, 2, and 3, and above −2° for sample 4. Qualitatively, Smith's[18] observation that the specific heat of ice tends toward constancy as the impurities in the ice are reduced appeared to be corroborated.

The degree of purity of the several samples was not known. Conductivity tests did not indicate great difference in purity, but this fact might be accounted for by the difficulty in transferring a sample from the container to the conductivity apparatus without contaminating the specimen. It was supposed from consideration of the refinement in method of preparing the samples that progress toward purity had been achieved, at least in sample 4.

Before proceeding further with the analysis of the experimental data it is necessary to consider the relation between the apparent specific heat of ice and the presence of dissolved impurities.

Relation Between Apparent Specific Heat of Ice and Dissolved Impurities.—Consideration of the form of specific heat curves referred to above appeared to indicate that over the range of temperature employed the specific heat of pure ice bears a linear relation to the temperature, and that the departure from this linear relation is attributable to the fusion of a portion of the sample due to contained impurity. As a basis for the analysis of the results this is assumed.

From experiment it is known that for dilute solutions of substances which lower the freezing point the lowering is very nearly proportional to the concentration. Since the samples of water used here were extremely dilute solutions of the impurities, it is assumed that over the range of temperature within which the impurities produce any significant effect upon the apparent specific heat, this proportionality of the concentration to the lowering of the freezing point holds true.

---

[18] Loc. cit., note 11.

Consider a specimen of ice containing a small amount of impurity which lowers the initial freezing point from that of pure ice. Let

l = the lowering of the initial freezing point
K = the concentration of impurity when sample is all melted
θ = any temperature below l
m = amount of unfrozen solution per gram of entire specimen at temperature θ
L = latent heat of fusion of ice
s = true specific heat of pure ice at temperature θ
S = apparent specific heat of ice specimen at temperature θ
H_f = total heat of fusion of the portion unfrozen at temperature θ, per gram of the specimen; $H_f = mL$.

Since the total amount of impurity is fixed, the concentration of the unfrozen portion of the sample is equal to $\frac{K}{m}$

Hence

$$\frac{\theta}{l} = \frac{K}{mK} \text{ or } m = \frac{l}{\theta}$$

By definition

$$H_f = mL$$

substituting for m,

$$H_f = \frac{lL}{\theta} \tag{1}$$

differentiating with respect to θ

$$\frac{dH_f}{d\theta} = -\frac{lL}{\theta^2} \tag{2}$$

The apparent specific heat of the specimen may be separated into four parts, namely, that due to the specific heat of the ice, that due to the specific heat of the water present, that due to fusion of a part of the ice, and that due to dilution of the solution present. Let these be designated, respectively,

$$\frac{dH_i}{d\theta}, \ \frac{dH_w}{d\theta}, \ \frac{dH_f}{d\theta}, \ \frac{dH_s}{d\theta}.$$

Since the amount of ice at temperature θ is $1-m$ per gram of specimen,

$$\frac{dH_i}{d\theta}=(1-m)\,s,$$

calling s_w the specific heat of water

$$\frac{dH_w}{d\theta}=s_w m.$$

The expression for apparent specific heat of ice may now be written

$$S=s\,(1-m)+s_w m-L\,\frac{l}{\theta^2}+\frac{dH_s}{d\theta},$$

or since $m=\frac{l}{\theta}$,

$$S=s+(s_w-s)\frac{l}{\theta}-L\frac{l}{\theta^2}+\frac{dH_s}{d\theta}.$$

For the dilute solutions here considered the last term may be entirely neglected. Since s_w-s is small compared to L the constant value $1.01-0.50=0.51$ is taken for this term. Taking 79.75 for L, the equation becomes

$$S=s+0.51\,\frac{l}{\theta}-79.75\,\frac{l}{\theta^2}.$$

Since by assumption $s=B+A\theta$, this equation becomes

$$S=B+A\theta+0.51\,\frac{l}{\theta}-79.75\,\frac{l}{\theta^2}. \tag{3}$$

In order to determine whether the experimental data satisfies equation 3 it is necessary to refer the observed mean specific heats to definite temperatures instead of the temperature intervals in terms of which they are derived.

The term $0.51\,\frac{l}{\theta}$ may be neglected, since for small values of θ it is small compared with $79.75\,\frac{l}{\theta^2}$ and for other values of θ it is small compared to S.

Equation 3 then becomes

$$S_\theta=B+A\theta-79.75\,\frac{l}{\theta^2} \tag{4}$$

Integrating between θ_1 and θ_2 gives the total heat for this temperature interval, which is written

$$H_2-H_1=B\,(\theta_2-\theta_1)+\frac{A}{2}\,(\theta_2^2-\theta_1^2)+79.75\left(\frac{l}{\theta_2}-\frac{l}{\theta_1}\right),$$

dividing by $\theta_2-\theta_1$ gives the mean specific heat

$$\frac{H_2-H_1}{\theta_2-\theta_1}=S_m=B+A\,\frac{\theta_2+\theta_1}{2}-79.75\;l\,\frac{1}{\theta_1\theta_2}.$$

If $\theta_1\theta_2=\theta^2$

$$S_m=B+A\,\frac{\theta_2+\theta_1}{2}-79.75\;l\,\frac{1}{\theta^2},$$

but

$$S_\theta=B+A\theta-79.75\;l\,\frac{1}{\theta^2},$$

and therefore

$$S_\theta-S_m=A\left(\theta-\frac{\theta_2+\theta_1}{2}\right)$$

if A is known the value of $S_\theta-S_m$ may be calculated from this equation. If this term is added as a correction to the observed values of S_m and θ be chosen equal to $-\sqrt{\theta_1\theta_2}$, there is obtained a series of values of S determined experimentally, referred to corresponding values of θ which may now be examined with respect to their agreement with equation 4.

The value of A used for this reduction was 0.00186, obtained from the preliminary graph of the data.

In Tables 3, 5, 7, and 9 are given the steps in reduction of the data, resulting in a series of values of S_θ given in column 8 referred to corresponding values of θ given in column 3. In column 9 are given values of S calculated from equation 4, using the values for B, A, and l as follows:

$B=0.5057$

$A=0.001863$

Sample.	l.
1	−0.00125
2	− .00120
3	− .00095
4	− .00005

The differences between the experimental and calculated values of S are given in column 10 headed $S_{\theta obs}-S_{\theta cal}$.

Consideration of the sources of error which may affect the values of S shows that $S_{obs}-S_{cal}$ is more affected by errors in temperature measurement the smaller the numerical value of θ or of $\theta_2-\theta_1$. Greater differences between the experimental and calculated values may therefore be expected at temperatures near zero than elsewhere. Making due allowance for this fact, examination of the values of $S_{obs}-S_{cal}$ shows that the agreement between experimental and calculated values is within the limits of experimental precision. In other words, the specific heat of ice samples numbers 1, 2, 3, and 4 in the range of temperature from −40° C to −.05° C is represented within the limit of experimental error by the equation

$$S_\theta=0.5057+0.001863\theta-79.75\frac{l}{\theta^2}$$

where l has the following values:

Sample.	l
1	−0.00125
2	− .00120
3	− .00095
4	− .00005

Discussion of Results.—The results of the determinations of specific heat are graphically represented in Fig. 6. The curve of calculated values for sample No. 2 is omitted because it is scarcely distinguishable from that of sample No. 1. The results in the range of temperature −10 to 0° C are shown in Fig. 7 plotted to a larger scale of temperature.

In arriving at the above results the assumption has been made that the measure of the departure of the specific heat of any specimen of impure ice from a linear function of the temperature can be expressed in terms of a constant representing the initial freezing point and depending upon the degree of purity.

The results upon four samples of ice all of high degree of purity, yet differing among themselves in purity, agree with this assumption, but it does not necessarily follow that the assumption is substantially correct. It is quite possible that the relation of

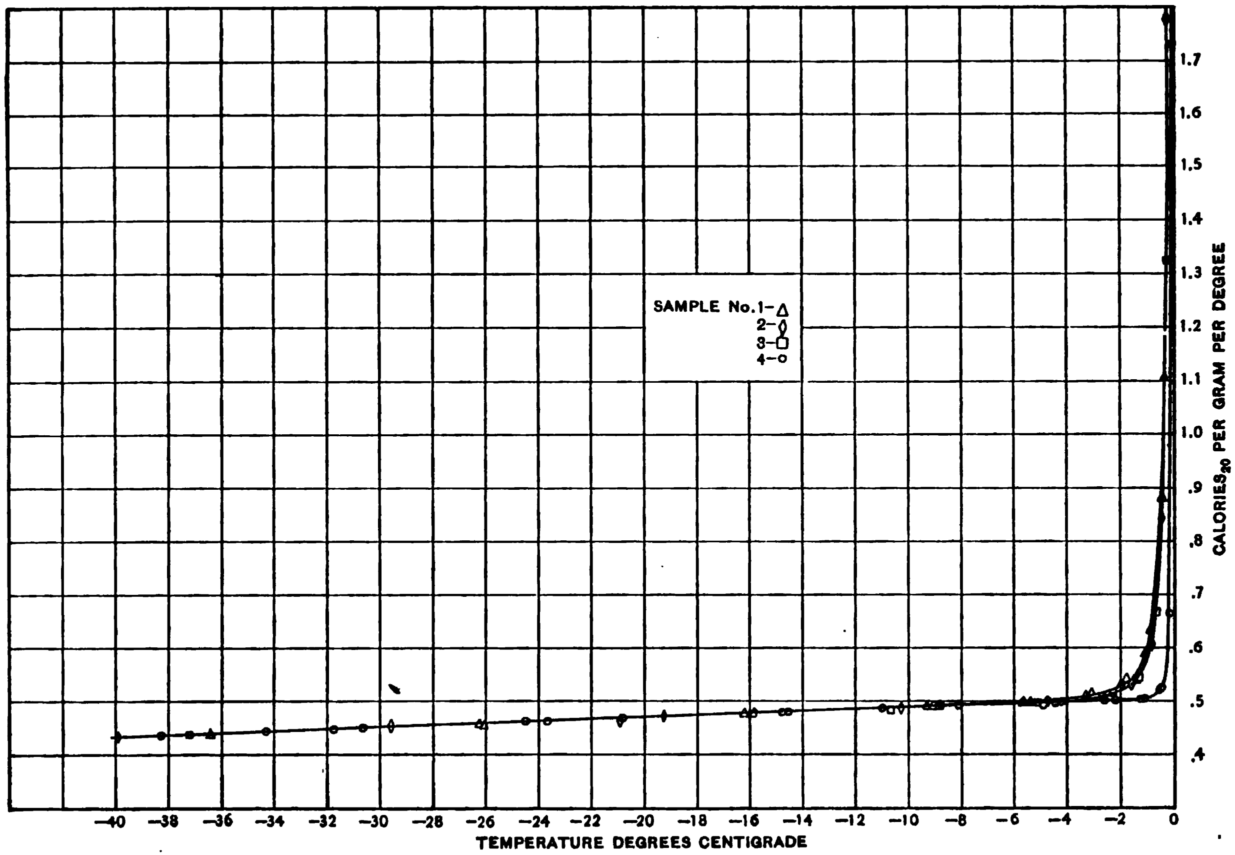


FIG. 6.—*Specific heat of ice, −40° to 0° C*

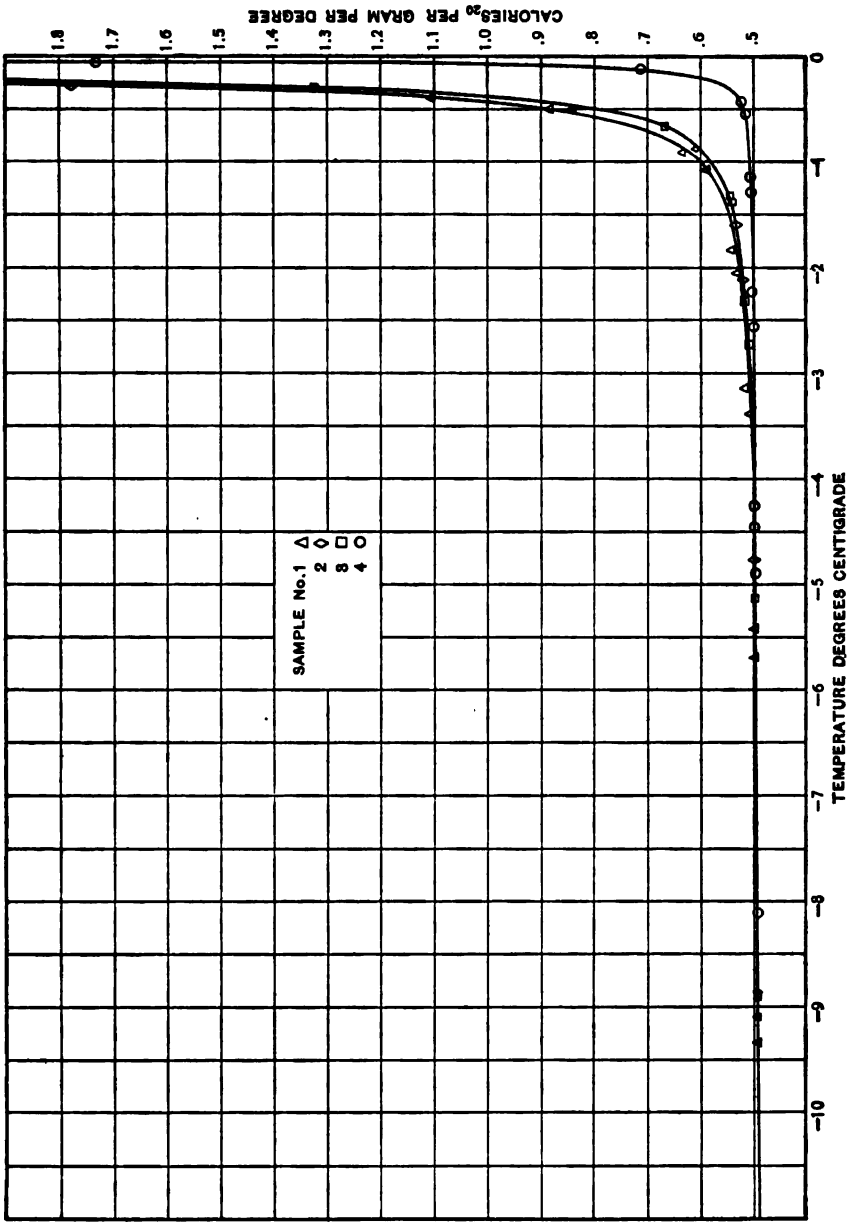


FIG. 7.—*Specific heat of ice, −10° to 0° C*

the specific heat of pure ice to temperature is other than linear, perhaps rapidly increasing near zero and the departure from its value with increasing contamination could yet be approximately represented in terms of l, the initial freezing point.

It is obvious that with the experimental means at our command there are two ultimate limitations, one as to the degree of purity of ice attainable, another as to the proximity to zero of the temperatures between which total heats can be measured. On account of these limitations no rigid conclusion from a purely experimental standpoint can be drawn as to the ultimate value which the specific heat of pure ice may attain in the range of temperature just before melting takes place; and therefore the separation of experimental determinations of total heat of ice into the elements of sensible heat and heat of fusion is to some extent arbitrary.

Conclusion.—In the present series of determinations it is found that over the greater part of the temperature range covered by the experiments, i. e., $-40°$ to $-2°$, the specific heat in 20° calories at a given temperature θ, of the purest ice experimented on is represented within the limit of experimental accuracy by the equation $s=0.5057+0.001863\ \theta$, and that in the range closer to zero, i. e., $-2°$ to $-0.°05$, the purer the ice the less the specific heat exceeds the value given by this equation. For the purest ice obtained the departure was not more than $\frac{0.004}{\theta^2}$.

In order to state the specific heat of ice in definite terms and permit the deduction from experimental data of a definite figure for the heat of fusion of ice, it seems justifiable, for technical purposes at least, to express the specific heat of pure ice in 20° calories per gram per degree by the equation

$$s=0.5057+0.001863\,\theta$$

HEAT OF FUSION OF ICE

Upon three of the samples used in the specific heat determinations observations were made to determine the heat of fusion. The data and results are given in Table 10.

TABLE 10

Heat of Fusion of Ice—Results of Present Investigation

Sample	Date	Initial temperature θ_1	Final temperature θ_2	Total energy	Correction for thermal leakage	Corrected energy	Ice θ_1 to 0°	Water 0° to θ_2	Calorimeter θ_1 to θ_2	Total sensible heat	Total latent heat
	1914			*joules*	*joules*	*joules*	*joules*	*joules*	*joules*	*joules*	*joules*
1.......	June 5	−3.6220	+4.7463	177812	+25	177837	3561.6	9367.8	8763.8	21693	156144
2.......	June 13	−0.2398	+4.7549	146330	−19	146311	202.3	8005.1	5233.9	13441	132870
4.......	Aug 8	−1.6353	+3.6024	167647	+96	167743	1586.6	6993.6	5475.8	14056	153687

Sample	Mass of ice	Heat of fusion	Heat of fusion	Correction for melting below θ_1	Corrected heat of fusion
	grams	*joules/g*	*cal$_{20}$/g*	*cal$_{20}$/g*	*cal$_{20}$/g*
1 (continued)................................	468.7	333.14	79.65	+.03	79.68
2 (continued)................................	399.8	332.34	79.45	+.40	79.85
4 (continued)................................	460.7	333.59	79.75	+.00	79.75
Mean................................	....	....	....	....	79.76

Since partial fusion of the sample, due to presence of impurities, takes place at temperatures below zero, as pointed out in the preceding discussion, it is necessary to apply a correction to the observed heat of fusion for the heat involved in the fusion which has already taken place at the initial temperature of the determination.

This correction is given by equation (1). $H_f = \frac{lL}{\theta}$.

The values of l have been given above. The corrections for previous fusion are given in next to the last column. The corrected values for heat of fusion of ice are given in the last column. The mean of the three samples gives for the heat of fusion of ice 79.76 20° calories per gram.

TABLE 11

Heat of Fusion of Ice—Results of Previous Determinations

Method	Samples	Kind of ice	Initial temperature of ice	Heat of fusion	Corrected heat of fusion	Corrected heat of fusion
			deg C	*cal$_{15}$/g*	*cal$_{15}$/g*	*cal$_{20}$/g*
Electrical	13	Plate Can	−0.72	79.65	79.66	79.74
Mixtures	8	Natural Can	− .72	79.61	79.62	79.70
Electrical	12	Can	−3.78	79.65	79.70	79.78
Mean						79.74

The results of the previous investigation [19] at the Bureau to determine the heat of fusion of ice, using the stirred water calorimeter, are given in column 5 of Table 11.

The earlier results were computed using an assumed value for specific heat of ice = 0.52 cal/g • deg. Correcting the results by using the newly determined value for the specific heat of ice, the figures given in column 6 are obtained. The results reduced to 20° calories are given in column 7. The mean of the three sets is 79.74 20° calories per gram.

The agreement of the results obtained by the widely different methods is within the limit of experimental error. The mean result of the two investigations is 79.75 20° calories per gram mass.

SUMMARY

By means of a calorimeter of the aneroid type—that is, employing the thermal conductivity of copper instead of the convectivity of a stirred liquid to distribute heat supplied electrically—the specific heat and heat of fusion of ice were determined.

The specific heat was determined upon four samples of ice, all of a high but yet of different degrees of purity.

A variation in the specific heat of ice is found which depends upon the degree of purity.

It is found that at a given temperature θ, between −40° and −2°, for the purest ice experimented on, the specific heat in 20°

---

[19] loc. cit., note 1.

calories per gram per degree is represented within the limit of experimental accuracy by the equation

$$s = 0.5057 + 0.001863\ \theta$$

and that from -2° to $-0^\circ.05$ the specific heat does not exceed the value given by the above equation by more than $\frac{0.004}{\theta^2}$.

The specific heat of impure ice at any temperature θ above -40° is greater than that of pure ice by $\frac{lL}{\theta^2}$ where L is the heat of fusion and l the initial freezing point.

The value found for the heat of fusion of ice is 79.76 20° calories per gram, which is within 1/4000 of the value previously determined at the Bureau by a different method employing a stirred water calorimeter.

TABLE OF TOTAL HEAT OF ICE AND WATER

For the convenience of those accustomed to the use of total heats expressed in terms of British thermal units[20] Table 12 has been prepared, giving in Btu per pound (weighed in air) the difference in total heat per pound, $H_{32} - H_t$, between ice at t° F and ice at 32° F; $h_{32} - H_t$, between ice at t° F and water at 32° F; $h_{t'} - h_{32}$, between water at 32° F and at t'° F.

If the specific heat s of ice in $\mathrm{cal}_{20}/\mathrm{g} \bullet \mathrm{deg\ C}$ at the temperature θ is expressed by the equation

$$s = 0.5057 + 0.001863\ \theta,$$

and the relation between units be taken as

$$1\ \mathrm{Btu/lb} \bullet \mathrm{deg\ F} = 1.001\ \mathrm{cal}_{20}/\mathrm{g} \bullet \mathrm{deg\ C},$$

the specific heat s' of ice in $\mathrm{Btu/lb} \bullet \mathrm{deg\ F}$ at the temperature t is given by the equation

$$s' = 0.5052 + 0.001861\ (t - 32).$$

---

[20] The British thermal unit used here is defined as the quantity of heat required to raise the temperature of 1 pound of water (weighed in air) 1° Fahrenheit at 60° F.

TABLE 12

Table of Total Heat of Ice and Water

Temperature of ice t deg. F	Difference in total heat per pound from ice at t° to		Temperature of water t^1	Difference in total heat per pound from water at 32° to water at t^1 h_t-h_{32}
	Ice at 32° $H_{32}-H_t$	Water at 32° $h_{32}-H_t$		
	Btu/lb	*Btu/lb*	*deg. F*	*Btu/lb*
−20	23.8	167.2	+32°	0.0
−18	22.9	166.3	34	2.0
−16	22.1	165.5	36	4.0
−14	21.3	164.7	38	6.0
−12	20.4	163.8	40	8.1
−10	19.6	163.0	42	10.1
− 8	18.7	162.1	44	12.1
− 6	17.9	161.3	46	14.1
− 4	17.0	160.4	48	16.1
− 2	16.1	159.5	50	18.1
0	15.2	158.6	52	20.1
+ 2	14.3	157.7	54	22.1
4	13.4	156.8	56	24.1
6	12.5	155.9	58	26.1
8	11.6	155.0	60	28.1
10	10.7	154.1	62	30.1
12	9.7	153.1	64	32.1
14	8.8	152.2	66	34.1
16	7.8	151.2	68	36.1
18	6.9	150.3	70	38.1
20	5.9	149.3	72	40.1
22	5.0	148.4	74	42.1
24	4.0	147.4	76	44.1
26	3.0	146.4	78	46.1
28	2.0	145.4	80	48.1
30	1.0	144.4	82	50.1
32	0.0	143.4	84	52.1
			86	54.1
			88	56.1
			90	58.0
			95	63.0
			100	68.0

The difference in total heat between ice at temperatures t and 32 is given by the equation

$$H_{32}-H_t=\int_t^{32} s'\,dt=-0.5052\,(t-32)-0.00093\,(t-32)^2$$

If the heat of fusion of ice is taken as 79.75 cal_{20}/g and the relation between units be taken as

$$1\ \mathrm{Btu/lb}=0.556\ \mathrm{cal}_{20}/g$$

the heat of fusion of ice is

$$143.4\ \mathrm{Btu/lb}$$

Example.—Required the number of Btu per pound required to change ice at 8° F into water at 68° F:

$$\begin{array}{lr} \text{Column 3, opposite } 8°, & h_{32}-H_8=155.0 \\ \text{Column 5, opposite } 68°, & h_{68}-h_{32}=\underline{\ \ 36.1} \\ \text{adding} & h_{68}-H_8=191.1 \end{array}$$

191.1 is the number of Btu per pound required.

Washington, D. C., February 23, 1915.

THE EMISSIVITY OF METALS AND OXIDES

IV. IRON OXIDE

By George K. Burgess and Paul D. Foote

I. INTRODUCTION

The present paper is a continuation of the study of the radiometric properties of metals and oxides. The experimental methods and bibliography of the subject have been given in the preceding three numbers of the series.[1]

The object of this investigation has been the determination of the monochromatic emissivity for a wave length $\lambda = 0.65\mu$ and the total emissivity of the oxide of iron formed by heating iron in air.

The oxide thus formed is usually considered to be magnetite, but Messrs. Sosman and Hostetter,[2] of the Geophysical Laboratory, have found that actually it may be almost any combination of FeO and Fe_2O_3. This fact may account in part for slight changes observed in the radiometric properties of the oxide surface.

In the present investigation the iron upon which the oxide formed contained about 0.12 per cent carbon. The following is a report upon the analysis of several samples of the oxide by J. R. Cain, of this Bureau:

> These oxides are magnetic and free from metallic iron (as shown by test with copper salt). Hence they are of the magnetite type. However, the material is not pure Fe_3O_4, inasmuch as the gain in weight on heating in air is greater than would correspond to the reaction: $2Fe_3O_4 + O = 3Fe_2O_3$, which represent the change normally taking place when magnetite is heated in air. There is, therefore, a large proportion of ferrous oxide, FeO. Assuming that only FeO and Fe_3O_4 are present, the composition

---

[1] Burgess and Foote, Bureau Standards Scientific Paper No. 224; Burgess and Waltenberg, Bureau of Standards Scientific Paper No. 242; Foote, Bureau of Standards Scientific Paper No. 243.

[2] The published paper will appear shortly.

may be calculated to be approximately 60 per cent FeO and 40 per cent Fe_3O_4. The radiating surface is not a single substance; at least FeO and Fe_3O_4 are present, and possibly solid solutions or compounds of these, with the possibility of existence of Fe_2O_3 on the surface directly exposed to air and of allotropic forms of the oxides or compounds within the investigated temperature range.

Iron oxide has proven an especially difficult material with which to obtain satisfactory measurements, on account of scaling and flaking from the iron surface upon which the oxide is formed. This oxide is extremely brittle and does not form in a tough thin coat, as in the case of the nickel oxide NiO. The latter oxide presents a smooth surface, showing some luster, while iron oxide is silky in appearance and pitted with minute depressions; its surface is entirely without luster—almost a dead matt. For this reason the emissivity is very high, the small depressions or granular structure producing the effect of a collection of minute black bodies. If a microscopic sample of the oxide is melted upon a platinum strip [3] and cooled below the melting point, the surface is then bright and smooth, similar to that of a polished crystal of magnetite. In this case the emissivity is much less. This type of surface, however, does not occur in general practice, and hence it was thought desirable to determine the correction necessary to apply to readings of optical and radiation pyrometers when sighted upon the type of surface of the oxide formed upon iron heated in the open. These corrections may therefore be used in technical practice when the temperatures of rails, ingots, etc., are measured radiometrically.

II. MONOCHROMATIC EMISSIVITY ($\lambda = 0.65\mu$)

A number of different methods were tried in the determination of the monochromatic emissivity. Iron tubes of various sizes and iron wedges did not prove satisfactory on account of the large temperature gradient through the oxide, flaking, and temperature variation of the surface. Measurements with the spectrophotometer, employed in a preceding paper, were not reliable. The only method which gave satisfactory quantitative results was that of the microscopic melts.[4] A small sample of the oxide formed by heating iron strips in air was flaked off and placed on a nickel or platinum strip heated electrically. Upon the oxide were placed minute samples of sodium chloride, sodium sulphate, and gold. The current through the heating strip was gradually increased until the standard sample melted, when at the instant of the melting the apparent temperature of the oxide surface was observed by a Holborn-Kurlbaum optical pyrometer, using light of wave length $\lambda = 0.65\mu$. The results are summarized in the following table:

---

[3] Burgess and Waltenberg, Bureau of Standards Scientific Paper No. 242.

[4] Burgess and Foote, Bureau of Standards Scientific Paper No. 224, p. 42.

TABLE 1

Emissivity of Iron Oxide by Microscopic Melts ($\lambda = 0.65\mu$)

Observations	Material	True temperature, ° C	Observed temperature, ° C.	Average deviation from mean	Emissivity observed
9	NaCl	801±1	801	±1.5	1.00
22	Na_2SO_4	884±1	882	±3	0.97
16	Au	1063±2	1058	±2	0.94

The emissivity E_λ is given by the equation:

$$\frac{1}{T} - \frac{1}{S_\lambda} = \frac{\lambda}{0.4343 C_2} \cdot \log E_\lambda$$

where T is the true absolute temperature, S_λ the observed absolute temperature, λ the wave length of the light in μ and $C_2 = 14450$.

Apparently the emissivity decreases slightly with increasing temperatures, as in the case of the oxide of nickel and other oxides. In the case of nickel oxide the curve, true temperature vs. E_λ for $\lambda = 0.65\mu$, was found to be a straight line in the range 800 to 1300° C. If it is permissible to extrapolate similarly the values obtained for iron oxide, neglecting the determination at 801° C, one obtains the following:

TABLE 2

Monochromatic Emissivity of Iron Oxide ($\lambda = 0.65\ \mu$)

True temperature, °C	800	900	1000	1100	1200
Emissivity	0.98	0.97	0.95	0.93	0.92

different times of heating or for different sized tubes by flaking and dropping off. The values represented by crosses were taken from the work of Burgess, Crowe, Rawdon, and Waltenberg[7] upon a section of a 100-pound rail cooling in the open. In this case the temperature under the oxide was determined by a thermocouple inserted in a small hole drilled parallel to the length of the rail and as near the outside as possible. The outside temperature of the oxide was measured by an optical pyrometer. It is therefore

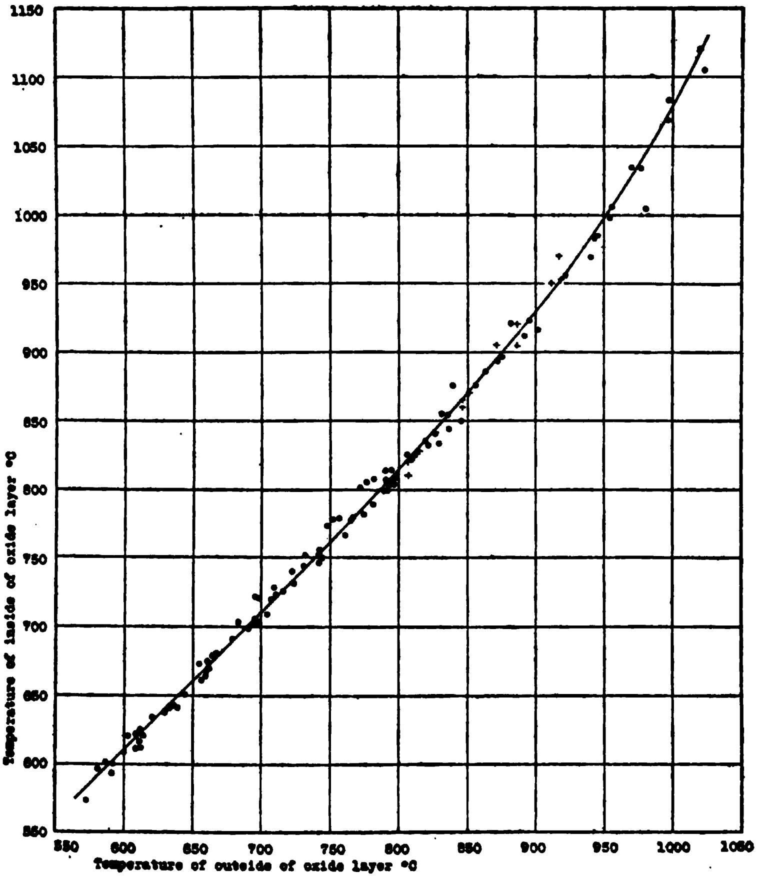


FIG. 2.—Temperature of outside of iron oxide layer versus temperature of inside of oxide layer

evident from a consideration of Fig. 2 that the temperature of the outside of the iron oxide is considerably less than that of the iron surface underneath.

---

[7] Burgess, Crowe, Rawdon, and Waltenberg, Bureau of Standards Technological Paper No. 38.

The large temperature drop from the metal surface to the outside of the oxide is due in part to the low thermal conductivity of the oxide and in part to what appears to be an actual air gap between the metal and the oxide caused by the peeling away of the oxide from the metal upon which it is formed.

VI. CONCLUSION

The monochromatic emissivity ($\lambda = 0.65\mu$) and the total emissivity of iron oxide formed by heating iron in air have been measured with an accuracy which is rather unsatisfactory, but which appears to be about as good as can be obtained when one considers the variable character of the surface, the constant scaling and flaking, and temperature variations occasioned by this flaking.

Iron oxide in the spectral region ($\lambda = 0.65\mu$) is almost "black," having an emissivity varying from 0.98 to 0.92 in the range 800 to 1200° C. The corrections necessary to apply to the readings of an optical pyrometer in this temperature range vary from 0 to 10° C. The total emissivity of iron oxide increases from 0.85 at 500° C to 0.89 at 1200° C. The corrections necessary to apply to the readings of a radiation pyrometer in this temperature range vary from 30 to 50° C.

The temperature of the outside of the oxide layer is considerably different from that of the inside in contact with the metal on account of in part the low thermal conductivity of the oxide and in part, undoubtedly, the actual separation of the outside oxide layer from the metal, thus forming an air gap between the two surfaces or between two surfaces of oxide, the outer one thick and the inner one thin. The drop in temperature through the oxide layer is approximately constant for various sized samples from small iron tubes to 100-pound rails, and increases rapidly with temperature rising to about 100° at an outside temperature of 1100° C.

WASHINGTON, March 31, 1915.

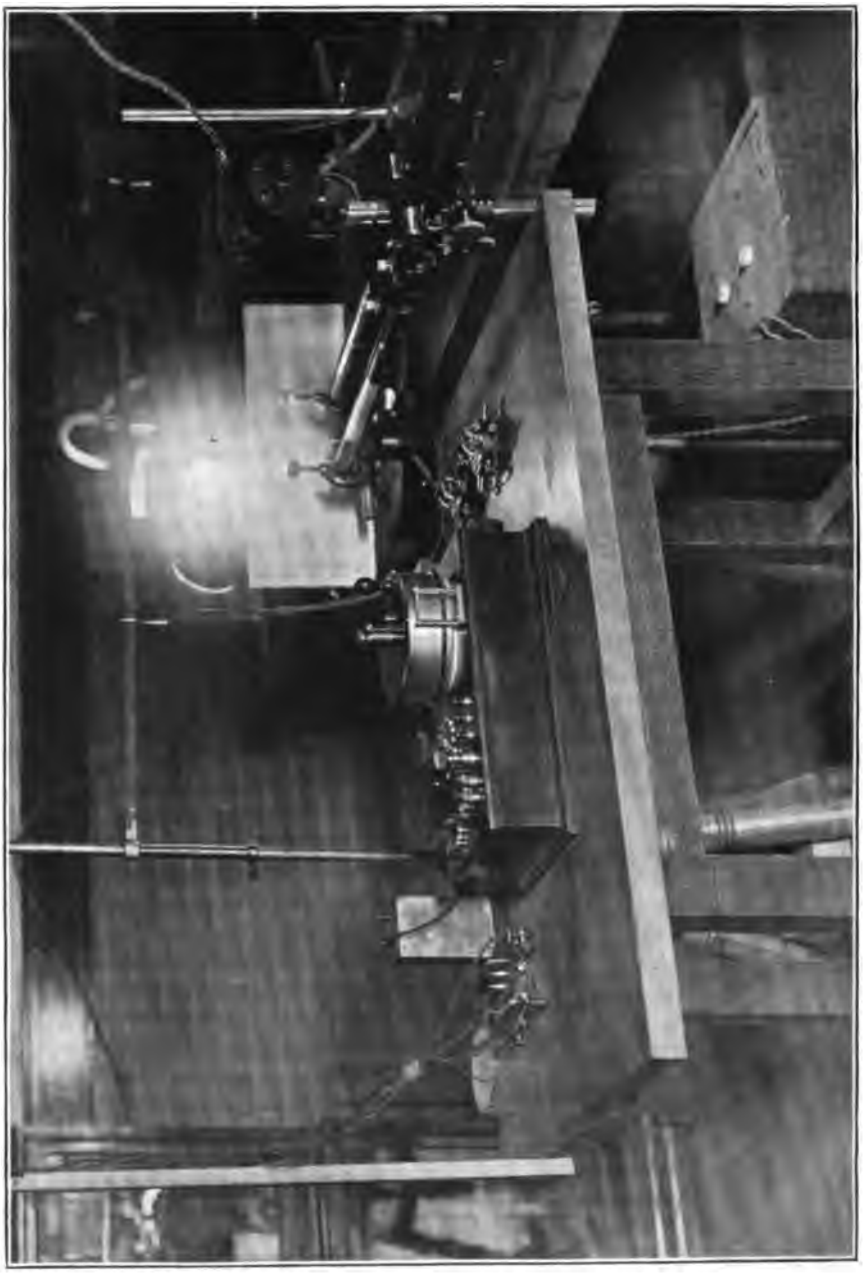


FIG. 24.—*Illustrating method of calibrating pyrometers by comparison upon a glowing nickel strip. The optical bench is used for varying the focusing distance*

CHARACTERISTICS OF RADIATION PYROMETERS

By George K. Burgess and Paul D. Foote

CONTENTS

Page
I. INTRODUCTION 92
II. PRINCIPLES OF RADIATION PYROMETRY 93
  1. Fundamental formulas 93
  2. Geometry of the radiation pyrometer 95
  3. The radiation pyrometer as a black receiver 99
  4. Thermoelectric receiver and galvanometer 104
  5. Statement of sources of error 107
III. TYPES OF RADIATION PYROMETER 109
  1. Mirror and thermocouple pyrometer 109
    (a) Adjustable focus 109
    (b) Fixed-focus pyrometers 114
  2. Mirror and spiral-spring pyrometer 115
  3. Lens and thermocouple pyrometer 117
  4. Cone thermoelectric pyrometer 119
  5. Notes on construction and behavior 119
IV. METHODS OF CALIBRATION 122
  1. Primary calibration 122
    (a) Graphite black body 122
    (b) Porcelain black body 125
  2. Secondary calibration 127
  3. Computation of calibration data 130
    (a) Effect of T_0 term 131
    (b) Determination of the exponent b 133
    (c) Calibration of Féry pyrometer with and without diaphragm 134
    (d) Effects of errors 134
V. METHODS OF USE 134
  1. Use with galvanometer or with potentiometer 134
    (a) Extrapolation on galvanometer scale 135
  2. Recording radiation pyrometers 137
  3. Methods of sighting on furnace 138
  4. Use of a radiation pyrometer with a source of insufficient size 140

	Page
VI. DISCUSSION OF ERRORS	143
1. Lag	143
(a) Causes of lag	145
2. Resistance of thermocouples used in radiation pyrometers and variation in resistance with temperature of the source	147
3. Effect of dirt and oxidation upon the condensing device	148
4. Effect of distance and size of source	150
(a) Féry mirror thermoelectric pyrometer	151
Mechanical construction	151
Convection currents	153
Stray reflection	153
Effect of size of image	154
Summary of focusing errors for a Féry pyrometer	160
(b) Féry fluorite lens pyrometer	161
(c) Foster fixed-focus pyrometer	161
(d) Thwing pyrometers	164
(e) Brown pyrometer	165
5. The temperature of the receiver	167
VII. APPLICATIONS	169
1. Determination of total emissivity of nonblack materials	169
2. The determination of temperatures	171
VIII. CONCLUSION	176

I. INTRODUCTION

Some 10 years ago the Bureau of Standards published an account of an investigation of the subject "Optical pyrometry," [1] in which were stated the principles involved in the measurement of high temperatures by optical methods and descriptions given of the instruments then in use, together with certain applications.

In the meantime there has been a marked development in the construction of instruments designed for the measurement of temperature, based on the use of the total radiation from hot substances. It is the object of the present paper to give an account of the principles of radiation pyrometry, considering mainly the various types of radiation instruments, their calibration, sources of error, including the various factors which may influence their indications, and their adaptability to the measurement of temperatures.

It will be shown that, when suitably designed, adequately calibrated, and correctly used there are available several trustworthy instruments of this type having many advantages in practical use. It will also be made clear that the radiation pyrometer is subject to many influences which may seriously limit the accuracy of its readings unless proper precautions are taken. Although some of the facts here emphasized are not unknown, it was considered desirable to give a rather complete discussion of the subject of radiation pyrometry even at the risk of some repetition of certain facts, the consequences of which have not always been sufficiently considered in pyrometric practice.

---

[1] Waidner and Burgess, Bureau of Standards Scientific Paper No. 11.

The discussion has, however, been limited, in the main, to those types of instruments which are portable and which are in general use as pyrometers, although it is recognized that there are other special energy receiving and measuring devices which have been of great service in the laboratory and which could be or have been adapted to the measurement of temperature.

The experiments here described and many of the problems attacked are the outcome of the accumulated experience of the pyrometric laboratory of the Bureau of Standards, and, therefore, represent in part the development of suggestions coming from others, including, in particular, Messrs. Waidner, Buckingham, and Kanolt.

II. PRINCIPLES OF RADIATION PYROMETRY

1. FUNDAMENTAL FORMULAS

The radiation pyrometer in the forms constructed at present is a temperature-measuring instrument, primarily based upon the idea involved in the Stefan-Boltzmann equation of energy transfer by radiation from a black body:

$$J=\sigma T^4=\int_0^\infty J_\lambda d\lambda \qquad (1)$$

Where J is the total amount of energy radiated per unit area and unit time from a black body[2] at an absolute temperature T, and

---

[2] The expression "black body" is synonymous with "complete," "total," or "integral" radiator or absorber of radiant energy. The absorption coefficient of a black body is unity; i. e., it absorbs all the radiation falling upon it, whatever be the intensity or wave length of this radiation. The absorption of radiation of any wave length will change the temperature of the black body and alter the spectral distribution of the emitted radiation in a definite manner, the spectral distribution of the energy absorbed having only an indirect relation to the spectral distribution of emitted energy.

σ is an empirical constant the value of which depends only on the units of measurement.

If a black body of area A_1 and absolute temperature T_1 is radiating to a black body of area A_2 and absolute temperature T_2, the two surfaces being parallel, in alignment, and distant apart by R, and if T_1 is considerably greater than T_2, and if A_1 and A_2 are small and R is large, the energy transferred by radiation from surface A_1 to surface A_2 in unit time is given by the following expression:

$$J=\frac{\sigma}{\pi}\left(T_1^4-T_2^4\right)\frac{A_1\,A_2}{R^2} \tag{1'}$$

If σ is determined in watts cm^{-2} deg^{-4}, the quantity J represents the energy transferred per unit time, in watts.

The total radiant energy of a material is represented by vibrations of all possible wave lengths from 0 to ∞. The energy associated with waves of lengths lying between λ and $\lambda+d\lambda$ may be expressed by J_λ, and for a black body the distribution of the energy in the spectrum, $J_\lambda=f\ (\lambda)$, is satisfactorily given by the Wien-Planck relation:

$$J_\lambda=C_1\ \lambda^{-5}(e^{c_2/\lambda T}-1)^{-1} \tag{2}$$

It would be possible to construct a radiation pyrometer which made use of either of the two displacement laws of Wien:

$$\lambda_{max}\ T=\text{constant} \tag{3}$$

$$J_{max}\ T^{-5}=\text{constant} \tag{4}$$

where λ_{max} is the wave length corresponding to the maximum intensity of radiation from a black body at an absolute temperature T, and J_{max} is the numerical value of the intensity at this wave length. Both of these laws have a theoretical as well as an experimental basis.

It would also be possible to construct a radiation pyrometer which utilized the radiation as heat confined in a narrow spectral region (equation 2), but the sensitiveness of such a spectral radiation pyrometer, as well as that of an instrument designed for locating the position or wave length of the maximum energy emission, or for the measurement of the intensity of this emission, at least in the forms practicable for the ordinary use required of pyrometers, is evidently very much less than that readily obtainable with the radiation pyrometers which use the radiant energy of the entire spectrum. These latter are often called total radiation pyrometers or sometimes total or integral receivers.

As distinguished from radiation pyrometers, instruments which are based upon the principle of estimation or comparison of radiation intensities in the visible spectrum, usually of a narrow spectral band, by means of the eye, are known as optical pyrometers. (See equation 2.) The characteristics and behavior of this type of instrument have been described elsewhere.[8] In what follows we shall consider only the behavior of total radiation pyrometers.

The energy received by a total radiation pyrometer may be measured in a variety of ways: calorimetrically, e. g., certain pyrheliometers; thermoelectrically, e. g., the thermopile; electrically, e. g., the bolometer; mechanically, e. g., the angular deflection of a bimetallic spiral spring or the elongation of a metallic strip; and radiometrically, e. g., the pressure of radiation exerted on delicate vanes mounted in vacuo, etc.

The thermoelectric and the mechanical (bimetallic spring) methods are the only total radiation methods which have been quite generally applied strictly for the purpose of temperature measurement.

The quantity of energy a body receives by radiation from another body depends on certain conditions relative to each of the two bodies, viz, area of surface, distance apart, emissive and absorbing power, and temperature.

2. GEOMETRY OF THE RADIATION PYROMETER

Let us first consider the question of the areas of the source and the pyrometer receiver. In general it is desirable to have the area of the receiver very small, partly for the purpose of requiring a small area of the radiating source, a matter of great practical importance. The effective sizes of both the receiver and source may be decreased merely by the use of diaphragms suitably placed, but usually, for practical purposes in such a diaphragmed system, the amount of energy received by the pyrometer would be too minute to be measured satisfactorily by either the thermoelectric or mechanical method. Some device is accordingly necessary for condensing the energy at the surface of the small receiver. This may be accomplished by locating the receiver at the focal point of a convex lens or concave mirror or at the apex of a hollow reflecting cone.

---

[8] Waidner and Burgess, Bureau of Standards Scientific Paper No. 11.

A fundamental condition, which should be satisfied by a radiation pyrometer, is invariability of the pyrometer reading when the distance from the source to the receiver is altered. This requires that the solid angle of the cone of rays entering the receiver remain constant. Such a condition may be secured by a proper arrangement of diaphragms.

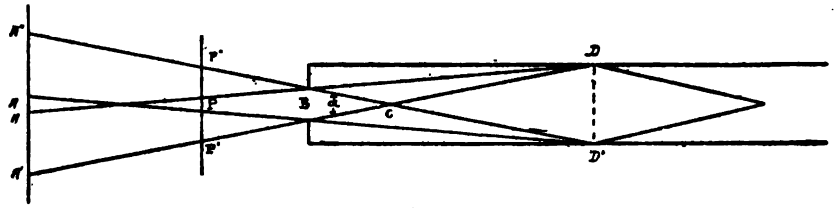


FIG. 1.—*Cone type of fixed focus radiation pyrometer*

The cone receiver is made independent of distance to the source, *A*, by locating at a short distance in front of the cone a small diaphragm opening, *B*, which acts as a secondary source, as shown in Fig. 1.

The amount of radiation falling on any element of area at *D* is proportional to the solid angle *HDA'*, which is independent of the distance from the point *D* to the source. This is true of any point on the base of the cone *DD'*. Hence, the total quantity of energy entering the cone is independent of the distance from the pyrometer to the source, provided the source is of sufficient size. The minimum-sized source for any distance is determined by the lines $A''\,D'$ and $A'\,D$, which intersect at *C*, forming the angle α, called the aperture of the instrument. Thus, for the distance *CA*, the diameter of the source must be at least $A'\,A''$, for the distance *CP*, the diameter must be $P'P''$, or for any distance x measured from the point *C*,

$$\text{Least diameter of source} = \frac{x\ (\text{diameter of diaphragm } B)}{BC}$$

When the device for condensing the energy upon the receiver is a concave mirror or a convex lens, two different designs of the instrument are possible.

1. The receiver and source are located at the conjugate foci of the mirror or lens.

2. The receiver and a front diaphragm of the pyrometer are located at the conjugate foci of the mirror or lens.

In case No. 1 the pyrometer must be provided with a focusing arrangement, whereby the distance from the lens or mirror to the receiver may be varied, depending upon the distance from the pyrometer to the source. It is possible to so construct the pyrometer that, as far as the geometrical optics is concerned, the readings of the instrument do not change with distance from the source to the pyrometer, Fig. 2.

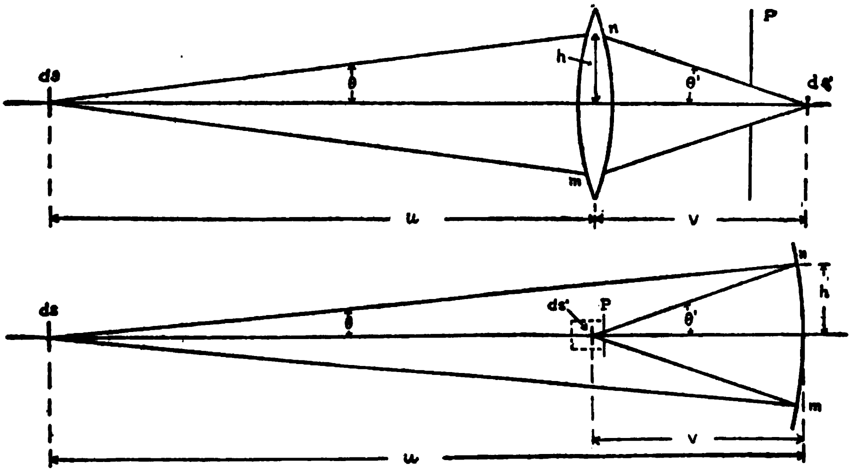


FIG. 2.—*Mirror and lens types of variable focus radiation pyrometer*

Let ds' = the area of the receiver. (This is fixed by the mechanical construction of the pyrometer. For example, it may be either a small receiving disk fastened to the thermocouple, or a limiting diaphragm immediately in front of the receiver and practically coincident with it.)

ds = the section of the source of which ds' is the image.

u = distance from source to mirror or lens.

v = distance from image to mirror or lens.

The total energy in the cone *mdsn* is given by the following equation:

(1) $J = 2\pi ids \int_0^{\theta} \sin\theta \cos\theta \, d\theta = \pi \, ids \sin^2\theta$, where i is the intensity of radiation of the source.

By the Helmholtz reciprocity theorem one may write a similar expression for the image space:

(2) $$J' = \pi \, i'ds' \sin^2\theta'$$

If there is no loss of radiation by absorption and reflection, (1) and (2) are equivalent. Therefore, since by the condition of magnification of the optical system $ds/ds' = u^2/v^2$, the intensity i is equal to the intensity i'. When radiation is lost at the lens or mirror by absorption (and reflection also in the case of the lens), i' is less than i, so that $i' = Ki$, where K is to all practical purposes a constant independent of u or v.

Hence, $J' = K \pi \, ids' \sin^2\theta' = i \cdot \text{constant} \cdot \sin^2\theta'$, since ds' is fixed in size. But J' represents the amount of radiation absorbed by the receiver, and this can be independent of the focusing distance only when $\sin^2\theta' = h^2/v^2 = \text{constant}$.

The angle θ', and hence $\sin^2\theta'$, is made constant by locating a properly-sized exit pupil, P, at a certain distance from ds'. When this is done, the energy falling upon the receiver is proportional only to the intensity of the radiating source. The readings of the pyrometer will then be independent of the focusing distance, provided the source is of sufficient size that its image completely covers the receiver ds'. The size of the opening P is determined such that for all focusing distances or variations in v the cone *mds'n* always intersects the mirror or lens surface.

The lens, or mirror (neglecting absorption and lost reflection), produces exactly the effect upon the receiver which would be obtained without the lens or mirror if the source having the size of the exit pupil P were located at P, or a source represented by *mn* located at the position *mn*, or a source of diameter $2x \cdot$ tangent θ' located at x, where x is the optical distance of the receiver from the source. It is obvious that a great advantage is gained by the use of a condensing device in diminishing the size of source required, especially at great focusing distances.

In case No. 2, the front diaphragm B, of the pyrometer and the receiver b, are placed at the conjugate foci of the lens or mirror DD', as illustrated in Fig. 3.

These latter instruments are of fixed focus, the front diaphragm B acting as a secondary source. The smallest-sized source which can be used without necessitating correction to the readings of the pyrometer must have a diameter $A'A''$ such that,

$$\text{Diameter of source} = \frac{x \text{ (diameter of diaphragm } B)}{BC}$$

where x is the distance from the source to the point c. The angular aperture of the instrument is α.

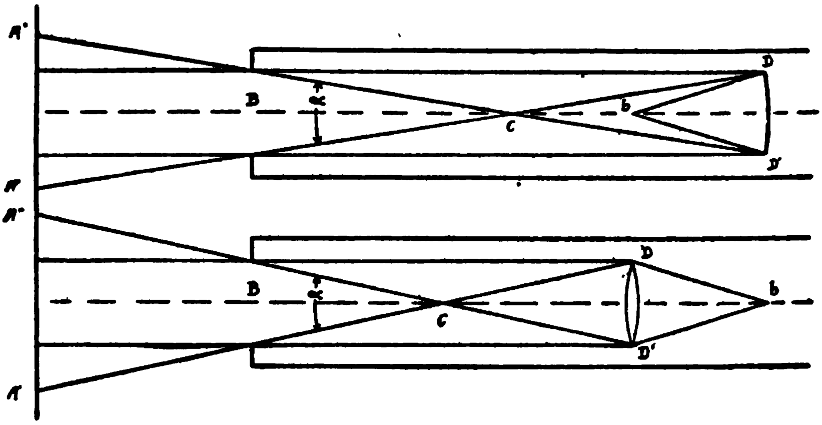


FIG. 3.—*Mirror and lens type of fixed focus radiation pyrometer*

3. THE RADIATION PYROMETER AS A "BLACK" RECEIVER

Energy receivers may be divided into three classes, as follows: (1) A "black" receiver is one which absorbs all the energy falling upon it and reflects none, whatever be the wave length of the incident radiation. Its absorption coefficient is accordingly unity. (2) A "gray" receiver is one having an absorption coefficient which is independent of the wave length of the incident radiation, the value of the coefficient being less than unity. (3) A "selective" receiver is one having an absorption coefficient which is a function of the wave length of the incident radiation.

If a radiation pyrometer is to be employed for the direct measurement of apparent temperatures of nonblack substances (see Sec. VII), it is essential that the receiver be either black or gray.

It is practically impossible to obtain reliable results with a selective receiver, as may be seen from the following ideal and extreme example: Suppose a material emitted selective radiation in a region confined exclusively between $\lambda = 1\mu$ and $\lambda = 2\mu$. A pyrometer having a selective receiver which absorbed radiation of wave lengths 3 to 4μ only would show no reading whatever when sighted upon this material, while one for which the region of selective absorption lay between 1 and 2μ would present a maximum indication. It is apparent that selective receivers can be adapted to the measurement of apparent temperatures on nonblack bodies only when the value of the reflection coefficient for a single wave length, R_λ, never becomes 1 (a condition which actually is always fulfilled), and when one possesses a complete knowledge of the equations $R_\lambda = f(\lambda)$ and $E_\lambda = f(\lambda, T)$, where E_λ is the emission coefficient of the nonblack body. Even assuming that these two functions are known for a given material, which is far from being true, corrections of an exceedingly complicated nature would then have to be applied to every reading unless some method of empirical calibration were devised, this empirical calibration of course being made for each and every material sighted upon.

Consequently, it is desirable to know under which type of receiver the ordinary radiation pyrometer may be classed. As an example, let us consider the Féry mirror instrument, one which apparently may show some deviation from the ideal black receiver. The receiving disk of this pyrometer is thoroughly blackened, and the observations of Coblentz[4] would indicate that a properly blackened surface is practically nonselectively absorbing, the absorption coefficient for all wave lengths being approximately 0.99, when the radiation is normally incident. The gold mirror, however, is selectively reflecting for wave lengths shorter than 1μ, as may be seen from the S-shaped curve of Fig. 4. The values of R_λ, which refer to normal incidence upon a gold surface, were taken from the Smithsonian tables and from recent papers of Hagen and Rubens, Tool, and Tate. The reflection coefficient for $\lambda = 0.4\ \mu$, is less than 0.30, rising to 0.97 at $1\ \mu$. From $1\ \mu$ to the extreme infra red ($25\ \mu$) the coefficient is more nearly constant, slightly increasing from the value 0.97 to 0.98_8.

---

[4] Coblentz, Bureau of Standards Scientific Paper No. 196.

It is desired to consider what effect the selective reflection of gold in the region $\lambda < 1\ \mu$ has upon the reading of the pyrometer, when the instrument is sighted upon a black body at various temperatures. If R denotes the fractional part of the total energy reflected by the gold surface when radiation from a black body falls normally upon the surface, and R_λ the ordinary monochromatic reflection coefficient of a gold surface at room temperature, for radiation at normal incidence, the value of R is given by the ratio of the product of R_λ and the Planck spectral distribution function J_λ, integrated from $\lambda = 0$ to $\lambda = \infty$ to the integral of J_λ between the same limits.

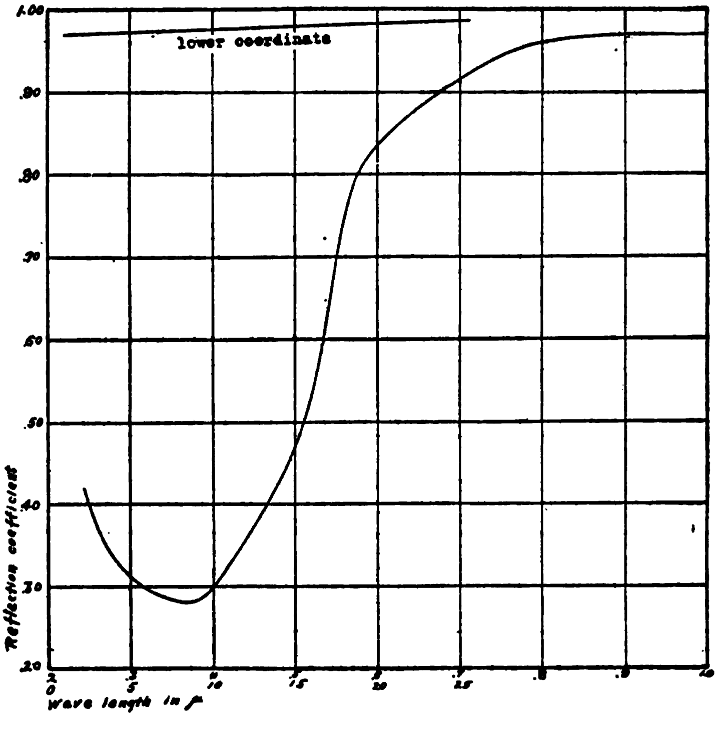


FIG. 4.—*Reflection coefficient of gold as a function of the wave length*

$$R = \frac{\int_0^\infty C_1 R_\lambda \lambda^{-5} (e^{c_2/\lambda T} - 1)^{-1} d\lambda}{\int_0^\infty C_1 \lambda^{-5} (e^{c_2/\lambda T} - 1)^{-1} d\lambda}$$

The above expression was evaluated mathematically, but since the function R_λ is difficult to express in a form which permits a simple computation of R, the integration is better performed graphically. Accordingly, curves were drawn of $R_\lambda J_\lambda$ vs. λ and of J_λ vs. λ and integrated by means of a planimeter. The ratio of the two areas thus obtained is the value of R for any given temperature of the black body. The values of R obtained in this manner are represented by Fig. 5. It is apparent from this curve that for a variation in temperature of the black body up to about 2500° C, the selective reflection of gold in the visible spectrum does not alter the amount of total energy emitted by the black body and reflected from the gold surface by more than 2 per cent (equivalent to a possible maximum error in absolute temperature measurement of five-tenths per cent, if the pyrometer were calibrated at low temperatures only). That a larger error is not incurred is of course due to the fact that the greater part of the energy incident upon the mirror is comprised of wave lengths outside the region

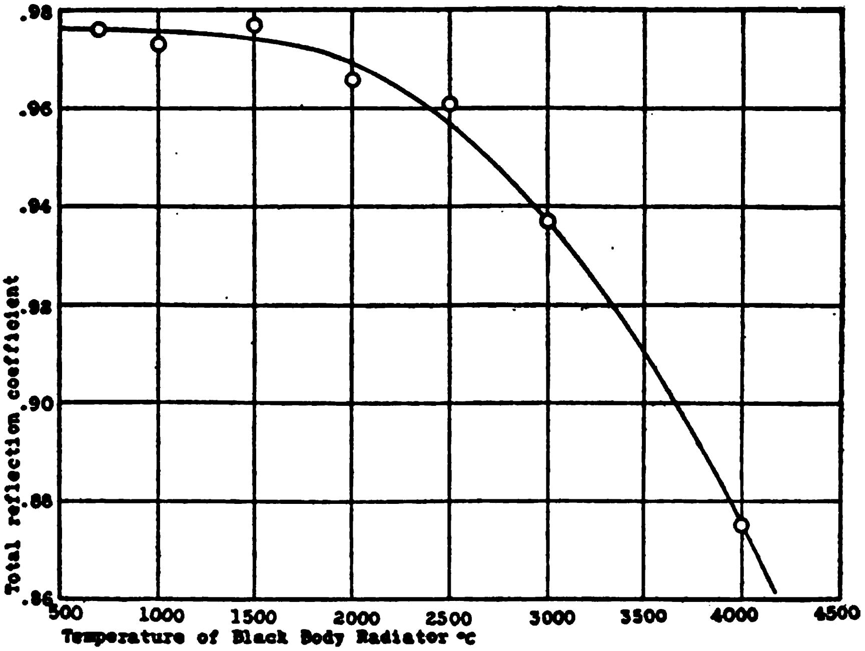


FIG. 5.—*Total reflection coefficient of gold*

of selective reflection of gold. Accordingly, when the pyrometer is sighted upon a black body at temperatures less than 2500° C, the instrument may be considered a gray receiver, approximately 3 per cent of the energy incident upon the mirror being lost by absorption at this surface (dissipated by conduction, convection, and radiation to the outside case), and an additional 1 per cent being lost by reflection at the blackened thermocouple disk. A gray receiver is obviously just as satisfactory for radiometric purposes as an ideal black receiver, the only effect of the nonblackness of the gray receiver being a constant percentage diminution in the effective energy absorbed.

A Féry mirror pyrometer, however, calibrated at temperatures in the region 500° to 1500° C could not be used to determine the apparent temperature of the sun, for example, without a careful consideration of the corrections for the selective reflection of the gold, because the maximum portion of the solar energy is confined to the region of the selectivity of gold. The apparent temperatures measured would be far smaller than the true apparent temperatures. Even at a temperature of the black body of 4000° C the gold mirror reflects but 87 per cent of the total energy falling upon it, and from the slope of the curve of Fig. 5 it is seen that at the sun's temperature (say 6000° C) the reflection coefficient will be somewhere in the neighborhood of 60 to 70 per cent. A reflection coefficient of 68 per cent means an error in temperature measurement of some 500° at 6000° C. Without doubt a silver or platinum mirror would be found much more satisfactory than the gold mirror for these high temperatures, but even these materials could not be used without applying large corrections.

In general, nonblack radiating materials at the temperatures ordinarily available in technical or laboratory work (less than 2000° C), even when selectively radiating, emit the greater portion of their energy in wave lengths outside the region of the selective reflection of gold. The radiation pyrometer accordingly acts as a gray receiver and may be employed for the determination of the total emissivity of these substances. In the case of gold and copper at high temperatures, however, the selective emission is high in the visible spectrum and low in the infra red. Since the selective reflection of the gold mirror of the Féry pyrometer is low in the visible spectrum and high in the infra red, the total emissivity of gold and copper determined by the Féry instrument would be too small, and the pyrometer can not in this case be considered a gray receiver. The pyrometer may, of course, be empirically calibrated to read the true temperatures of glowing copper or gold, or a table may be prepared for correcting the apparent temperatures measured by the Féry to true temperatures, but these apparent temperatures are not those which one would obtain with a black or gray receiver.

4. THERMOELECTRIC RECEIVER AND GALVANOMETER

The proper design of the thermoelectric receiver and indicating galvanometer of a radiation pyrometer for technical use is quite different from that of a thermopile as usually employed for radiometric investigation in the laboratory. In the case of the latter instrument experience appears to show that the most satisfactory working conditions are obtained when the resistance of the galvanometer is approximately equal[5] to that of the thermopile. In general the thermopile has a low resistance—less than 10 ohms—so that in order to obtain a galvanometer resistance of this magnitude the entire galvanometer electrical circuit is usually constructed of copper. Since copper possesses a large temperature coefficient of resistance, the sensibility of this type of galvanometer will vary with change of room temperature. Also changes in the resistance of the thermopile and lead wires due to fluctuations of room temperature will seriously alter the galvanometer readings. Accordingly, while such a system may be suitable for laboratory use, where a calibration may be made frequently, it is not to be preferred for technical work. The galvanometer readings, therefore, should not be influenced by changes of room temperature. Furthermore, the galvanometer should have a short period, be dead beat or suitably damped, and have a readily controlled zero, as well as be sufficiently robust and sensitive.

Of late the design of moving coil or d'Arsonval galvanometers has progressed so rapidly that even with the portable pivot type suspension coils the sensibility is so high that a swamping resistance of zero temperature coefficient may be placed in series with the coil, and still the instrument will be of sufficient sensitivity for pyrometric practice. The swamping resistance is located inside the galvanometer, so that in what follows the resistance of the galvanometer is understood to include this resistance, as well as that of the coil; in other words, it is the resistance measured across the galvanometer binding posts. The effect of the swamping resistance is to reduce to practically zero the temperature coefficient of the galvanometer [6] and the dependence of the deflection upon variations in resistance of the thermocouple circuit. The elimination of the effect of change of resistance in the thermocouple and lead wires is of extreme practical importance. It is also desirable that the galvanometer read true emf's of the pyrometer and that it will still read correct emf values when used with pyrometers of different resistances. All of these conditions are satisfied when the resistance of the galvanometer is high in comparison with that of the couple and leads, as is evident from the following discussion:

---

[5] This condition need not be satisfied exactly, variations of 50 to 200 per cent being permissible, cf. Coblentz, Bureau of Standards Scientific Paper No. 229, p. 145.

If R_1 is the resistance of the thermocouple and the lead wires, R_2 that of the galvanometer, and e_0 the true emf of the thermocouple, then the emf, e, indicated by the galvanometer, becomes $e = e_0 \frac{R_2}{R_1+R_2}$. If the galvanometer has been properly calibrated to read the emf at its terminals, the value e_0, actually indicated by the galvanometer when connected to the pyrometer, will be too low by the factor $R_2/(R_1+R_2)$. It would be possible, of course, to allow for this factor in the calibration of the instrument if R_1 remained fixed. But even though R_1 varies considerably in a single pyrometer, and especially for different pyrometers, the effect of this variation may be swamped by making R_2 at least 100 times R_1. The factor then becomes constant for all practical purposes and nearly equal to 1. A number of galvanometers are available which are sufficiently sensitive for use with a radiation pyrometer and which have a high resistance, 100 to 500 ohms, and a negligible temperature coefficient, are robust and portable, and highly satisfactory for technical purposes.

---

[6] Other devices are also employed for this purpose, such as varying automatically the strength of the magnetic field, cf Thwing, J. Frank. Inst., 165, pp. 363-370; 1908. Various shunting arrangements in the electrical circuit are also employed.

The thermoelectric power of the couple should be as high as possible, but since the couple must be robust and able to resist mechanical shocks which it is likely to receive, certain couples showing extremely high thermoelectric power can not be used. Iron-constantan, copper-constantan, and several other metals and alloys, some of secret composition, have been employed satisfactorily.

The couple should be of very small wire and rather long in order that heat conduction from the hot to cold junction be a minimum. The cold junction should be carefully screened from radiation from the furnace and the entire couple thoroughly protected from air currents. In general the hot junction has a small metal receiving disk soldered to it, although it is sometimes possible to simply flatten the couple at the junction in order to form a satisfactory receiver. The heat capacity of the receiving system should be small in order to have a quick-acting pyrometer. The receiver should be carefully blackened, so that very little of the energy falling upon it is reflected. The resistance of the couple should be small in comparison with that of the galvanometer. The size of the receiver should be small in order to obtain by the mirror or lens condensers a sufficiently large image without requiring too large a source. And, finally, the hot junction and receiver must not be overheated. It would be possible to actually melt the receiver by the use of a too highly condensing system of mirrors.

The following statement summarizes the essential conditions to be satisfied by the well-constructed thermoelectric radiation pyrometer. Some of the conditions are somewhat contradictory, in which case an adjustment must be made. For example, it might be necessary in a given case to sacrifice low resistance of the couple in favor of high emf.

Galvanometer.—(1) High resistance (this refers to the resistance across the terminals of the instrument); (2) negligible temperature coefficient; (3) satisfactory sensibility; (4) robust and portable; and (5) short period and critically damped.

Receiving system.—(1) High thermoelectric power; (2) low resistance; (3) robust mechanically; (4) the couple wires and the receiving disk small in size; (5) cold junction screened from radiation and air currents; (6) heat capacity small (quick acting, i. e., no lag); (7) black receiver; and (8) maximum temperature attained by receiver should not be too great.

Pyrometer as a whole.—(1) Indications independent of distance from the source to the pyrometer; (2) indications independent of the size of the source above the minimum size required by the instrument; (3) size of source required by the instrument, small; and (4) pyrometer strong mechanically and convenient to handle.

The use of the thermoelectric radiation pyrometer and the characteristics of the mechanical type receiver will be discussed later in the paper.

5. STATEMENT OF THE SOURCES OF ERROR

The errors which may occur in total radiation pyrometry may be classified as follows: (1) Limitations or approximations of the fundamental formulas; (2) imperfections of the radiating source or uncertainties in its radiometric properties; (3) effects of the intervening medium, i. e., air more or less charged with water vapor and gases such as CO and CO_2; (4) construction of the pyrometric receiver; and (5) errors of the measuring or recording instruments. Of these, No. 1 can be exactly determined and allowed for; No. 2 can usually be eliminated when suitable precautions are taken; No. 3 is uncertain, but, in general, is small; No. 4 is of the greatest practical interest and will be considered in detail; and, finally, No. 5 may readily be made negligible by a suitable choice of instruments.

In the ideal radiation pyrometer, the energy J, received from the radiating source at an absolute temperature T, by the receiver at an absolute T_0, is proportional to the factor $(T^4 - T^4_0)$, viz,

$$J = \text{const.}\ (T^4 - T^4_0),$$

as follows directly from the Stefan-Boltzmann radiation law (equation 1, p. 93). Various factors enter, however, into the actual construction of the radiation pyrometer which slightly alter this ideal relation. For example, consider the thermoelectric type of radiation pyrometer, in which the energy of the radiator is indirectly measured by the emf developed in a thermoelectric circuit. Here, the emf developed is not exactly proportional to the temperature of the receiver, and the temperature of the receiver is not exactly proportional to the energy received. While the proportionality relation may exist to all practical purposes over a very small temperature range of the radiating source, e. g., $T = 1000$ to 1001, etc., it does not hold for the wide variation in emf or in T which is met with in practice.

Again, mechanical defects in construction, some of which appear impossible to remedy, may cause deviations from the ideal condition. Stray reflection, selective reflection, and convection currents in the pyrometer must necessarily vary in magnitude, depending upon the temperature of the radiating source. The temperature of the hot junction of the thermocouple, T_0, increases necessarily with T^4, the relation being but approximately linear. The loss of energy expressed as a fraction of the energy incident at the receiver is entirely different for different values of T_0, not merely because of changes in radiation from the receiver, but mainly because of the different rates of energy loss by conduction and by convection currents, i. e., departure from Newton's law of cooling.

For these reasons the radiation pyrometer does not follow exactly the Stefan-Boltzmann radiation law. The deviation from this relation, however, is so small that a quite similar expression can be used. Thus, for the thermoelectric radiation pyrometer the relation between the emf developed, e, and the absolute temperatures of the radiating source, T, and of the receiver, T_0, is expressed by the equation,

$$e = a\ (T^b - T^b_0)$$

where a and b are empirical constants. The basis for this equation is mainly experimental. Usually it is possible to neglect the T_0 term at higher values of T, so that $e = aT^b$. This expression may be written $e = (aT^4)\ T^{b-4}$, where the term T^{b-4} is a small correction factor necessary to take care of the departure of the instrument from the ideal conditions. This form of equation gives to the performance of the pyrometer a somewhat theoretical significance.

If, in the above equation, b is arbitrarily chosen of value 4, the calibration computed from the theoretical Stefan-Boltzmann relation, on the basis of the determination of the emf at one standardization temperature, will in general be greatly in error.

Among the sources of error to which the various forms of total radiation pyrometer are subject are:

Lag, or the time required for the pyrometer to come to equilibrium and so give a constant reading when sighted on a hot body. For certain instruments this effect may vary with distance and size of source as well as with temperature.

For the thermoelectric receivers, the resistance of the thermocouple wires and its variation with the temperature of the radiator.

Dirt or dust on and oxidation of the optical system.

The effect of distance of pyrometer to radiator, as influenced by absorption of the intervening medium and by the angular size of the radiator.

Optical imperfections of the pyrometer and its geometry as related to that of the radiator or source.

Lack of "blackness" of the source and of the pyrometer.

These various sources of error will be considered in detail in Section VI, after describing the usual types of instrument and their calibration.

III. TYPES OF RADIATION PYROMETER

The important characteristics in the construction of the several radiation pyrometers available will be briefly considered.

1. MIRROR AND THERMOCOUPLE PYROMETER

(*a*) ADJUSTABLE FOCUS.—This type of instrument is represented by the Féry thermoelectric pyrometer, Fig. 6.

The image of the source sighted upon must be sharply focused, by adjusting the focusing pinion, upon a limiting diaphragm immediately in front of the thermocouple receiver. This adjustment is greatly facilitated by means of an ingenious device due to Féry. Two semicircular mirrors, inclined to one another at an angle of 5° to 10°, Fig. 7 (a), are mounted in the thermocouple box, an opening of about 1.5 mm at the center of the mirrors forming the limiting diaphragm immediately in front of the couple. The observer views the image of the furnace formed by the large concave mirror *MM'* and reflected by the focusing mirrors *xx'* and *yy'* through the small telescope at *D*. If the image is not correctly focused at

O, the line of intersection of the two small mirrors, the image appears broken in half, as shown by (*b*). Correct focus is obtained when the two halves of the image are in alignment (*c*). Thus, until correct focus is obtained certain straight lines of the source appear broken in the image. This breaking of a line is quite evident from a consideration of (*d*).[7] Suppose that the pyrometer were incorrectly focused upon a line source, the image falling at position *AB* instead of at *O*. The image of the line reflected from the mirror *yy'* lies at *A''B''* and that reflected from the mirror *xx'* at *A'B'*. To the observer at *D*, the projections of these images appear as two distinct arrows, thus, → →. As the pyrometer is brought nearer into the correct focus, by turning

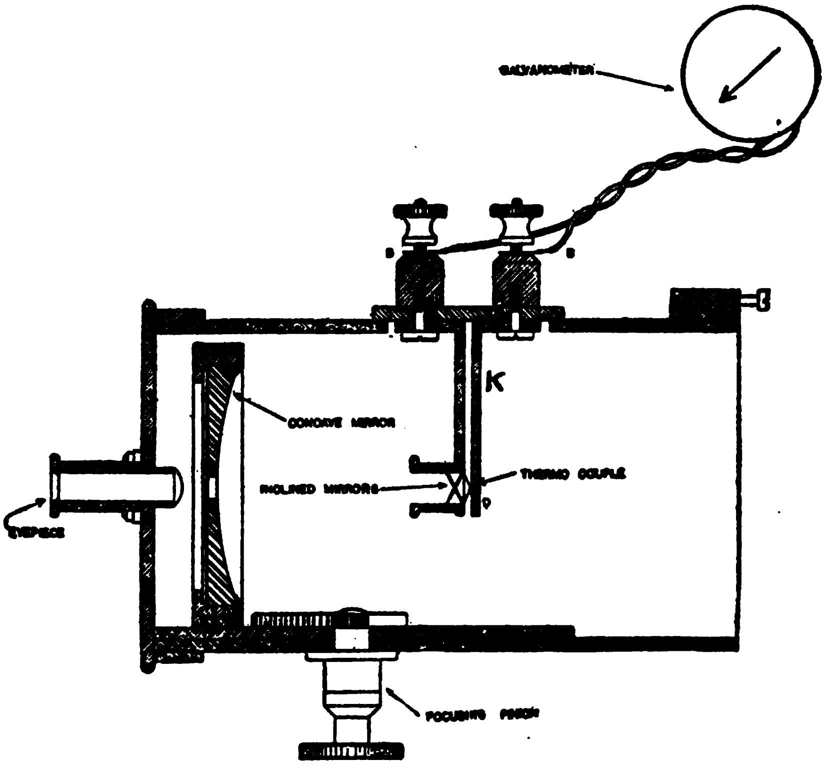


FIG. 6.—*Féry thermoelectric mirror type radiation pyrometer.*

---

[7] This drawing has been distorted intentionally, for the purpose of more clearly illustrating the principle rather than the actual course of the rays in a given instrument.

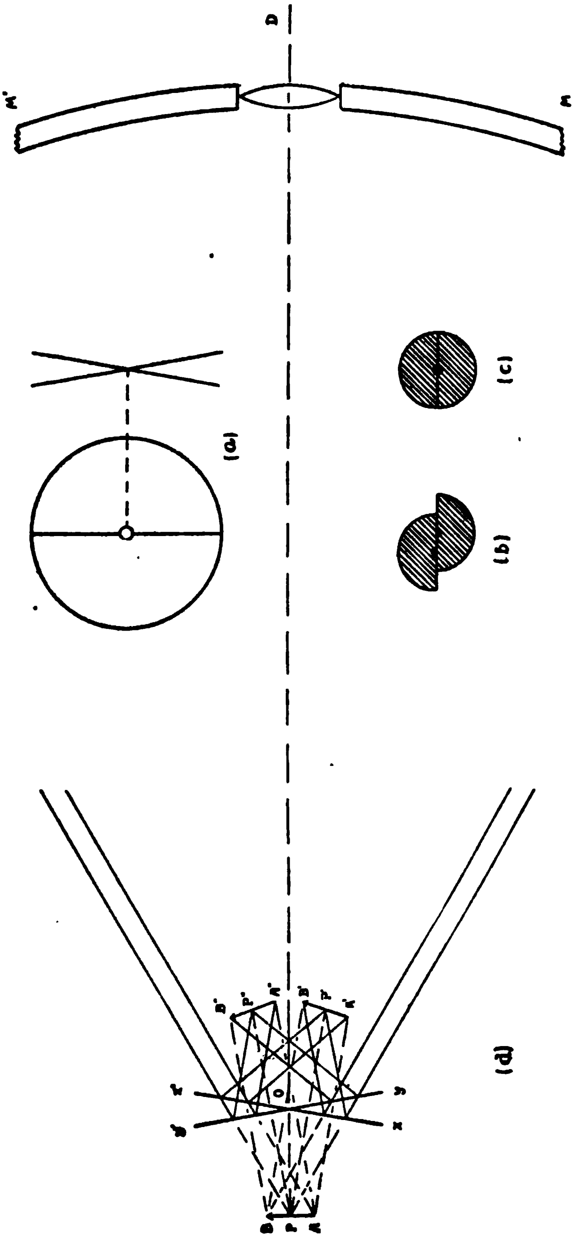


FIG. 7.—Optical system of the Féry mirror type radiation pyrometer

the pinion screw and thus moving the large concave mirror in the direction OD, the points P' and P'' of the reflected images move along the lines $P'O$ and $P''O$, coinciding at O when the correct focus is obtained. The two arrows are then superposed, forming a single image.

In the earlier type of the Féry pyrometer the large concave mirror was constructed of glass silvered on the outside surface. On account of the oxidizing of a silver surface in air the silver was later deposited on the back of the mirror. Glass itself is a very good reflector for the infra-red rays, hence, with a mirror silvered on the back, the heat rays are reflected in part from the front air-glass surface and in part from the back glass-silver surface. These two groups of rays are brought to the same focal point by making the radii of curvature of the two surfaces slightly different. For all practical purposes, however, the same radius of curvature may be used for each surface if the distance between the front and back surfaces of the glass be small, i. e., 1 to 2 mm.

In some of the later types of the instrument gold mirrors have been employed. Until recently the gold has been deposited upon the front surface of glass by a special "burning-in" process similar to glazing of porcelain[8] but such surfaces show large local aberrations, so that at present an electrolytic deposit of gold upon copper is preferred. Even these surfaces show pitting and local aberration to some extent. The small focusing mirrors are either of glass silvered or gilded or of copper gold plated. The concave mirror is usually about 6.8 cm diameter, 7.5 cm focal length, and 15 cm radius of curvature. The thermoelement is usually iron *vs.* constantan or Heil's alloy *vs.* constantan.

This type of pyrometer is generally used at a minimum temperature of 500° or 600° C., although it would be possible to empirically calibrate the instrument for a much lower range, provided the indicating device were made sufficiently sensitive to measure the small emf's developed at low temperatures. The upper temperature range is limited, by the excessive heating of the receiving system, to about 1500° C. However, the cover to the front of the pyrometer is provided with a sectored diaphragm which may be adjusted to reduce the radiation falling upon the receiver by any desired amount, and in this manner the upper temperature range of the instrument is practically unlimited.

---

[8] Fery, Revue Scientifique, (5), 8, pp. 264-272; 1907.

The pyrometer, without the front diaphragm, develops an emf of usually several millivolts at 1000° C., the magnitude of course depending upon the construction of the individual instrument.

It has been objected[9] that this type of pyrometer requires a large source upon which to sight, and as generally used such is the case. But it is quite simple to reduce the size of the limiting diaphragm of this pyrometer, just as Gillette reduced the aperture of the Thwing, and thus permit much smaller sources to be employed, at a sacrifice of course in both cases, of the magnitude of the emf developed. Such a procedure can not be called an "improvement of the pyrometer,"[10] but it is rather an adaptation of the instrument for some special work. The diameter of the opening in the receiving diaphragm of the Féry pyrometer is usually about 0.15 cm. The relation between the diameter of source D_s, diameter of image D_i, and focal length of the concave mirror, f, is given by the expression:

$$D_s = D_i\left(\frac{u}{f} - 1\right)$$

where u is the distance from the source to the mirror. Thus, for a mirror of focal length 7.6 cm, the following table shows the diameter of source required for several focusing distances in order that the image cover the 0.15 cm receiver. If the receiver had an opening of half this magnitude sources of half the diameter indicated would be required.

u	Diameter of source
cm	cm
70	1.2
80	1.4
100	1.8
150	3.1
200	4.2
300	6.3
500	10.7

---

[9] Gillette, J. Phys. Chem., 15, p. 274; 1911.

[10] Gillette, J. Phys. Chem., 15, p. 302, item 8; 1911.

The selection of a satisfactory sectored diaphragm for use with the Féry pyrometer at higher temperatures is rather difficult. Originally a blackened diaphragm was used, but this absorbed a considerable amount of energy and reradiated from the back face to the receiver. To overcome this effect a gold-plated diaphragm is employed. However, reflection from this gold surface back to the source may alter the properties of the radiation of the source as noted on page 155. This effect is in general of small, if any, consequence, but for certain special work it must be considered. Certainly the gold-plated diaphragm is to be preferred to the blackened diaphragm unless the latter were water-cooled.

(*b*) FIXED FOCUS PYROMETERS.—The front diaphragm of the Foster pyrometer (diameter 2.5 cm), which acts as a secondary source, is placed about 51 cm in front of a mirror of 2.5 cm diameter and about 2.4 cm focal length. The diameter of the source sighted upon must be at least one-tenth the distance from the source to the wing nut at the middle of the pyrometer tube. The mirror is usually of glass gilded on the front surface. The deposit of gold is very thin and may be removed by touching with the finger. Gold-plated copper mirrors would be more satisfactory. The method of mounting the thermoelement is shown in Fig. 8.

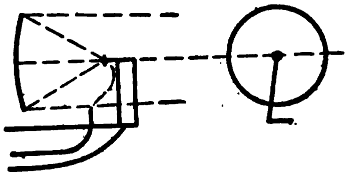


FIG. 8.—*Method of mounting thermocouple in the Foster pyrometer*

The Brown pyrometer is similar in principle to the Foster pyrometer, the distinguishing feature being the construction of the tube or case, which is made collapsible in sections for convenience of carrying. The mirror is of glass silvered on the front surface, having a focal length of 2.2 cm. The thermocouple is rather large, and could be improved by reducing the size of the wires and possibly style of mounting. A camera "finder" for convenience of sighting is mounted on the tube casing, but this, however, is not always properly adjusted. The couple is mounted about 76 cm from the front opening of the pyrometer. The arrangement of diaphragms is poor, each end of each tube section of the case acting as a diaphragm. As a result points on the outer edge of the mirror are screened almost entirely from the radiation entering the front opening of the pyrometer. However, considering the extreme cone of rays which can fall upon the receiver, it may be seen that the diameter of the source must be at least one-twelfth the distance from the source to a point 29 to 30 cm back from the front opening of the pyrometer.

2. MIRROR AND SPIRAL SPRING PYROMETER (FÉRY "SPIRAL" PYROMETER)

The construction of this instrument is quite similar to that of the thermoelectric type except that the couple is replaced by a bimetallic spring spiral, Fig. 9, carrying an aluminum pointer *P*, which turns over a dial *D*, graduated in degrees centigrade, in response to the differential expansion and uncoiling of the spring when radiation is concentrated upon it. In some of the instruments the spring is trimetallic, the thermal expansion coefficient of the intermediate metal being itself intermediate to that of the two outer metals. Thus, gold, platinum, and invar have been employed. The spiral is similar to that used in the metallic thermometer of Breguet. A strip 0.02 mm thick and 2 mm wide is coiled by several turns into a spiral 2 mm in diameter. The center of the spiral is connected by a shank to a small disk and on this disk is usually mounted the pointer. (In Fig. 9 a slightly

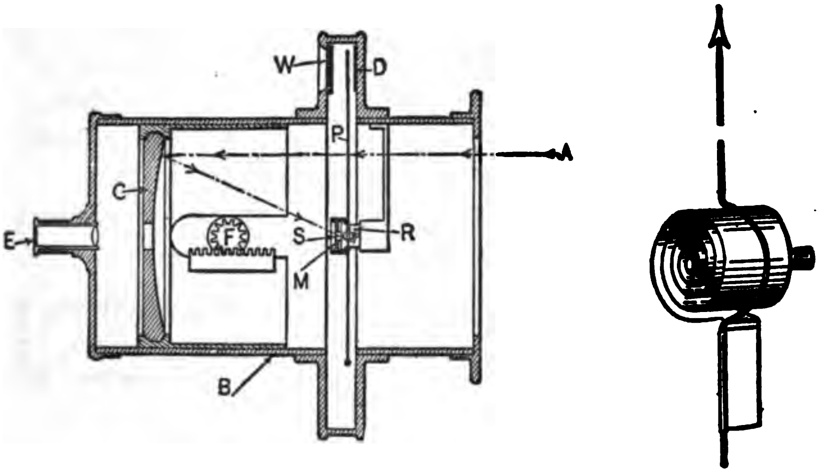


FIG. 9.—*Féry spiral pyrometer*

different mounting is shown. The shank is fixed and the pointer is mounted at the other end of the spiral.) Usually a mirror is placed behind the spiral so that the radiation which passes through and between the turns of the spiral is reflected back upon it.

It is of interest to consider the method of spacing of the temperature scale engraved on the instrument. Suppose that the scale is first spaced linearly or in terms of angular deflection of the pointer:

Let d = angular deflection.
T_0 = absolute temperature of spiral.
T = absolute temperature of furnace.
J = energy falling upon spiral.

The angular deflection of the pointer is approximately proportional to the temperature of the spiral; the temperature of the spiral is approximately proportional to the energy absorbed by it; this energy is approximately proportional to the fourth power of the absolute temperature of the furnace; or,

$$d \propto T_0 \propto J \propto T^4$$
$$\therefore\ d = \text{const} \cdot T^4$$

Hence, determining the deflection corresponding to any one furnace temperature fixes the constant in the above relation and permits the computation of the temperatures corresponding to all other deflections.

Actually, the pyrometer does not exactly follow the fourth-power law but rather the relation:

$$d = c \cdot T^b$$

where b is an empirical constant slightly different from 4. If a calibration is made at a number of different temperatures, the exponent b may be determined from the slope of the best straight line drawn through the observations, plotting log d vs. log T.

The spiral pyrometer has an especial advantage in being self-contained, requiring no accessories such as lead wires, galvanometer, etc., but its accuracy is not equal to that of the thermoelectric instruments. (See p. 146.) The readings depend somewhat upon the position in which the pyrometer is held and upon the previous condition of the instrument. For example, tilting the case to the right or left alters the reading, and slightly different readings may be expected when (1) the pyrometer has been sighted upon a source at a higher temperature immediately before taking a certain reading, and (2) when the initial source sighted upon was at a lower temperature.

3. LENS AND THERMOCOUPLE PYROMETER (FÉRY LENS THERMOELECTRIC PYROMETER)

Fig. 10 illustrates the construction of the laboratory type of Féry lens pyrometer. The image of the furnace is focused upon the junction of an iron-constantan thermocouple located behind the shield *C*. In the center of the shield there is a small opening of less than 1 mm which admits the radiation to the thermocouple junction. The image is viewed at the ocular *O* through the sectional openings of the shield. The ocular is first carefully focused upon the thermocouple junction, and the objective is then adjusted by the pinion *P*, either for distinctness of image or, preferably, until the effect of parallax is eliminated.

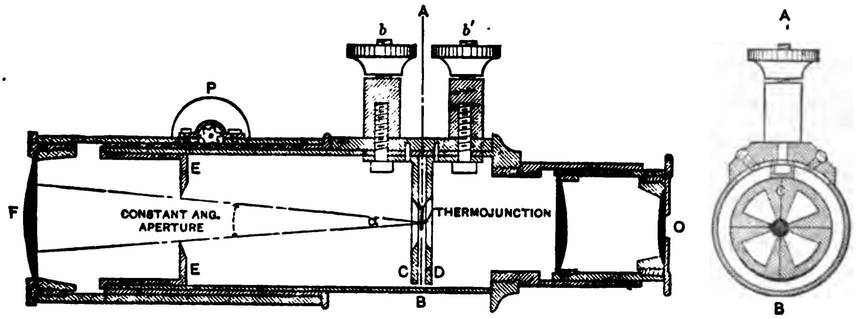


FIG. 10.—*Fery lens type thermoelectric radiation pyrometer*

The difficulty in construction of this pyrometer is in realizing a material for the lens which is transparent for all radiations, so that the pyrometer may be calibrated directly in terms of the modified Stefan-Boltzmann law. This is effected by the use of a fluorite lens, which for temperatures above 900° C satisfies the conditions of not altering seriously the radiations transmitted through it; that is to say, the ratio of the radiation absorbed or reflected to the radiation transmitted is approximately constant.

At low temperatures a large proportion of the energy exists in the form of long wave lengths, and as fluorite has an absorption band in the infra-red (near 6 μ) it will absorb a considerable portion of the radiation (at still longer wave lengths fluorite exhibits metallic reflection), and therefore the Stefan-Boltzmann law can no longer be assumed. With a sufficiently sensitive galvanometer or potentiometer this pyrometer could, however, still be empirically calibrated for low temperatures.

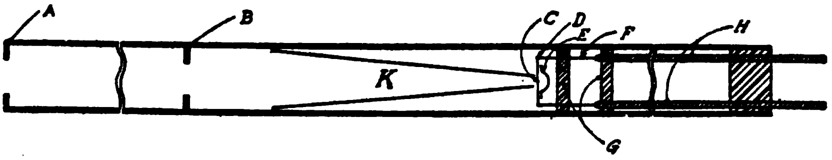


FIG. 11.—*Receiving system of Thwing cone pyrometer*

A, Front diaphragm; *B*, Center diaphragm; *C*, Thermocouple hot junction; *D*, Thermocouple cold junction; *E*, Hemispherical mirror; *F*, Heavy copper wire; *G*, Hard rubber plates; *H*, Brass rods and connecting terminals; *K*, Hollow aluminum cone.

The laboratory form of apparatus described above is not well suited for use in technical practice, and fluorite is difficult to procure of sufficient size, and, moreover, is extremely costly. An industrial pyrometer is made by substituting for the fluorite lens a much larger one of glass, and for the delicate galvanometer one of the same type and sensibility as is used in thermoelectric work; the resulting instrument is robust and sufficiently sensitive for all practical uses, and as made has a range from 800° to 1600° C, although the upper limit could readily be extended by the use of sectored diaphragms or rotating sectored disks.

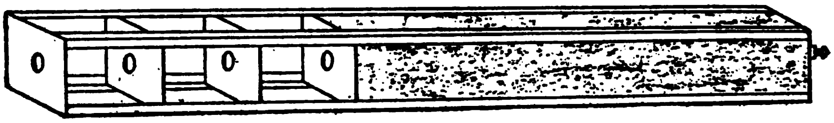


FIG. 12.—*Ventilated box type of Thwing pyrometer*

The indications of the industrial form of this pyrometer will not obey Stefan's law, but the instrument may be readily calibrated empirically and the temperatures engraved on the scale of the pyrometer galvanometer.

The emf developed by the fluorite lens instrument is of the order 50 microvolts at 1000° C. Emf's of this magnitude may be conveniently and accurately measured in the laboratory by use of a specially designed potentiometer.

4. CONE THERMOELECTRIC PYROMETER

The construction of the Thwing pyrometer is essentially that discussed on page 96. Fig. 11 represents a section of the receiving system. Thwing has also designed a ventilated type receiving tube or box in which the diaphragms are open to the air, Fig. 12. Air circulation prevents the heating of the diaphragms, so that this instrument may be employed for permanent installation. Also, as will appear later, stray reflection is reduced to a minimum in this type of construction.

The aperture of the Thwing tube pyrometer is usually about 3.5°, requiring a source of diameter at least one-fifteenth of the distance measured from the source to a point 19 cm back of the front diaphragm, i. e., to the point midway between the diaphragms *A* and *B*, of Fig. 11, but, of course, any desired aperture may be obtained by altering the size or distance apart of these diaphragms. The emf developed by the usual Thwing pyrometers is much smaller than that of the Féry-mirror, Foster or Brown instruments, having the order of about 0.2 to 0.5 millivolts at 1000° C and about 1 millivolt at 1500° C.

5. NOTES ON CONSTRUCTION AND BEHAVIOR

All of the above-described pyrometers are provided by the makers with indicating galvanometers. Their proper design and construction are also of importance. The question of galvanometers and other points arising in the use and construction of these pyrometers are discussed on pages 104 and 134.

(*a*) In several Féry pyrometers examined the mirror support or carriage has proven unsatisfactory. The carriage should be rigid, so that the mirror is always perpendicular to the axis of the pyrometer. In one instrument the play was sufficient to cause a large deviation in the reading when the pyrometer was used in different positions. This is due to the difficulty of centering the image upon the receiver when the mirror is tilted or out of alignment.

(*b*) The blackening used inside the tube case of the Thwing pyrometer is not as satisfactory as that used in the Brown or Foster instruments. On the other hand, the Brown and Foster pyrometers could be improved by cutting a screw thread inside the tubes, such as that used in the Thwing.

(*c*) What is the effect of locating the thermocouple receiver of the Foster or Brown pyrometer at a position slightly different from the focus conjugate to the front opening? The result of this shift in position of the receiver may be a decrease in emf, but geometrically the reading of the instrument is still independent of the distance from a sufficiently large source. As a particular example, consider the following case:

$u = 50$ cm = distance of front diaphragm to mirror.
$v = 5.6$ cm = distance of receiver to mirror.
$f = 5.0$ cm = focal length of mirror.
$d_o = 0.2$ cm = diameter of receiver.
$d_s = 1.79$ cm = diameter of opening in front of diaphragm.
$d_m = 2.5$ cm = diameter of mirror.

d_s is the image of d_0, and the size of d_0 determines the limiting size of the front diaphragm of the pyrometer.

Suppose the same size receiver were incorrectly located 6 cm instead of 5.6 cm from the mirror, all diaphragms and other construction remaining the same, then:

$v = 6$ cm = distance from receiver to mirror.
$u = 30$ cm = distance from mirror to image of receiver.
$d_s = 1$ cm = diameter of image of receiver.

The source may now be considered as lying at the point $u = 30$, and having a diameter of 1 cm. The front diaphragm, however, will be seen to act as a stop, preventing radiation from all parts of this new secondary source from reaching all parts of the concave mirror. This results in a decrease in the amount of energy received by the couple.

The solid angle subtended by the mirror at the receiver decreases when the receiver is moved from $v = 5.6$ to $v = 6$ cm. This also results in a decrease in amount of energy falling on the couple.

If the couple receives radiation directly on its front surface, the amount of energy thus received increases when the couple is moved from $v = 5.6$ to $v = 6$.

The net result of these three effects is a decrease in the reading of the pyrometer when the couple is located too far from the mirror.

Actually, stray reflection and the aberrations of the mirror seriously modify the effects described in the above statements, so that frequently the couple may show the highest emf when located at a position slightly different from the focal point conjugate to the front diaphragm. The best adjustment is probably obtained by experiment, but whatever be the location of the receiver, if approximately near the ideal position, the readings of the pyrometer will be no more dependent upon the distance to the source than when the receiver and front diaphragm are exactly at the conjugate foci of the concave mirror.

(*d*) It is frequently stated that great care must be taken to prevent heating of the front diaphragm of the fixed focus pyrometers on account of the expansion of the opening, and as a consequence an increase in the amount of energy falling upon the receiver. The front diaphragm should not become heated for reasons which will be discussed on pages 161–167, but any error arising from expansion is negligible. Thus, suppose the front diaphragm is constructed of brass and is uniformly heated to 125° C, or to 100° above room temperature. The increase in area is proportional to twice the linear coefficient of expansion and to the rise in temperature.

$$\text{Hence, increase in area} = 2 \times 0.000019 \times 100$$
$$= 0.0038 \text{ or } 0.38 \text{ per cent.}$$

If all this excess energy transmitted by the increased opening fall upon the receiver, it would result in an error of less than 0.1 per cent in absolute temperature—i. e., about 2° at 2000° C. Actually, the front openings of the Foster and Brown are larger than the size required by the image of the receiver, so that a slight increase in the diameter of the opening merely increases the diameter of the image of the opening formed at the receiver, and thus affects very little the amount of energy reflected from the mirror and falling upon the receiver itself. The radiation coming directly to the side of the receiver opposite the mirror, forming a very small proportion of the total energy received, would theoretically be increased by an increase in the size of the diaphragm. For these two instruments, therefore, barring stray reflection, the possible increase in emf of the receiving thermocouple due to expansion of the front diaphragm would probably not be greater than 0.001 per cent in the case cited, or, expressed in temperature, would be an inappreciable amount.

IV. METHODS OF CALIBRATION

1. PRIMARY CALIBRATION

The primary calibration of a radiation pyrometer is made by sighting upon a black body. This is realized experimentally by uniformly heating the walls of a hollow opaque inclosure and utilizing the radiation coming from the inside through a very small opening. Since, in general, the radiation pyrometer requires a rather large source at a convenient sighting distance, the problem of the construction of a suitable black-body furnace is much more difficult than is the case with one designed for calibrating an optical pyrometer of small aperture. Two slightly different types of black-body radiators have been employed in this work, one constructed of graphite and the other of Marquardt porcelain. (See also Sec. V, 3.)

(*a*) GRAPHITE BLACK BODY.—A solid Acheson graphite cylinder 16 by 8 cm is turned on a lathe into the form of a crucible with three diaphragms, as illustrated by *D*, Fig. 13.

The walls of the three inclosures thus formed are about 1 cm thick, each inclosure having the dimension of about 5 by 6 cm. This radiator is mounted in an electrically-heated resistance furnace of the Heraeus type, 60 by 8 cm inside dimensions. The back wall of the graphite radiator is placed 40 cm from the front opening of the furnace, this position giving the most satisfactory temperature distribution as determined experimentally. The temperature of the black body is measured by two thermocouples *E* and *F*, protected by glazed Marquardt porcelain tubes inserted into the back wall of the graphite cell and projecting 1 or 2 cm into its interior. The back of the furnace is closed with a kaolin diaphragm, 2 or 3 cm of powdered Marquardt (*G*) and a plug of asbestos wool. In front of the cell are located two or more kaolin or Marquardt porcelain diaphragms, which aid in mitigating air circulation. As a further aid in preventing air circulation it is sometimes advantageous to incline the entire furnace at an angle of about 30°. The furnace tube is of Marquardt porcelain wound with platinum tape 5 cm wide. An auxiliary heating coil 15 cm long wound with 1 cm platinum tape has been placed in the front part of the furnace, considerably improving the temperature distribution. If the pyrometer to be calibrated is of the focusing type, it is placed at any convenient distance *A*, say 100 cm from *D* and usually focused on the opening of the inmost diaphragm. This opening is made of sufficient diameter so that its image just covers the receiver. The sizes of all other diaphragms are then determined geometrically by drawing the cone of rays to the pyrometer as shown in the figure.

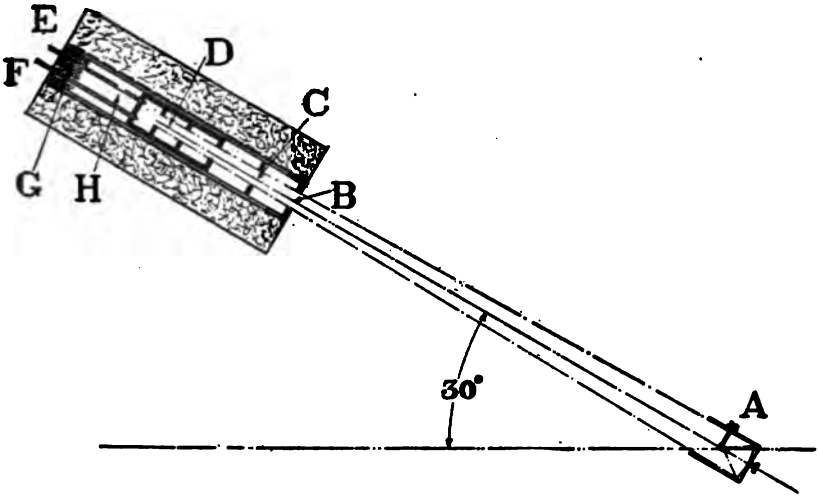


FIG. 13.—*Graphite black body and electric furnace*

Graphite has a high thermal conductivity, and hence the temperature distribution over the black body is easily made quite satisfactory; with practically no special adjustment of the auxiliary heating coils, variations of temperature greater than 5° C at 1300° C will not occur over a 10-cm length of the graphite cell.

Graphite in the open has an emissivity very nearly that of a black body. When constructed in the present form of a series of three diaphragmed hollow inclosures, one would expect the radiation from the inner inclosures to be about as "black" as could be realized experimentally. The ends of the porcelain thermocouple tubes, having a very low emissivity in the visible spectrum, are indistinguishable from the surrounding graphite walls at 1300° C. Optical pyrometers, which have been calibrated by sighting into a Lummer-Kurlbaum black-body furnace[11] or with a graphite black body totally immersed in pots of molten metal,[12] the latter method furnishing almost ideal black-body conditions, will give temperatures agreeing with the indications of the thermocouples

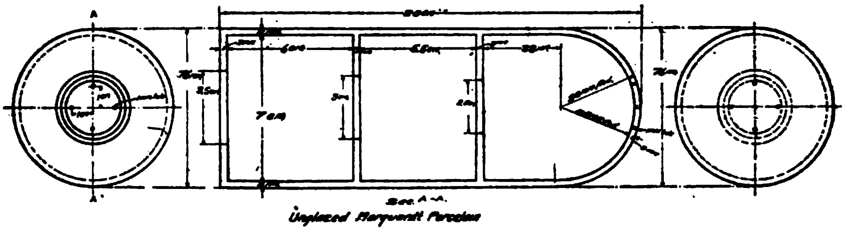


FIG. 14.—*Porcelain black body*

E and *F* to within ±5°, when sighted on the inner diaphragm *D*. And finally the two inner diaphragms of the graphite black body are easily made indistinguishable to the eye from each other or from the openings of the diaphragms. All these conditions would indicate that black-body conditions have been satisfactorily met.

Graphite, however, usually contains impurities in small quantities, but sufficient to corrode the furnace and even contaminate the thermocouples, protected in glazed tubes, after prolonged use. The graphite oxidizes rather quickly even in an atmosphere quite free from drafts, and the black body crumbles away and frequently collapses after several days' continuous heating. There is also some question relative to the effect of the combustion products CO and CO_2 upon the indication of the pyrometer, although it is not likely that serious error can occur from this source, since one would expect the hot CO and CO_2 gases to radiate in the region of their absorption.

---

[11] Waidner and Burgess, Bureau of Standards, Scientific Paper, No. 55, p. 165.

[12] Kanolt, Bureau of Standards Technologic Paper, No. 10, p. 8; Foote, Metallurgical and Chemical Engineering, **11**, pp. 97–98, 1913.

(*b*) Porcelain Black Body (Fig. 14).—With the idea of eliminating any possible effect of furnace gases, and in order to do away with corroding effects of the impurities in the graphite, a black body has been constructed of Marquardt porcelain. The diaphragms are of thinner walls but similarly arranged, as in the case of graphite black body. The inner radiating inclosure was made hemispherical in order to secure reflection from the side walls, it being thought possible that such construction might be of some advantage. The thermocouples are used without the protecting tubes and are threaded through small holes in the back of the radiator. This type of radiator also furnishes satisfactory black-body conditions and is somewhat more convenient to use than the graphite form, although the temperature distribution is not quite as uniform, apparently, on account of the much lower thermal conductivity of porcelain. It is possible for a black body of this type to emit radiation of even greater intensity than that of a perfect black body at the temperature of the thermocouples on account of reflection from the somewhat hotter side walls. Whatever undesirable reflection takes place may be minimized by blackening the interior with nickel oxide. However, Coblentz[13] from spectrobolometric measurements has observed practically no difference between a white porcelain diaphragmed black body and

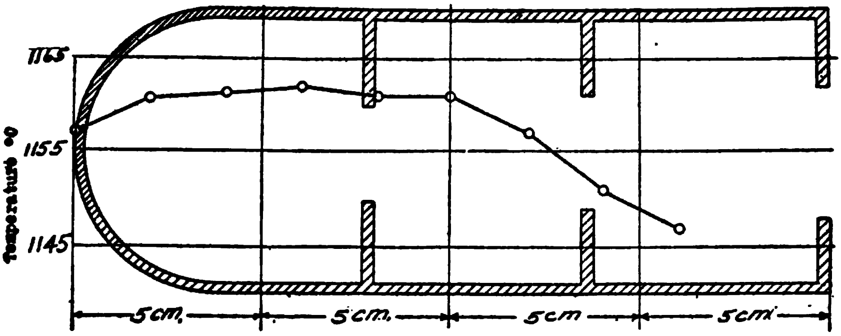


Fig. 15.—*Curve showing temperature distribution in porcelain black body*

the same radiator blackened. If any difference exists, it appears to be too small to be detected by the radiation pyrometers so far examined.

---

[13] Coblentz, Bureau of Standards Scientific Paper No. 204.

In Fig. 15 is given a curve illustrating the temperature distribution through a portion of the porcelain radiator. The temperatures were obtained by exploration with a thermocouple. For a distance of 12 cm the variation from 1160° is in no place greater than 3°. No special care was taken to secure high temperature uniformity, this distribution representing a case picked at random during an ordinary commercial test. With special attention given to the adjustment of the auxiliary heating coil, the variations can be somewhat reduced.

The thermocouples used in the measurement of the temperature of either type black body are of Pt – 10 Rh 90 Pt, and are carefully calibrated from time to time in terms of the melting points of zinc, antimony, and copper. The parabolic equation $e = a + bt + ct^2$ is used for interpolation between the points 300° to 1200° C. Above 1200° C corrections must be added to the temperatures determined by extrapolation of this thermocouple scale, in order that this empirical scale conform with the established gas thermometer scale, as follows:

Temperature	Correction to add
°C	°C
1200	± 0
1300	+ 2
1400	+ 6
1500	+14

In the calibration of a radiation pyrometer the thermocouple temperatures are always checked by the readings of an optical pyrometer of the Holborn-Kurlbaum type, sighted from the position of the radiation pyrometer, into the black body.

By use of the above-described apparatus, the radiation pyrometer may be directly calibrated from 500° to 1500° C. Satisfactory observations above this temperature range have not as yet been obtained, though it is hoped eventually to develop a method for use at much higher temperatures.

The following table presents the calibration of one of the Bureau's Féry mirror-pyrometers No. 111 by (1) the graphite radiator and (2) the porcelain radiator. The differences in the calibrations by the two methods are less than the experimental errors involved; and while the deviations are all in the same direction this may readily be due to the pyrometer receiver itself, as will be shown in the discussion of the source of errors of pyrometers in Section VI. Although this individual instrument happens to be the least subject to the various errors affecting pyrometers of any of its type so far examined, one is not justified in using it to compare minutely the merits of the two radiators.

Féry Mirror-Pyrometer No. 111 (Emf vs. Temperature)

International millivolts	Degrees centigrade		Graphite-porcelain Δt
	Graphite radiator	Porcelain radiator	
0.5	638	633	+5
1.0	820	814	6
1.5	943	939	4
2.0	1040	1036	4
2.5	1119	1116	3
3.0	1187	1185	2
3.5	1248	1248	0
4.0	1304	1304	0
4.5	1354	1354	0
5.0	1400	1400	0
6.0	1484	1484	0

2. SECONDARY CALIBRATION (Illustrated in operation by Fig. 24)

The use of an electric furnace and black body of the type described above has two disadvantages: First, it requires several hours, at least 20 hours for 1500° C, in order to reach temperature uniformity and satisfactory black-body conditions, and second, the size of the radiating source is rather small for certain types of pyrometers.

Once having obtained a primary calibration upon a standard radiation pyrometer, the pyrometer to be tested may be compared with the standard by sighting upon a uniform source which is not emitting exactly black-body radiation, provided this radiation is not of a too selective nature.

A source which satisfactorily meets this requirement is obtained by electrically heating sheet nickel in air. A firm and uniform coat of black nickel oxide (NiO) forms on the surface of the nickel thus heated. Such a strip can be used almost indefinitely up to 1300° C, and rapid changes of temperature can be made from 500° to 1300° C. The cooling from 500° to room temperature must be done slowly, or flaking of the oxide will occur. In the present work a strip of nickel usually 17 cm long (exposed section), 13 cm wide, and 0.015 cm thick was mounted vertically between water-cooled brass-clamp terminals. The size of the strips was limited by the current necessary to operate them, 1500 amperes being the maximum current available with the 10-kilowatt transformer employed. A strip of this size furnishes a source of circular area and diameter of 12 cm, which is uniform to within 2° at 1200° C over its entire surface. A careful survey of an area 8 cm in diameter on a 12 by 14 cm strip by means of a Holborn-Kurlbaum optical pyrometer showed the following temperature distribution:

Zone radius	Temperature
cm	°C
0-1	1046.5
1-2	1046.0
2-3	1046.0
3-4	1045.5

The advantage gained in using a strip several cm longer than its width is marked. The temperature variation across the width of the strip is practically nil, the main variation occurring along the lower edge. Thus, the temperature gradient along a vertical section of the strip is not symmetrical, the bottom of the strip being cooler for several cm than the top. Using a strip 17 by 13 cm, the center of the 12-cm uniform temperature area is located 6.5 cm from either side, about 7.5 cm from the top, and 9.5 cm from the bottom.

Whereas heretofore it has been practically impossible to calibrate certain radiation pyrometers requiring a large source, with this type of radiator an intercomparison with a standard pyrometer can be satisfactorily made in an hour. This is illustrated by the following calibration (an ordinary commercial test) of a Foster radiation pyrometer. Comparison was made with the Féry pyrometer No. 111 at 18 different temperatures. All of the observations are here presented, and the precision of the method may be judged from the deviations of the readings from the mean curve computed and drawn through them. Any systematic errors, or errors in the calibration of the standard pyrometer, can not be determined from this data.

Foster Fixed-Focus Pyrometer No. 116

Observed emf millivolts	Observed temperature	Computed temperature	Δt computed −observed
	°C	°C	°C
0.584	554	558	+4
0.760	612	612	0
1.035	686	685	−1
1.300	743	742	−1
1.505	789	785	−4
1.740	828	826	−2
1.913	852	853	+1
2.009	865	863	−2
2.227	898	898	0
2.277	908	905	−3
2.553	944	942	−2
2.850	974	978	+4
3.137	1008	1010	+2
4.102	1107	1103	−4
4.712	1151	1153	+2
5.248	1197	1194	−3
5.655	1225	1223	−2
6.282	1257	1264	+7
Average deviation..................			2.4

In the use of this nickel-oxide source for the calibration of pyrometers it is essential that the instruments compared be of similar type, so that the departure from blackness of the strip will affect each pyrometer in the same manner. Large errors would be involved in the comparison of an optical and radiation pyrometer by this method unless the observations were corrected for the monochromatic and for the total emissivity of nickel oxide.[14]

---

[14] Burgess and Foote, Bureau of Standards Scientific Paper No. 224. This paper gives a complete discussion of these corrections.

3. COMPUTATION OF CALIBRATION DATA

The thermoelectric type of radiation pyrometer obeys the relation $e = a(T^b - T_o^b)$, where e is the emf developed when the pyrometer is sighted on a black body at an absolute temperature T, the temperature of the receiver being T_o, and a and b are empirical constants. (See p. 107.) In general, T_o^b is negligible in comparison with T^b, so that one may write $e = aT^b$. The constants a and b must be determined empirically for each instrument.

Although two calibration points serve to determine a and b, observations are usually made at five or more different temperatures, and the best curve is drawn through all the points. Since an exponential curve of the correct form is difficult to adjust graphically, the curve is rectified into a straight line by plotting log e vs. log T. Thus, expressed in log form, the equation for the pyrometer becomes,

$$\log e = \log a + b \log T$$

which is a linear relation between log e and log T, the slope of the straight line determining the constant b. It is at once seen that in the process of rectification points which should possess equal weight on the e vs. T plot can not have equal weight on the log e vs. log T plot. It is possible to determine the reweight factors of the points on the log plot, but in general sufficient accuracy is obtained by slightly favoring the points at the higher temperatures, in drawing the best straight line through the observations. It would also be possible to adjust the straight line on the log plot by least squares for a minimum of $\Sigma\delta \log e$, weighting each point differently to correct for the rectification, but the present accuracy of a radiation pyrometer does not warrant the procedure.

Having drawn the best straight line on the log plot, a plot or table of e vs. $t°$ C may be obtained by computation of the exponential equation, the values of a and b following directly from the log plot; or, more conveniently, by reading from the log plot the values of log e corresponding to various values of log T with $t (= T - 273)$, varying by, say, 50° intervals over the temperature range desired.

With the present equipment of the bureau a calibration may be experimentally determined to 1500° C. For the calibration of a pyrometer at higher temperatures the straight line of the log plot is linearly extrapolated. Since the equation of the radiation pyrometer, although somewhat modified empirically, is still fundamentally based upon the Stefan-Boltzmann relation, which admits of extrapolation over any range, inasmuch as it defines a temperature scale, it is not likely that serious errors can occur in the extrapolation of the equation of the radiation pyrometer over a few hundred degrees interval above 1500° C. This statement, however, may be questioned, and can only be verified experimentally.

(*a*) EFFECT OF THE T_0 TERM.—In the use of the above method of plotting the observations in terms of log e and log T, it is frequently noticed that the curve through the observed points departs somewhat from the linear relation at the smaller values of T, the curve tending to curl up at the lower end when log e represents the abscissa and log T the ordinate. This is due to the effect of the T_0 term in the expression $e=a(T^b-T^b_0)$. On taking the logarithm of this expression and expanding log $(T^b-T_0^b)$, one obtains:

$$\log e=\log a+b\left\{\log T-\frac{0.4343}{b}\left[\left(\frac{T_0}{T}\right)^b+\cdots\right]\right\}$$

$$\text{Let } p=-\frac{0.4343}{b}\left(\frac{T_0}{T}\right)^b$$

Then $\log e=\log a+b(\log T+p)$ which is a linear relation between log e and $(\log T+p)$.

Hence, if instead of plotting log T vs. log e, one plots $(\log T+p)$, the best straight line may then be drawn, as before, through the points thus obtained. The slope of this line is b and the value of a follows from computation. Since the quantity p is only a small correction factor, one may assume the value of b in the correction term to be equal to 4, so that

$$p=-\frac{0.4343}{4}\left(\frac{T_0}{T}\right)^4$$

If the pyrometer is so constructed that T_0 does not vary much from room temperature, the value 300° may be used for T_0 so that

$$p=-0.1086\left(\frac{300}{T}\right)^4$$

In such a case the correction p is most conveniently determined from a plot of p vs. log T, as illustrated in Fig. 16, curve A.

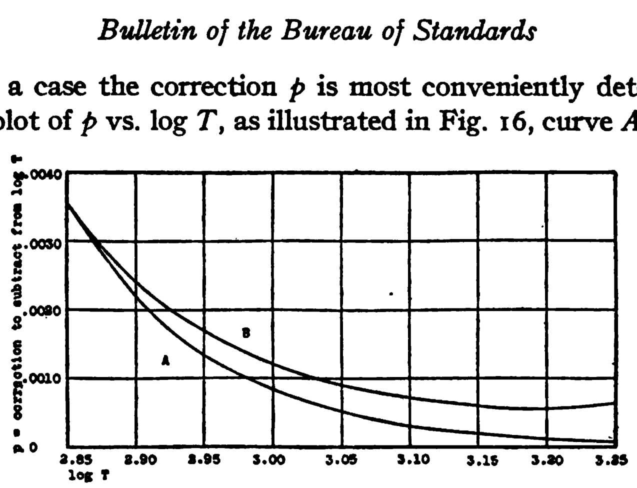


FIG. 16.—*Correction to apply to log T in plotting vs. log e*

The corrections are negative, so that log T is decreased by the value p in plotting each observation. After the best straight line

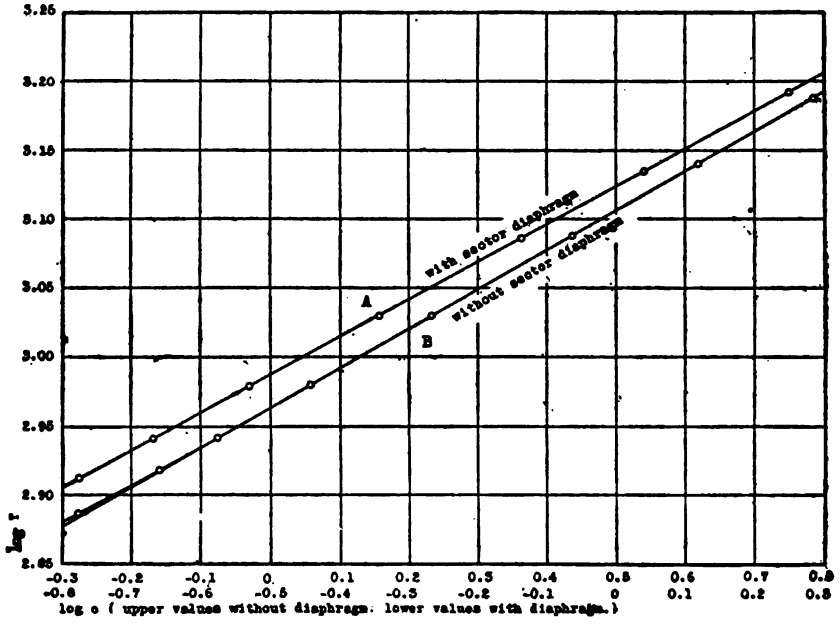


FIG. 17.—*Method of plotting data obtained in calibration of radiation pyrometer*

has been drawn on the log T vs. log e plot, each value of log T having been decreased by the correction p, a new curve may be drawn on the same plot by adding the numerical values of p, for several values of log e, to the values of log T, read from the straight line. The new curve represents the actual calibration of the instrument and is in general slightly concave to the log e axis at lower values of log T. This method of plotting the observations is illustrated in Fig. 17. The actual observations are represented by circles. The points are first replotted by correcting each value of log T by p, and the best straight line is drawn. Then the numerical values of p are added to the ordinates of the straight line and a new curve is drawn representing the final calibration.

(*b*) DETERMINATION OF THE EXPONENT b.—The value of this exponent depends upon the construction of the individual instrument. Any defects or pecularities in the pyrometer may seriously alter this constant, as will be shown later. It has sometimes been assumed [15] that the exponent b for radiation pyrometers has a value of 4 within at least 1 per cent. That this assumption is not generally justified may be seen from the following table, which presents the values of b determined in the calibration of several radiation pyrometers submitted to the Bureau for test. The method of calibration employed was one of the two primary methods discussed above.

Exponent	Type and number (arbitrary reference numbers)	Exponent	Type and number (arbitrary reference numbers)
3.85	Féry 100 without diaphragm	3.82	Féry 113 without diaphragm
3.91	Féry 100 with diaphragm	3.85	Féry 113 with diaphragm
4.14	Féry 101	3.50	Féry 114
4.12	Féry 102	to	
4.10	Féry 103	4.50	
3.96	Féry 104	3.55	Féry 105a without diaphragm
3.75	Féry 105	3.69	Féry 105a with diaphragm
3.80	Féry 106	3.28	Féry 109a without diaphragm
3.90	Foster fixed focus 107	3.63	Féry 109a with diaphragm
4.13	Féry 108	3.82	Foster fixed focus 116
3.84	Féry 109 without diaphragm	4.22	Féry 117 without diaphragm
3.99	Féry 109 with diaphragm	4.26	Féry 117 with diaphragm
4.17	Fluorite lens Féry 110	4.32	Thwing 118 box receiver
3.78	Féry 111 without diaphragm	3.56	Thwing 119 tube receiver
3.90	Féry 111 with diaphragm	3.68	Thwing 120 tube receiver
3.87	Féry 112		

---

[15] Féry and Drecq, J. de Phys., 1 (5), pp. 551–9; 1911. Numerous other references might be quoted in this regard.

(*c*) CALIBRATION OF FÉRY PYROMETER WITH AND WITHOUT DIAPHRAGM.—The effect of the sector diaphragm used with the Féry pyrometer for extending its temperature range should reduce by a constant factor the amount of energy falling upon the receiver. Thus, if the calibration were represented by $e=aT^b$ without the diaphragm and by $Ke=aT^b$ with the diaphragm, the two curves on the log e vs. log T plot would be parallel, since the slope (b) of the line remains unaltered. In general, however, one finds that the two best curves are not exactly parallel, which means that the constant b does not have the same value with and without the diaphragm. The range of b with and without diaphragm for the same instrument is noted in the table of exponents above. That the constant b may be slightly different in the two cases is evidently quite possible when one considers that b differs from the value 4 mainly because of convection and conduction losses inside the receiver. These losses may be quite different for a given temperature T, when the energy falls directly on the receiver and when it is reduced possibly 10 times by the sector diaphragm. It is quite possible also that the heating, however slight, of the sectored diaphragm and reradiation from it is in part responsible for variations in the value of b. Thus, in general, for the highest accuracy, the calibration of the pyrometer with diaphragm should be made just as though one were using an entirely different instrument. (Cf, Fig. 17.) This is especially important if one desires to extrapolate the temperature scale above the points obtained in the calibration.

(*d*) EFFECTS OF ERRORS.—Errors of calibration, such as lag, or time required to reach the reading of equilibrium condition, the effect of distance from source, and the size of source sighted upon, etc., will be treated quantitatively in Section VI.

V. METHODS OF USE

1. USE WITH GALVANOMETER OR WITH POTENTIOMETER

In the ordinary use of a thermoelectric radiation pyrometer a galvanometer is employed for the measurement of emf, but for the highest accuracy a potentiometer especially adapted to the measurement of small electromotive forces is desirable. Potentiometers are now available for the measurement of emf's as small as 0.0001 millivolts, and are so designed that internal thermal emf's are reduced to a minimum. The emf and battery circuit of the potentiometer should be provided with reversing switches for the elimination of stray thermal emf's in the lead wires and connections. Special attention should be given to the design of the reversing switches in order that they may not be a source for extraneous thermal emf's. In general, the same precautions apply for the use of a potentiometer with a radiation pyrometer as apply for its use in exact thermocouple work. It is often advisable to ground the pyrometer case, the potentiometer, galvanometer, and accessories.[16]

In the use of a potentiometer the resistance or length of the lead wires from the pyrometer, the resistance of the thermocouple, and the variation with temperature in the resistance of the pyrometer circuit produce no effect whatever upon the emf reading.

These sources of error are, however, inherent in the galvanometric method, and in addition there are the sources of error intrinsic to the galvanometer, such as change of sensibility with room temperature, etc. For practical purposes, however, galvanometers are available which are very satisfactory. Many pyrometer galvanometers have a resistance of over 100 ohms and a very low temperature coefficient. The sensibility is sufficient for ordinary pyrometric practice, and still the galvanometer is portable and mechanically robust.

The deflection potentiometer [17] appears not to have been used for radiopyrometric measurements, but without doubt this type of instrument would prove very desirable for laboratory work. The deflection potentiometer combines the potentiometric and galvanometric methods, the first figure of the value of the emf usually being given by a dial reading and the other figures by the galvanometer scale. The instrument is compact, portable, and accurate, and can be operated very rapidly.

(*a*) EXTRAPOLATION ON GALVANOMETER SCALE.—Frequently it is desired to calibrate a pyrometer and galvanometer indicator for a high temperature range. Thus, the Féry pyrometer and galvanometer may be designed for a range 600° to 1300° C using the pyrometer without the sector diaphragm, and a range of 1300° to 2000° C when the diaphragm is employed. Three scales are, therefore, engraved by the makers upon the galvanometer, a scale of emf's and two temperature scales, one, say, 600°–1300° C and the other 1300°–2000° C. Suppose that the highest temperature at which an observation may be taken during the calibration is 1400° C. The low range scale, using the pyrometer without the diaphragm, could thus be standardized, since the interpolation of the temperature scale of the galvanometer between observed points is permissible, especially if a large number of temperatures from 600° to 1300° C are used. Interpolation of the high range scale from 1300° to 1400° C may also be allowed, but direct extrapolation from 1400° to 2000° C of the temperature scale engraved on the instrument can not be justified. The extrapolation, however, may be effected in the following manner: The temperature-emf relation is obtained for the pyrometer, preferably by measurement on a potentiometer of the emf developed by the instrument, on open circuit, i. e., without the galvanometer, both with and without the diaphragm. These readings may be taken at the same temperatures at which the pyrometer and galvanometer are tested, the galvanometer being temporarily cut out of the circuit while the potentiometer is used.

---

[16] Measurement of High Temperatures, Burgess and Le Chatelier, Chapter IV, 3d ed., 1912, Wiley, N. Y.

[17] Brooks, Bureau of Standards Scientific Papers Nos. 79 and 173.

As follows from the discussion, on page 130, the graph log e vs. log T may be linearly extrapolated. Thus, from observations made with the diaphragm opening, in the range 500° to 1400° C, it is possible to obtain values of e for the temperatures 1400° to 2000° C by extrapolation of the log plot. The following six steps may then be observed in the extrapolation of the galvanometer scale:

1. Plot of both temperature scales of the galvanometer vs. the galvanometer emf scale.

2. Plot of emf of pyrometer on open circuit vs. emf by galvanometer on closed circuit, using all observations, both with and without the sector diaphragm.

3. Extrapolation of log T vs. log e plot to the higher temperatures.

4. Reading on emf scale of the galvanometer corresponding to the true emf's at the higher temperatures. These values are obtained from (2).

5. Reading of the galvanometer high range temperature scale corresponding to the values on the galvanometer emf scale, obtained from (4).

6. Plot or table of galvanometer temperature scale 1300° to 2000° C vs. correct temperatures.

Other methods of extrapolation might be employed, but the one described above has proven convenient and satisfactory, especially since, in general, corrections have to be applied to both the emf scale and the temperature scales of the galvanometer.

2. RECORDING RADIATION PYROMETERS

All the types of radiation pyrometer herein described may be made self-registering or automatically recording; and it is possible to obtain on a single sheet the distinctive, permanent records of several such instruments.[18] For many technical processes, particularly those requiring a uniform temperature or regulation within narrow temperature limits, and those involving regular heating or cooling over definite temperatures or time intervals, the use of recording instruments is oftentimes highly desirable, as it is also in many cases in which a control over the operators is desired as well as of the operation. The recording apparatus may also be arranged with a signal or alarm adjusted to call attention when any desired temperature is reached or departed from.

The recording instrument is distinct from the pyrometer proper, and in general consists essentially of a galvanometer of suitable sensibility and range and of a recording mechanism which is preferably operated without interference with the galvanometer sensibility. The various types of recorders used with thermocouples are available for this purpose. In the case of certain radiation instruments the relatively small emf's developed necessitate slight modifications in the recorder. In many of the radiation instruments, however, the emf is nearly as great as that developed by the rare metal thermocouples, so that very little, if any, alterations need be made to the recorder. Unless photographic registration is used, the galvanometer and recording mechanism are usually inclosed in the same containing box, which should be dust proof and its interior readily accessible. Some of the more recently developed recording mechanisms are driven by an electric motor, and the stability, range, and sensibility in some cases have been greatly improved by using a recording potentiometer. In this latter system the pen, which gives a continuous record, is mechanically distinct from the galvanometer needle, which is practically always kept at zero deflection. Clockwork, combined with an electromagnetic or mechanical control acting directly on the deflecting needle, is a common method of control giving a discontinuous record.

---

[18] Descriptions of types of recording instruments, some of which are suitable for use with radiation pyrometers, are given in "The Measurement of High Temperatures," 3d ed., 1912, by Burgess and Le Chatelier; Recording Pyrometers (Trans. Faraday Soc., **10**, p. 139, 1914), by C. R. Darling; and in the descriptive catalogues of several instrument makers.

The recorder may be located at a station distant from the pyrometer and, if the two are suitably designed and calibrated, may be operated in parallel with an indicating instrument, which may be placed near the pyrometer.

3. METHODS OF SIGHTING ON FURNACE

Two methods of sighting a variable focus pyrometer such as the Féry may be employed. The instrument may either be focused on the front opening of the furnace, Fig. 18, *A*, or upon some plane in the interior of the furnace, *B*. In general, method *B* is to be preferred for calibration purposes. The exact realization of black-body conditions for method *A* would require, in general, that the front diaphragm, *a*, be as hot as the furnace interior, which condition is practically impossible of being fulfilled even in the case of a furnace with compensating heating coils. Diaphragms *c*, *d*, *e*, and *f* may be located as shown in the figure, these being properly cut to fit the diverging cone of rays *iag*. The inner diaphragms will be more nearly the temperature of the radiator and, hence, will assist in maintaining black-body conditions. The radiator *ihg* is best constructed of some material of high emissivity, such as graphite. In the technical use of a pyrometer, such as the measurement of the temperature of a kiln, the inside of the front wall, *a*, is frequently as hot as the interior, and the opening is small in comparison with the rest of the heated inclosure, so that in such a case method *A* of focusing is quite satisfactory. When method *B* is used for calibration, the diaphragms *c*, *d*, *e*, and *f* are cut to fit the converging cone of rays from the pyrometer. The radiator *bgh*, located in the center of the furnace, is easily maintained at a uniform temperature over its entire surface, and since the opening, *b*, is rather small in comparison with the area of surface *bgh*, black-body conditions are sufficiently well realized even when the radiator is constructed of white porcelain, as was seen from the discussion in Section IV, 1.

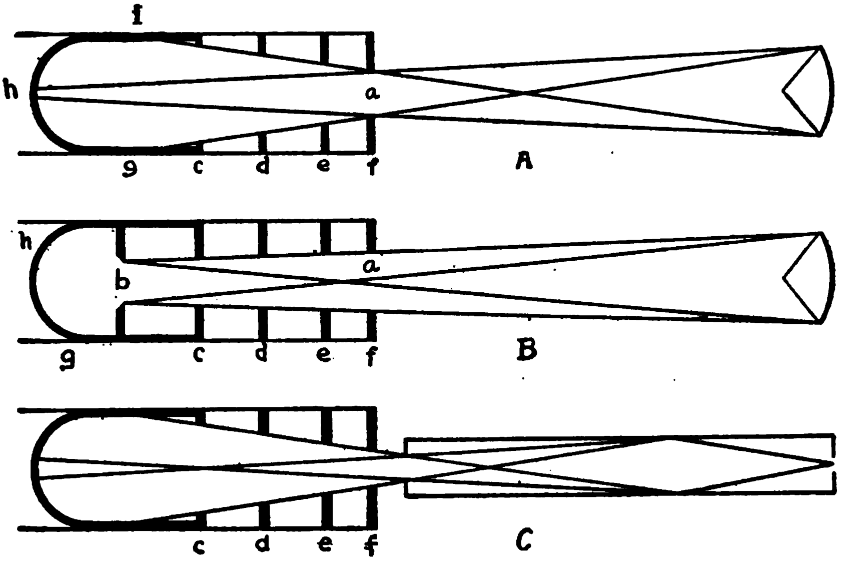


Fig. 18.—*Methods of sighting pyrometer on a furnace*

Method *C* should be employed for calibrating a fixed focus pyrometer such as the Thwing or Foster. The diaphragms *c*, *d*, *e*, and *f* are cut to fit the diverging cone of rays from the instrument. The radiator should be constructed of a material having a high emissivity.

Various modifications of a black-body furnace have been proposed. For example, the diaphragm *a* of method *A* might be silvered on the inside, in order that the reflection from its surface may compensate for the small amount of radiation it contributes to the radiator.

In technical practice, diaphragmed furnaces are not generally available, and the methods described in Section VII, 2, may then be employed.

4. USE OF A RADIATION PYROMETER WITH A SOURCE OF INSUFFICIENT SIZE

It is sometimes necessary to use a radiation pyrometer at such a distance from a small source that the aperture of the instrument is not completely filled. Thus, with the Féry pyrometer, the image of the source formed at the receiver may be smaller than the limiting diaphragm immediately before the couple. The most satisfactory method of using the radiation pyrometer under such conditions is to construct a new limiting diaphragm of the proper size and recalibrate the pyrometer, sighting upon a black body.

Millochau[19] has suggested another method which may be employed if high accuracy is not desired. Assuming the 4th power law to hold one obtains

$$K\,\delta_1 = T^4 \tag{1}$$

where K is a constant, δ_1 the galvanometer deflection, and T the absolute temperature of the furnace. The following empirical relation is assumed to hold when an absorbing screen of glass, gelatine, or mica is placed between the source and the pyrometer:

$$\left(\frac{c}{T^4}+a\right)\delta_2 = T^4 \tag{2}$$

where c and a are constants, and δ_2 the galvanometer deflection. Whence, from (1) and (2)

$$T^4 = \frac{c\,\delta_2}{K\,\delta_1 - a\,\delta_2}, \tag{3}$$ the constants c, a, and k being known.

Equation (3), although derived for the case in which the source is large enough to completely fill the pyrometer, is just as applicable when the source is of insufficient size. Thus, in order to obtain the temperature of a very small source, one obtains a reading δ_1 of the galvanometer without the absorption glass before the pyrometer and a reading δ_2 with the absorption glass, whence, from equation (3), the correct temperature may be computed, since K is a constant known from the calibration of the pyrometer, and a and c are constants depending upon the particular absorbing screen employed.

---

[19] Millochau Compt. rend., 158, pp. 171–174; 1914.

It is evident that equation (2) is purely empirical and can have little physical significance. It is, therefore, very doubtful if this method may be trusted with accuracy over a large temperature interval. The arbitrary form of equation (2) is apparent from the following discussion:

Let J_λ = energy emitted by black body at wave length λ to $\lambda + d\lambda$.

P_λ = transmission coefficient of the absorbing screen

J'_λ = energy transmitted by screen from λ to $\lambda + d\lambda$

J = total energy transmitted by screen of all wave lengths

$J_\lambda = C_1 \lambda^{-5}\left(e^{\frac{c_2}{\lambda T}} - 1\right)^{-1}$, Planck's law.

$J'_\lambda = P_\lambda J_\lambda$.

$$J = \int_0^\infty P_\lambda C_1 \lambda^{-5}\left(e^{\frac{c_2}{\lambda T}} - 1\right)^{-1} d\lambda$$

(4) galvanometer deflection $= \delta_2 \alpha J' = B\int_0^\infty P_\lambda C_1 \lambda^{-5}\left(e^{\frac{c_2}{\lambda T}} - 1\right)^{-1} d\lambda$ with screen (B = const)

According to equation (2):

$$\delta_2 = \frac{T^4}{\frac{c}{T^4} + a} \times \text{ const.} = \frac{T^8}{c + aT^4} \times \text{const.} \tag{5}$$

Thus, the integral expressed in equation (4) must have the form $\frac{T^8}{c + aT^4} \times$ const. This puts a very arbitrary limitation on the transmission coefficient $P_\lambda = f(\lambda)$. Since different materials have entirely different values of $P_\lambda = f(\lambda)$, one would not expect equation (2) to hold at all accurately. Accordingly this method must be used with extreme caution.

Probably a more satisfactory method to be employed with small sources is to compute the actual size of the image formed at the receiver (see p. 113) and correct the observed deflection, making use of the assumption that the galvanometer deflection is proportional to the area of the image as long as the image is smaller than the limiting diaphragm. Thus, if the area of the opening to the receiver of the Féry pyrometer were 1 mm², and the area of the image of the source formed by the gold mirror were 0.5 mm², the correct temperature would be given by the value corresponding to a deflection twice that of the actually observed deflection. Errors due to aberrations of the gold mirror will affect the measurements to some extent. However, the assumption that the galvanometer deflection is proportional to the area of the image as long as the image is smaller than the limiting diaphragm before the receiver is fairly well justified by the section *AB* of the curve in Fig. 25. Here the relation, diameter of the image vs. emf is parabolic, and hence, if (diameter of image)² were plotted against emf, the relation would be found linear.

As a check upon this method of using a Féry pyrometer the following rough measurements were made: The area of image required by the pyrometer was 1.77 mm². The source remained at approximately a constant temperature 1260° C, and its size was altered by means of water-cooled diaphragms.

Area of image	Observed emf	$\frac{\text{Area of receiver}}{\text{Area of image}}$	Observed emf× ratio of areas	Temperature computed
mm²	Millivolts			°C
0.302	1.04	5.86	6.09	1290
.695	2.33	2.55	5.94	1280
.807	2.78	2.19	6.09	1290
1.564	4.32	1.13	4.88	1200
1.77	5.64	1.00	5.64	1260
Mean....	....	....	....	1265

The above-computed temperatures have a wide range, but, without doubt, if sufficient care were taken, the accuracy could be increased, possibly to ±20° C. This method certainly may be recommended in preference to that of Millochau, but neither method will give results as satisfactory as may be obtained by replacing the limiting diaphragm with one of smaller opening and then recalibrating the instrument by sighting upon a black body.

VI. DISCUSSION OF ERRORS

The sources of error mentioned in Section II, page 107, may now be considered in detail, including the determinations of their magnitudes and methods of elimination or correction.

The question of nonblackness of source and of receiver has already been treated in Section II, page 99. The effects of the T_0 term in the Stefan-Boltzmann equation and of the modifications in this equation on the calibration are given in Section IV, page 130. The errors of the electrical measuring devices, galvanometers, and potentiometers, do not need to be emphasized here. We shall limit ourselves to a study of the errors of the pyrometric receiver.

1. LAG

When a radiation pyrometer is exposed to the radiation from a source at a constant temperature, the pyrometer does not immediately indicate the temperature of the source, but exhibits a certain time lag during which the receiving system is heating up. Eventually, the receiver emits or loses by conduction, radiation, and convection as much heat as it receives, and a condition of equilibrium is maintained between the source and the receiver. Theoretically, this condition is reached only after an infinite time, but it is possible to so construct a pyrometer that for all practical purposes this state of equilibrium is attained within a few seconds, although few pyrometers show such good behavior. The following examples illustrate the lag effect in various amounts.

The behavior of Féry pyrometer No. 108, one of the earlier forms of the gold-mirror thermoelectric instruments, is illustrated in Fig. 19, curve *A*. The pyrometer was sighted into a black body at a temperature of 1335° C. At least 10 minutes were required for the instrument to reach this maximum temperature indication. Thus, if the reading had been taken after an exposure of 1 minute to the radiating source, the observed temperature would have been 1300° C, or in error by 35° C. A very much greater error would be incurred for a 30 seconds' exposure.

Féry pyrometer No. 109, sighted upon a furnace at a temperature of about 1313° C, showed a maximum reading of 1313° C in 20 seconds (curve *B*). The time required for this pyrometer to reach its maximum indication is satisfactory, but the instrument shows a defect which is quite common in radiation pyrometers, namely, a rapid decrease in the emf immediately following the maximum emf. Thus, for an exposure of 20 minutes the temperature reading has dropped from 1313° to 1305° C. As seen

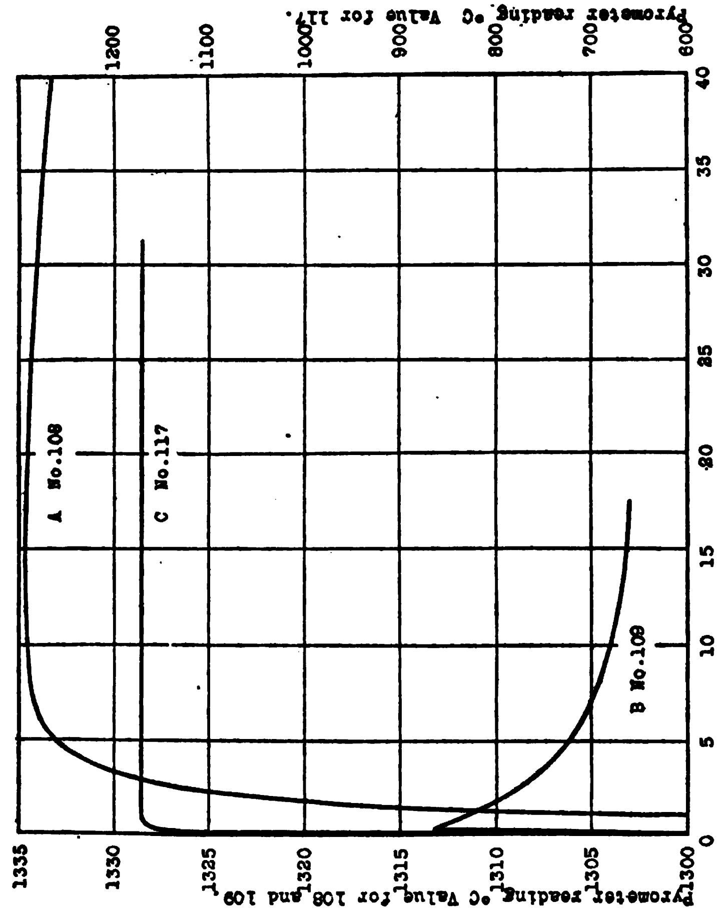


FIG. 19.—*Typical curves illustrating the time lag in the indication of a radiation pyrometer*

from the curve, the real point of equilibrium is reached after 20 minutes' exposure. It would be possible to calibrate this pyrometer so that the correct temperature would be indicated after 20 minutes' sighting upon the radiating source. With this particular instrument, however, it is better to make the calibration for general use for a 20 seconds' exposure, and, if necessary, corrections may be applied whenever in actual use longer exposures are required.

In the case of pyrometer No. 108, the temperature reading even after 40 minutes' exposure is but 2° less than the maximum reading at 10 minutes. It is, therefore, not of serious importance whether the calibration is made for a 10, 20, or 30 minute exposure.

Curve *C* illustrates the behavior of Féry pyrometer No. 117. The maximum reading is obtained in about 30 seconds and is maintained thereafter. These three curves are typical of the action of all the pyrometers examined.

In general, the Thwing radiation pyrometers are very quick acting because of the simple type of receiver employed. The Féry pyrometers are complicated by the focusing device. These mirrors which are very essential for the convenience of focusing and operation may cause a great amount of trouble, as will be shown clearly on page 154.

(*a*) CAUSES OF LAG.—The lag of the reading of a radiation pyrometer may be decreased by reducing the heat capacity of the receiver to a minimum. The thermocouple wires should be of as small a diameter as is consistent with satisfactory mechanical strength, and the receiving disk, when used, should be constructed of very thin blackened tin or silver foil. The drop in emf after the maximum reading is attained should be practically eliminated This drop is usually caused by heat conduction along the thermocouple wires from the hot to the cold junction, and by reradiation or convection from the sides of the receiving box, various diaphragms, focusing mirrors, etc. The effect of conduction may be reduced by locating the cold junction as far as possible from the hot junction, and by employing wires of small size. The couple should not be encumbered with a heavy supporting device The cold junction in general must be very well screened from the direct radiation of the source.[20] Silvered glass focusing mirrors used in some of the Féry pyrometers do not form a satisfactory screen. The heavy copper mirrors used in the latter type of the Féry instrument serve this purpose much better. The high heat conductivity of copper allows the heat absorbed by the diaphragm to be carried away to the outside of the receiving box where it is dissipated by radiation and convection. In the case of glass diaphragms, however, the heat absorbed is not carried to the outside so rapidly. The temperature of the diaphragm rises and the convection currents and radiation losses are set up inside the receiving box, which may affect both the hot and cold junctions of the thermocouple in a variety of ways. For the ideal pyrometer all diaphragms should be water-cooled, but the use of fairly heavy diaphragms of a good heat-conducting material such as copper, making tight contact with the outside case of the instrument or of the receiving box, will greatly reduce the effect of reradiation. All diaphragms should, of course, be cut with a bevel edge around the opening.

---

[20] There are special instances where the type of construction may allow exposure of the cold junction. This construction will not be considered in the present paper.

Besides the effect of lag due to the high heat capacity of the receiving system, the mechanical pyrometer such as the Féry spiral is subject to all the errors of hysteresis or elastic lag of the expansion device—i. e., the spiral spring. Some of these errors may be partially eliminated by setting the pointer at zero immediately before each temperature reading. If the zero reading is not frequently adjusted, very large errors may occur. Thus, Fig. 20 illustrates the shift of the position of the pointer for a period of 20 hours, when the front opening of the pyrometer was closed by the diaphragm at constant room temperature. While a 20-hour test is rather severe for this instrument, it will be noted that the shift even during a few moments is pronounced. This effect, combined with the elastic lag of the spring in attaining its maximum expansion when exposed to the radiating source and the thermal lag of the receiver, makes the instrument somewhat unsatisfactory for scientific work, although its compact and convenient form offsets in a measure these objections, especially for technical use.

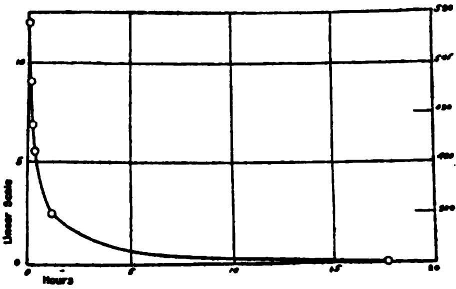


FIG. 20.—*Hysterisis exhibited by the spring of the spiral pyrometer*

No generalizations can be drawn as to the magnitude of the lag for the various types of pyrometers. Apparently no two pyrometers of the same type are constructed exactly alike. If a radiation pyrometer is to be used for highly accurate work, the effect of lag should be thoroughly examined, so that the calibration and all readings may be made with an exposure for which the change of lag with time is a minimum. Thus, pyrometer No. 108 may be best calibrated and used for exposures of 10 to 30 minutes, pyrometer No. 109 for exposures of either 20 seconds (maximum reading) or 20 to 30 minutes, and pyrometer No. 117 for exposures of 30 seconds to 30 minutes. Under certain conditions of use the pyrometer might be calibrated for a definite exposure such as one minute, but, as seen from Fig. 19, any wide variation in use from the one-minute time of exposure would give rise to a large error, especially in the case of No. 108. It is usually more satisfactory to calibrate and use a pyrometer with an exposure sufficient to give a maximum reading. This would preclude No. 108 for the measurement of rapidly varying temperatures.

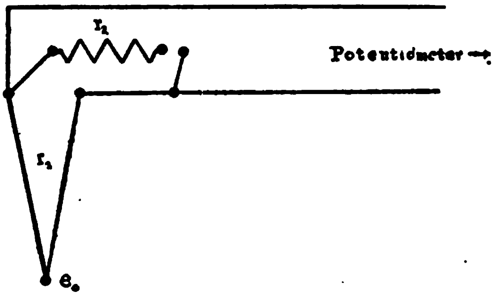


FIG. 21.—*Method of measuring resistance of thermocouple of radiation pyrometer*

2. RESISTANCE OF THERMOCOUPLES USED IN RADIATION PYROMETERS AND VARIATION IN RESISTANCE WITH TEMPERATURE OF THE SOURCE

If the radiation pyrometer is to be used with a galvanometer, it is desirable that both the resistance of the thermocouple and its variation in resistance with the temperature of the source be small. These conditions are well satisfied by all the radiation pyrometers examined. Fig. 21 illustrates the method employed for determining the resistance for various temperatures of the source. The emf, e_0, developed by the thermocouple on open circuit when the pyrometer was sighted upon the furnace at any definite temperature, and the potential drop, e, across the resistance, r_2, connected to the couple terminals, were measured by a potentiometer.

The resistance of the couple, r_1, is given by the expression:

$$r_1 = r_2\left(\frac{e_0}{e} - 1\right)$$

The resistance of the couples of Féry mirror pyrometer No. 111, Thwing cone pyrometer No. 120, and a Brown pyrometer were found to be about 0.8, 0.6, and 0.2 ohms, respectively, with variations from these values of 2 or 3 per cent for a range in temperature of the source of 500° to 1200° C. Since only a very short length of the couple is heated by the source, large variations in resistance would not be expected. The small changes in resistance of the couples observed will produce no effect upon the reading of a galvanometer indicator, even if the resistance of the galvanometer be as low as 10 ohms.

3. EFFECT OF DIRT AND OXIDATION UPON THE CONDENSING DEVICE

Whipple has observed that a slight film of oxide on the surface of the concave gold mirror of the Féry or Foster radiation pyrometers does not seriously alter the amount of radiation reflected (96 per cent),[21] as the greater part of the energy exists in the form of long wave lengths, which the tarnished mirrors reflect without difficulty. This statement has frequently been interpreted to mean that in spite of dirt accumulation, stains, and scratches the gold surface remains unchanged in its reflecting power, but obviously such constancy is not the case. Pyrometers subjected to severe use in steel mills and other industries soon become coated with dust and dirt.

As an actual example of the errors to be expected from this source, the following table is given for a Féry mirror thermoelectric pyrometer No. 108. The instrument which was submitted to the Bureau for test was calibrated (1) in the condition received and (2) after the accumulation of dirt had been removed from the mirror. This case is not at all exceptional; many other instruments have been received which were just as dirty.

---

[21] Whipple, Engineering, **90**, p. 142; 1910. (See also the present paper, p. 103.)

Indicator reading	Without diaphragm (temperature °C)			With diaphragm (temperature °C)		
	Dirty mirror	Clean mirror	Δ	Dirty mirror	Clean mirror	Δ
600	706	600	106	....	....	....
700	813	702	109	....	....	....
800	920	804	116	....	....	....
900	1026	906	120	1050	920	130
1000	1133	1008	125	1158	1025	133
1100	1240	1110	130	1262	1125	137
1200	1346	1212	134	1370	1229	141
1300	1452	1314	138	1478	1336	142
1400	....	....	....	1590	1445	145
1500	....	....	....	1700	1554	146
1600	....	....	....	1815	1662	153
1700	....	....	....	1930	1777	153
1800	....	....	....	2051	1893	158

The differences in calibration with clean and with dirty mirrors, columns 4 and 7, vary from 100° to 160° C in the temperature range 600° to 2000° C. Since the ordinary calibration of the pyrometer is made for clean mirrors, the instrument as used with dirty mirrors may read too low by over 100° C. It is interesting to note that the exponent b in the emf relation $e = aT^b$ was approximately the same in the two calibrations, the main change being a shift of the straight line on the log e vs. log T plot, parallel to itself. The approximate constancy of b would be expected, since the dirt merely alters the reflection coefficient of the mirror and diminishes the energy falling upon the couple, and hence the emf, by approximately a constant factor. Accordingly, only the constant a can be seriously affected.

The importance of keeping the mirror free from dirt is therefore evident. When necessary, the mirror may be taken from the telescope and carefully washed with water, care being taken not to rub the surface of the earlier-type gilded mirrors on glass, as the gold is easily removed. Carefully polishing the mirror with soft rouge will frequently prove beneficial.

The statement of Whipple above, viz, that oxide films do not alter the reflection coefficient of the gold, is open to question. It would be interesting to measure the reflection coefficient of a freshly polished gold mirror for several wave lengths in the visible spectrum and infra-red, and to determine the coefficients of the same mirror after it had been exposed to the atmosphere for different lengths of time, one to five years.

The effect of dirt upon the surface of the aluminum cone of the Thwing pyrometer has not been tested, but without doubt the calibration would be altered in a manner quite similar to that observed in the case of the Féry pyrometer.

4. EFFECT OF DISTANCE AND SIZE OF SOURCE

As is seen on page 95, the simple geometrical optics of a radiation pyrometer indicates that the readings should be independent of the distance from the pyrometer to the radiating source, provided the uniform source were of sufficient size required by the constant aperture of the instrument. Enlarging the size of the source above this limiting value should produce no effect. Actually this ideal condition by no means obtains, and herein lies the most serious error to which radiation pyrometers are subject.

The causes for the dependence of the readings of a radiation pyrometer upon the distance from the instrument to the radiating source are two in number, (1) atmospheric absorption and (2) construction of the pyrometer. The errors due to atmospheric absorption are small in comparison with those occasioned by an unsatisfactory construction of the pyrometer. Quantitative measurements have not been made in the present work upon the absorption of the atmosphere. Dr. F. E. Fowle, of the Smithsonian Astrophysical Observatory, however, is at present making very extended measurements upon the absorption coefficient of air containing water vapor, from the ultra-violet to about 20 μ in the infra-red, and when these results are completed it will be possible to determine just what variation in the reading of the pyrometer will occur when the instrument is used close to the source and at several meters distance. It is possible that atmospheric absorption and absorption due to water vapor and gases such as CO and CO_2 which occur in the vicinity of most furnaces may account for discrepancies of 1 or more per cent in the emf or linear scale readings of a radiation pyrometer close to a source and at 3 meters distance. The effect of atmospheric absorption is to diminish the reading of the pyrometer as the distance to the source is increased.

Errors arising from the size of the source, when this size is greater than that required by the aperture of the pyrometer, may be ascribed to the construction of the instrument. The various types of instrument examined will be discussed separately.

(*a*) Féry Mirror Thermoelectric Pyrometer—*Mechanical Construction.*—Many of the pyrometers of this type are not of constant aperture. Fig. 22 illustrates a typical example of this mechanical defect, observed for pyrometer No. 109. Referring to Section II, page 98, the energy J falling upon the receiver at A is proportional to $I \sin^2\theta$. In order that J may be independent of the focusing distance, $\sin^2\theta$ must be constant. As constructed, the entire mirror is effective for all possible focal distances. The diameter of the mirror is 6.8 cm and its focal length 7.6 cm. Hence, for a focusing distance of 80 cm (mirror to source), $\sin^2\theta = \sin^2\theta_1 = 0.396$, and for 300 cm, $\sin^2\theta = \sin^2\theta_2 = 0.432$. Thus, the energy received by the thermocouple at A is 9 per cent greater for a focusing distance of 300 cm than for a focusing distance of 80 cm. The difference in emf of the thermocouple for these two distances would be approximately 9 per cent, and if the pyrometer were calibrated at 80 cm distance and used at 300 cm, the temperatures thus measured would be too high by approximately $\frac{1}{4} \times 9$ per cent = 2 per cent of the absolute temperature, i. e., 35° at 1500° C.

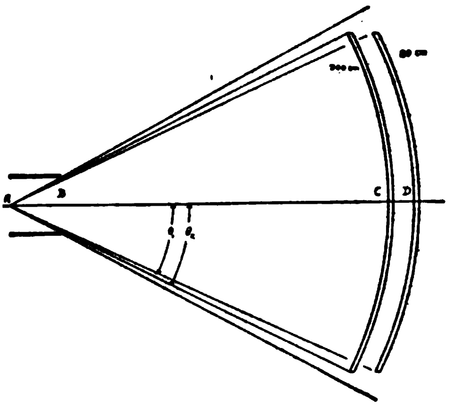


Fig. 22.—*A common mechanical defect in the construction of the Féry pyrometer. The instrument is not of constant aperture in this illustration*

This particular pyrometer may be made of fixed aperture for any focusing distance greater than 80 cm, by locating at B a diaphragm having a 9 mm opening. The angle of aperture is then θ_1. The size of the proper diaphragm opening is determined so that the cone of rays diverging from A and passing through the limiting diaphragm B will always completely intersect the surface of the gold mirror at the shortest focusing distance u employed (i. e., the longest distance v).

In the discussion of the independence of focusing distance of a constant aperture pyrometer (p. 98) it was assumed that the receiver is of very minute size. In actual practice the receiver, or entrance diaphragm immediately in front of the receiving surface, is about 1.5 mm diameter. An area of this size can not be considered theoretically as a point receiver. The discussion, therefore, rigorously applies not for the total energy falling upon the receiver, but for the intensity of the energy at a point in the center of the receiver. It would be possible similarly to compute the intensity at a number of points within the 1.5-mm image, but for practical purposes such a procedure is unnecessary, the simple discussion as given being sufficient.

Another factor altering the rigorousness of the simple geometrical relation is the aberration of the gold mirror. Besides showing the usual amount of spherical aberration for a mirror of this size and focal length, the mirror of every pyrometer examined shows all sorts of local aberrations, so that the image is fuzzy and distorted. This is especially true of the gilded mirrors on glass, but even the gold-plated copper mirrors in the later type instruments show this imperfection. It is impossible to say what effect these distortions have upon the reading of the pyrometer for various focusing distances. The Taylor Instrument Companies are experimenting with gold-plated speculum metal mirrors, and it is hoped that better mirrors may be obtained as a result of their investigations.

Still another factor modifying the simple geometrical relations, as discussed on page 98, is the shading effect of the small box containing the thermocouple receiver and of the supports (K, Fig. 6). It is easy to compute the shading effect of this system for a point of the source on the axis of the pyrometer and to determine the variation of the magnitude of this shading for various focusing distances, but to determine the integral effect of the shading upon the amount of energy falling on the receiver would require a consideration of the shading for every point of the source (image of the receiver) for several focusing distances, the source of course being of a different size at each distance. A rough drawing will show qualitatively that the mean intensity of the total image decreases as the focusing distances decreases, in consequence of the shading of the thermocouple box and mounting. Hence, on account of this shading effect, the pyrometer reading will increase with increasing focusing distance.

Convection Currents.—The question of convection currents of hot air passing directly from the source to the receiver is not of serious importance. The effect, which is very small, usually tends to produce readings which increase with decreasing focusing distance. It has been suggested that convection currents of this nature might be reduced by placing a mica window on the front of the pyrometer. This would, however, reduce the readings of the instrument, due to reflection of a part of the radiation at the surfaces of the window, which, of course, is undesirable. Also, selective reflection of the mica window might seriously modify the relation discussed on page 107. Furthermore, the absorption of energy may heat the mica and give rise to new convection currents within the pyrometer.

As the pyrometer is brought nearer the source the temperature of the case and entire telescope increases somewhat, on account of convection from the source and absorption of the radiation by the metal case. This increase of temperature may either increase or decrease the pyrometer reading, depending upon whether it affects the hot or cold junction of the thermocouple. With ordinary care the Féry pyrometer will not become hot enough to have any serious error from this cause.

Stray Reflection.—Errors resulting from stray reflection may cause considerable trouble. The Féry pyrometer is blackened inside, but most of the stray reflection occurs at nearly grazing incidence, and under such a condition a surface blackened even with a thick deposit of soot from the acetylene flame becomes a fairly good reflector. The thermocouple box of all the instruments examined produced a rather intense caustic, caused by reflection from the side walls. Stray reflection can be reduced by cutting a sharp screw thread, preferably of small pitch, upon all the surfaces likely to give any trouble. The thermocouple box should certainly have such a screw thread cut on the inside, and, undoubtedly, it would be worth while to cut the threads on the inside of the pyrometer case. Stray reflection for a source of given size generally causes the pyrometer readings to increase with decreasing focusing distance.

Effect of Size of Image.—It is obvious in the use of the Féry pyrometer that the source should be of a size sufficient for its image to cover the thermocouple receiver or the limiting diaphragm immediately in front of the receiver. (See p. 140 for a discussion of the use of the Féry with sources which are too small to meet this condition.) Usually this limiting diaphragm is the hole in the focusing mirrors, although in some of the instruments the limiting diaphragm is placed between the focusing mirrors and the thermocouple. As long as this opening is covered by the image of the source, it has usually been assumed that the reading of the pyrometer is independent of distance or size of source.

In general, the reading of the Féry pyrometer decreases with increasing focusing distance and with decreasing size of source, even though the image of the source always covers the receiver, although some instruments show this effect much more markedly than others. It is even possible to obtain a positive reading, as shown by Dr. Kanolt, when the pyrometer is sighted on a hole in a heated surface, even though the image of the hole covers the opening to the thermocouple receiver. The cause for this effect is best determined from a consideration of the size of the image of the source.

Errors which are surprising in magnitude, and which completely outclass those resulting from the preceding causes, may arise in the variation of the size of the image produced by the gold mirror of the Féry pyrometer. The size of the image may be altered by (1) varying the focusing distance, the size of the source remaining constant, and (2) by varying the size of the source, the focusing distance remaining constant.

In order to satisfactorily study the errors resulting from variations in the size of the image, it was necessary to devise a method whereby both the size of the source and the focusing distance could be conveniently and rapidly changed.

A large nickel strip (see p. 128) was employed as the radiator, in front of which, at a distance of 1 or 2 cm, were placed diaphragms of various openings. A number of polished brass diaphragms were first tried, but on account of the reflection of heat from the brass back to the strip the temperature of the strip was found to change seriously with openings of different size, as would be expected. Blackening the diaphragms reduces this effect of heat reflection, but the diaphragms then become so hot that they act as secondary sources of radiation. It was therefore necessary to employ water-cooled and thoroughly blackened diaphragms. For convenience of operating a series of diaphragms was constructed, as shown in Fig. 23. By a sliding arrangement and spring catch the various openings may be quickly shifted and centered in front of the strip. The water cooling maintains the temperature of the diaphragms at about that of the room, so that there is no effect of radiation from them, and the thoroughly blackened surface of the diaphragms absorbs practically all the heat falling upon it, so that the radiation loss from the surface of the strip behind the diaphragm is the same as that from the exposed surface of the strip. Consequently there is no variation in either the apparent or true temperature of the strip when the size of the diaphragm opening is changed. Except for very close distances from the pyrometer to the strip, the diaphragm openings act as the real source of the radiation. In this manner five sizes of the radiating source were obtained, circular areas having diameters of 1.95, 3.5, 5.6, 8.5, and 12 cm.

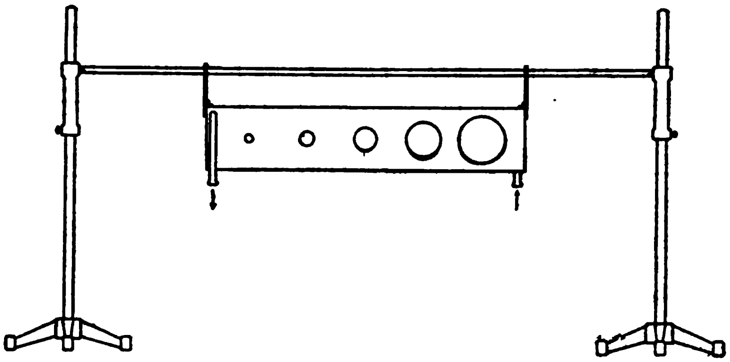


FIG. 23.—*Water-cooled diaphragms for producing sources of various sizes*

The pyrometer was mounted upon a carriage which rolled on parallel tracks similar to an ordinary photometer bench (Fig. 24). The strip and water-cooled diaphragms were properly adjusted at one end of the bench, and the apparatus aligned so that the image of the diaphragm opening employed was always centered upon the pyrometer receiver for all focusing distances (usually 80 to 300 cm).

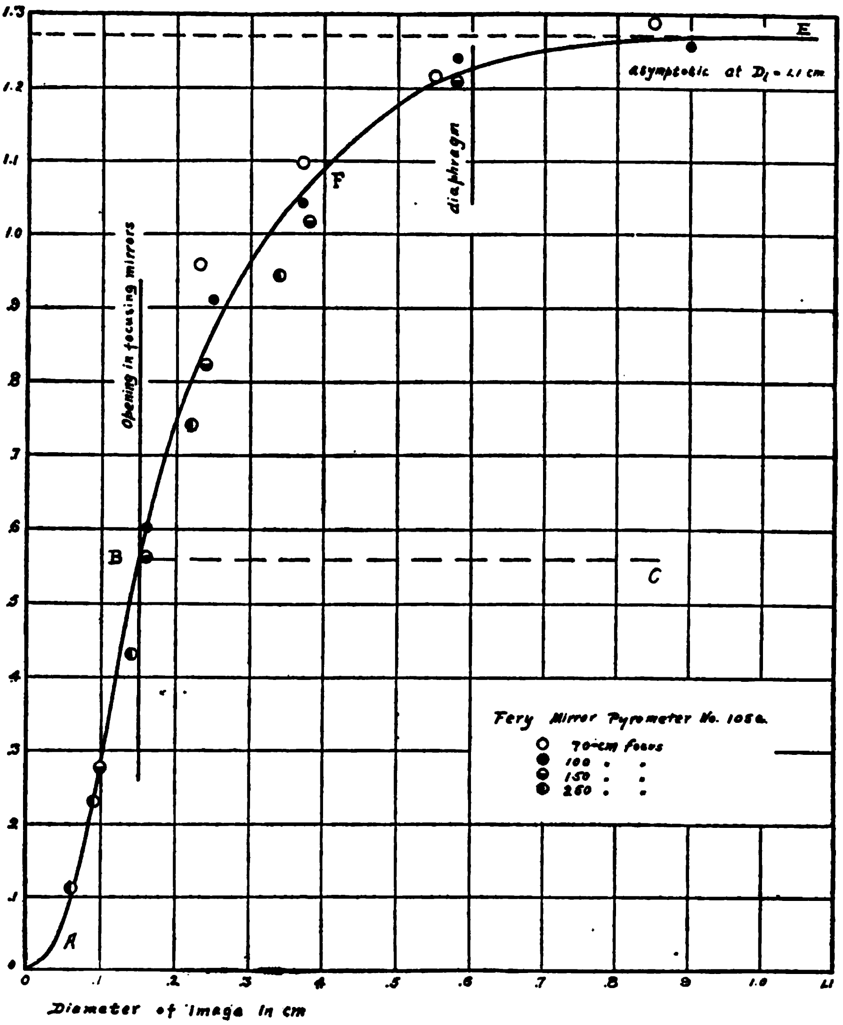


FIG. 25.—*Emf as a function of the size of image at the receiver. Féry mirror pyrometer 105a*

The relation between the diameter of the source D_s, the diameter of the image D_i, and the focusing distance u (measured from the source to the concave mirror) is given by the equation

$$D_s = D_i\left(\frac{u}{f} - 1\right)$$

where f is the focal length of the mirror, usually about 7.6 cm.

Thus, for a given diameter of the source at any distance u, the diameter of the image may be computed. Fig. 25 represents the variation in emf with the diameter of the image of a uniform temperature source for Féry mirror pyrometer 105a. The limiting diaphragm immediately in front of the thermocouple receiver is the focusing mirror system, the opening in these mirrors being 0.15 cm. A 0.6-cm diaphragm is located immediately in front of the focusing mirrors, and the inside diameter of the thermocouple box is 1.1 cm. The focusing mirrors are of thin glass silvered on the back surface. In the present case the diameter of the image was varied from 0.05 to 0.9 cm by employing sources of diameters 2 to 11 cm at focal distances of 70 to 250 cm. In the section *AB* of the curve the image was not large enough to cover the 0.15 opening in the focusing mirrors. For this range the emf is approximately proportional to the area of the image or to the (diameter)^2, so that *AB* is a parabola. If the focusing mirrors formed a perfect diaphragm completely shutting out all radiation except that passing through the opening, the point *B* would represent a maximum reading and the curve would continue horizontally along the line *BC*. Actually the emf increases up to the point *E*, where the image has a diameter of about 1.1 cm, the inside diameter of the thermocouple box, although for all points from *B* to *E* the size of the image was sufficient to cover the receiver. The large increase along *BE* is due to the heating of the silvered glass focusing mirrors, and the amount of this heating increases with the size of the image, until the image completely fills the inside of the thermocouple box. This heat is communicated to the thermocouple by radiation and by convection currents set up within the receiver. The errors in measurement resulting from variations in size of image are readily apparent. For example, suppose the instrument were calibrated by sighting at 150 cm distance upon a black body having an opening of 3 cm and were used for the measurement of the temperature of a source 11 cm in diameter at a distance of 100 cm. In the first case the image diameter is about 0.15 cm and in the latter 0.9 cm, corresponding on the curve to the points B and E, respectively. The emf at E is 227 per cent of that of B, so that as used the instrument would indicate emf's in error by 127 per cent. Suppose this instrument obeys the law $e = aT^4$, then by differentiation

$$\frac{\delta e}{e} = 4\frac{\delta T}{T}$$

which states that the fractional error in the absolute temperature is one-fourth the fractional error in emf. Hence, an error of 127 per cent in emf is equivalent to an error of 32 per cent in the absolute temperature. Thus, for a source of 11 cm diameter, 100 cm distance, and at a temperature of 1500° C, this pyrometer would read about 2070° C, or a temperature too high by 570° C.

The example cited is an extreme case, and the variation of emf with diameter of the image was greater for this instrument than for any other examined. Actually, a pyrometer would not be calibrated with the minimum-sized image required by the optics of the instrument. It is rather difficult to center such an image, so that usually the image is made large enough to overlap the opening of the receiver. Probably an image diameter of approximately 0.4 cm is more often employed in calibration, and in general use, a variation in image diameter of from 0.2 to 1.1 cm. Thus calibrated, this pyrometer would indicate emf's too small by 32 per cent when the smaller image is used and too great by 17 per cent with the larger image.

Fig. 26 illustrates the variation of emf with image diameter for the Féry pyrometers No. 111 and No. 117. The size of image was varied by both altering the size of the source and of the focusing distance, as explained by the key inserted on the plot. The fact that the points obtained by either method coincide equally well with the curve precludes the possibility of any experimental errors, such as temperature gradient across the nickel strip used as a source or the change in temperature of the strip due to the use of various sized diaphragms. It also indicates that the error due to the atmospheric absorption is small, at least in comparison with errors involved in the heating of the focusing mirrors. These curves represent the behavior of the two best acting pyrometers examined, but it is apparent that their behavior is far from being satisfactory. Thus, in the case of No. 111, the emf for a 1.1-cm image is 133 per cent of that obtained with an image of 0.15 cm, the diameter of the limiting diaphragm in front of the receiver, and for No. 117 this relation is 162 per cent. If these instruments were calibrated with an image diameter of 0.4 cm, the emf developed by No. 111 would be too great by 2 per cent for a 1.1-cm image and too small by 11 per cent for a 0.2-cm image, and by No. 117, too great by 2 per cent for the larger image and too small by 18 per cent for the smaller image, with errors of one-fourth these percentages when referred to absolute temperatures.

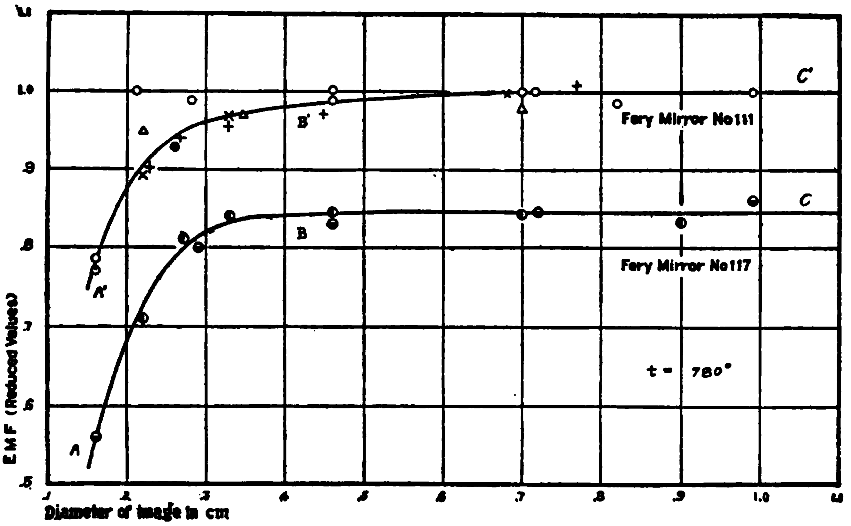


FIG. 26.—*Emf as a function of the size of image at the receiver. Féry mirror pyrometers Nos. 111 and 117*

○, 100 cm distance (size of source varied); ◐ + ×, 8.5 cm source (distance varied); △, 200 cm distance (size of source varied); ●, 12 cm source at 300 cm distance; ◒, source diameter varied from 1.95 to 12 cm.

Pyrometer No. 111 has a metal diaphragm located between the silvered glass focusing mirrors and the receiver, while in No. 117 this diaphragm is absent, but the focusing mirrors are of gold-plated sheet copper. These two instruments show a smaller effect of variation in size of image than does pyrometer No. 105, for the reason that the metal diaphragm in No. 111 and the copper mirrors in No. 117 are good heat conductors, in contrast with the glass-mirror diaphragm of No. 105, and allow part of the heat to be carried to the walls of the thermocouple box, where it is dissipated by radiation and convection on the outside, away from the thermocouple.

Summary of Focusing Errors for a Féry Pyrometer.—The principal error results from a variation in the size of the image of the source formed by the large condensing mirror, and is due to the heating of the limiting diaphragm or the focusing mirrors immediately in front of the thermocouple receiver. The amount of this heating increases with increasing size of image. On account of this fact, the pyrometer readings for a source of constant size decrease with increasing focusing distance; and for a constant focusing distance the readings increase with increasing size of source. With ordinary use of the pyrometer, errors of this type may amount to several hundred degrees in extreme cases, and, in general, to 50° or more, unless certain specified methods of procedure are employed, for example, the use of an image of a definite size for every measurement. Other errors to which the pyrometer is subject are of minor importance.

The following table presents a summary of the various errors the magnitudes of which depend upon the focusing distance. For the best construction of the Féry mirror pyrometer, it might be advisable to purposely make the instrument one of variable aperture in order that the error, arising thereby, may be used to annul the errors caused by convection currents and atmospheric absorption, etc.

Effect Upon the Pyrometer Reading of Increasing the Focusing Distance

Reading increases on account of—	Reading decreases on account of—
1. Variable aperture 2. Shading of concave mirror by thermocouple box	1. Atmospheric absorption 2. Convection currents from source to receiver 3. Stray reflection in receiver and telescope tube 4. Reradiation to couple from side walls of pyrometer 5. Image of source becoming smaller

(*b*) FÉRY FLUORITE LENS PYROMETER.—Fig. 27 illustrates the effect of size of image upon the readings of the Féry lens type pyrometer. The readings increase with increasing diameter of the image just as is the case with the mirror instrument. The diameter of the receiver is less than 0.1 cm, but on account of the radiation and convection from the shield and limiting diaphragm to the receiver the readings continue to increase even with an image diameter of 0.9 cm.

It is interesting to note that Féry and Drecq[22] employed an instrument of this type for the measurement of the temperature of the radiator used in the determination of the constant σ of the Stefan-Boltzmann law. The value of σ determined by Féry and Drecq is 6.51×10^{-12} watt cm^{-2} deg^{-4}, while the unweighted mean of all determinations by various investigators is about 5.75. The latest determination of this constant is 5.61.[23] If the pyrometer used by Féry had been calibrated with a large image, as was quite likely the case, and used with a small image, the temperature measured would be too small, and hence the value of σ too great.

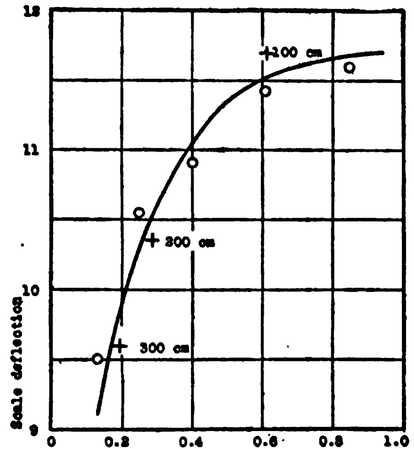


FIG. 27.—*Emf as a function of the size of image at the receiver. Féry fluorite lens pyrometer*

(*c*) FOSTER FIXED-FOCUS PYROMETER.—This pyrometer is so constructed that, as far as the geometry of the instrument is concerned, the front diaphragm opening acts as the radiating source provided the actual source has a diameter at least one-tenth the distance from the actual source to the wing nut of the pyrometer. Thus if the source were enlarged to a diameter more than one-tenth this distance the readings of the instrument should be unaffected. Similarly, for any definite size of source, the readings should remain constant, as the distance from the wing nut to the source is increased up to the point where this distance becomes ten times the diameter of the source. For still greater distances the front diaphragm is not sufficiently covered by the source, so that a rapid decrease in the readings will occur. Actually the readings of this pyrometer decrease when the distance from the source increases, even though the aperture of the instrument is always completely filled, on account of (1) atmospheric absorption, (2) radiation from the side walls and diaphragms of the front part of the pyrometer case to the thermocouple receiver, and (3) stray reflection from the side walls. The largest error is probably due to stray reflection and could be decreased by cutting a screw thread inside the entire length of the receiving tube.

---

[22] Féry and Drecq, J. de Phys., 1 (5) pp. 551-9; 1911.

[23] Coblentz, Phys. Zs., 15, pp. 762-764; 1914.

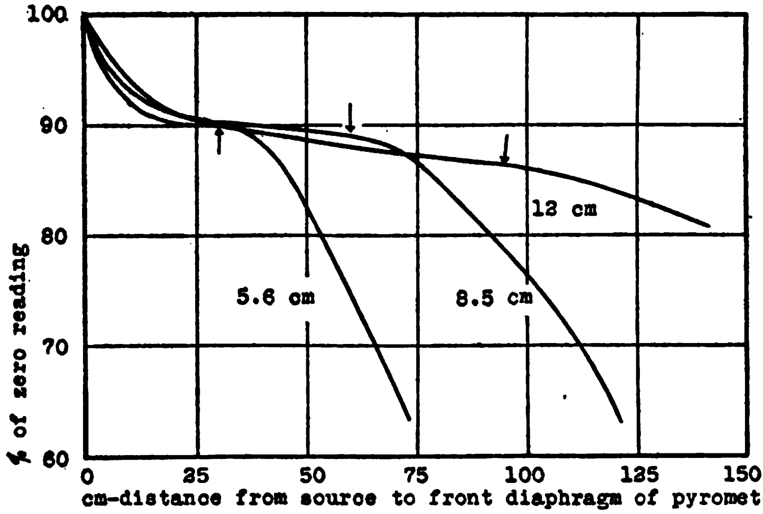


FIG. 28.—*Emf as a function of the sighting distance. Foster pyrometer*

However radiation from the heated side walls and diaphragms of the front part of the pyrometer may amount to a considerable fraction of the entire radiation received by the couple, even though the temperature of the case does not rise above 100° C. This radiation effect may be reduced by water-cooling the pyrometer case and by the use of heavy metal plate diaphragms (preferably copper), making good contact with the walls in order that the heat they receive may be readily conducted to the outside.

Both the effects of reflection and radiation from the side walls depend upon the size of the source, as well as the sighting distance. Fig. 28 illustrates the decrease in reading of a Foster pyrometer with increasing sighting distance (measured from the source to the front of the pyrometer) for three different sources having diameters 5.6, 8.5, and 12 cm. The arrows mark the point at which the given-sized source is insufficient to cover the front diaphragm of the pyrometer. It is only with the readings from zero distance up to this point that one is here concerned. The readings were taken rapidly, just as in actual practice, and hence the curves do not show the variation with distance as prominently as they would if several minutes were allowed for each determination. Also, at zero distance the water-cooled diaphragm in front of the nickel strip does not act as the real source, although this fact was not considered in drawing the curves. Thus, using the pyrometer under these favorable conditions, a 12 per cent variation in emf was observed over a range in distance at every point of which the aperture of the instrument was filled.

It is evident that the most satisfactory sighting distance from a source of definite size, both for calibration and use of this pyrometer, is that distance at which the slope of the curve, distance vs. emf, is a minimum, i. e., where the variation of emf with distance is small. The ratio of the best distance to the diameter of the source is roughly constant, as seen from the following table obtained from the curves of Fig. 28:

Diameter source	Best distance—		$\frac{\text{Best distance}}{\text{diameter source}}$.
	To front diaphragm	To wing nut	
cm	cm	cm	
5.6	19–27	44–52	7.9–9.3
8.5	28–44	53–69	6.2–8.1
12.0	60–70	85–95	7.1–7.9

Thus, for the best results, this individual pyrometer should be calibrated and used at a distance from the source (measured to the wing nut) approximately seven to eight times the diameter of the source. Distances greater than ten times the diameter of the source are barred because of the aperture of the instrument, and distances smaller than seven to eight times the diameter of the source should not be used on account of stray reflection and radiation from the walls of the pyrometer tube. Even after imposing this limitation upon the use of this pyrometer, the instrument for the highest accuracy should not be used with sources or sighting distances materially different from those used during the calibration, on account of atmospheric absorption and the fact that the amount of stray reflection from a source of constant angular dimensions (measuring from the wing nut) increases as the source is brought nearer the front opening of the pyrometer.

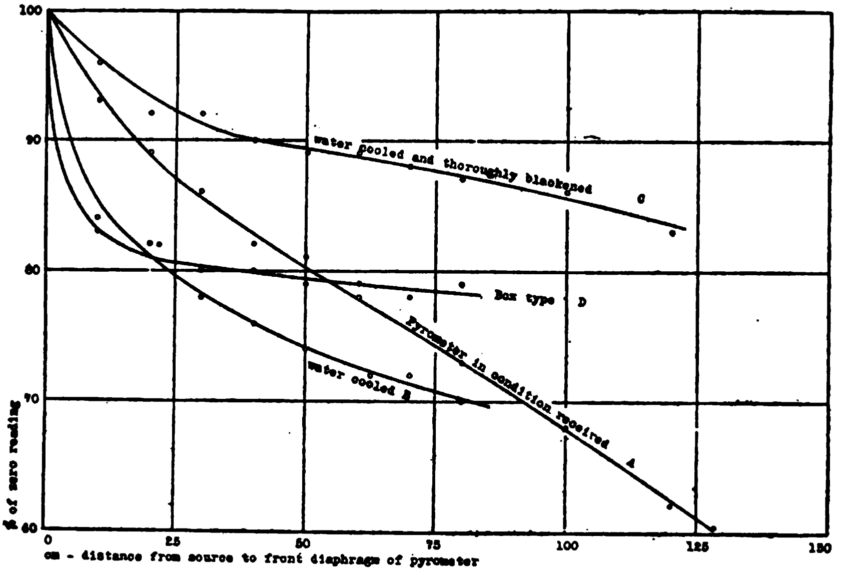


FIG. 29.—*Emf as a function of the sighting distance. Thwing pyrometers*

In general, therefore, it may be concluded that the readings of the Foster fixed-focus pyrometer decrease with increasing distance from the source, but that the errors to which this instrument is subject are very much smaller than those observed with the Féry pyrometer, as constructed at present.

(*d*) THWING PYROMETERS.—The Thwing pyrometers are, in general, subject to the same errors affecting the Foster pyrometer. The readings decrease with increasing sighting distance, as illustrated in Fig. 29. The readings here plotted were obtained with a source 12 cm in diameter, which is of sufficient size to fill the aperture of the instruments at every distance represented. Curve *A* refers to a tube form pyrometer No. 119 in exactly the condition received. The instrument was then remodeled by completely water-cooling the receiving tube and the front diaphragm. In order to illustrate the importance of thoroughly blackening the inside of the pyrometer tube, the black paint was removed from the front part of the case, and in this condition curve *B* was obtained, representing the variation of emf reading with sighting distance. When the inside of the water-cooled tube was thoroughly blackened with smoke from an acetylene flame, curve *C* was obtained. The water-cooling prevented the pyrometer from becoming warm even when 1 or 2 cm from the radiating strip, so that curve *C* represents exposures for any length of time whatever, and hence can not be compared with curve *A* or with the curves of Fig. 28 obtained with the Foster pyrometer for short exposures. Since water-cooling maintains a uniform temperature of the pyrometer tube for all sighting distances, the variation in emf illustrated by curve *C* can not be due to stray radiation from the side walls. The decrease in emf must, therefore, be caused by atmospheric absorption, and more especially by stray reflection. The entire interior of the tube of the Thwing pyrometers is cut with a shallow screw thread; and even though this was thoroughly blackened with soot from the acetylene flame it is still probable, from a consideration of curve *D*, that a greater part of the variation with distance is due to stray reflection.

Curve *D* represents the variation of emf with distance for a box-type (Fig. 12) pyrometer. In this instrument there are no side walls to produce stray reflection, and hence the error from this source is practically nil. The decrease of the first 20 cm is due mainly to heating and reradiation from the front diaphragm, this blackened diaphragm becoming almost red-hot when the source is close. The variation in emf from 25 to 100 cm is less than 2 per cent. Apparently, then, if the front diaphragm of this pyrometer were thoroughly water-cooled, the readings would be practically independent of distance, except in so far as any effect of atmospheric absorption obtains.

(*e*) BROWN PYROMETER.—Fig. 30 illustrates the variation of emf with sighting distance for a Brown pyrometer. The points close to the source were taken as quickly as possible on account of the rapid heating of the thin wall of the pyrometer tube, the heating in a few seconds being sufficient to cause fuming of the inside-painted surface. Consequently the emf's observed at these

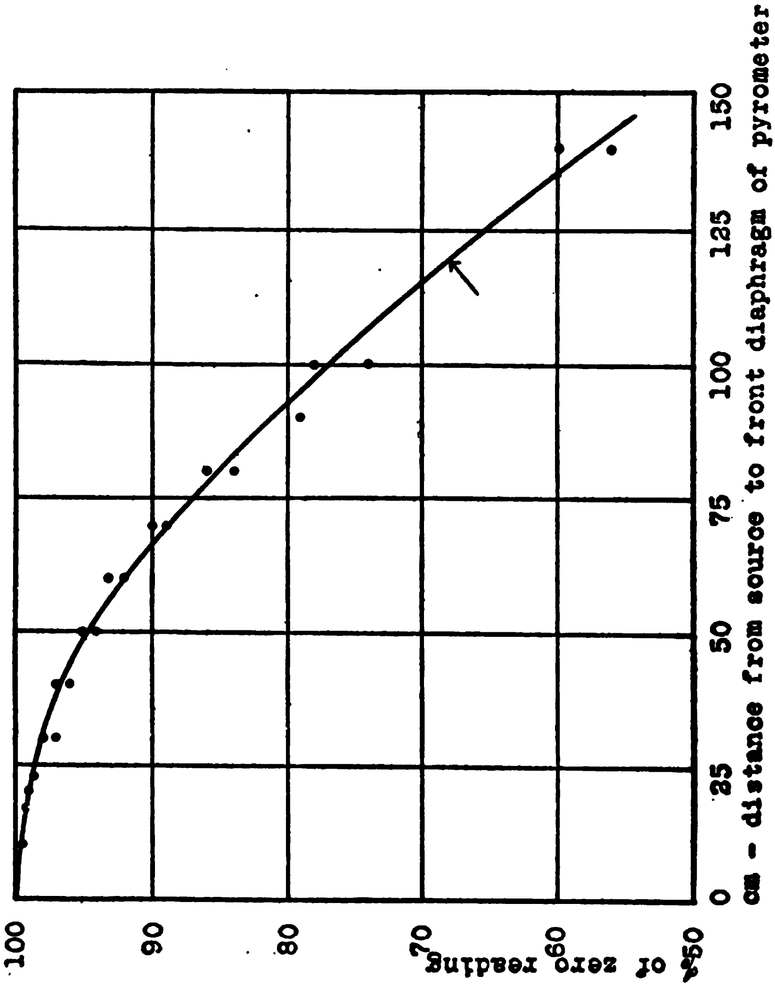


Fig. 30.—*Emf as a function of the sighting distance. Brown pyrometer*

positions are all too small. Even under this condition the variation in emf observed, from zero distance up to the maximum distance allowable by the aperture of the instrument, is about 65 per cent of the zero reading. Thus, if the pyrometer were calibrated on a 12-cm source at 115 cm distance and used at distances close to the source, the instrument would indicate emf's too great by over 50 per cent, i. e., absolute temperatures too great by over 12 per cent.

On account of the construction of this pyrometer with no real front diaphragm the errors due to stray reflection are of considerable magnitude.

5. THE TEMPERATURE OF THE RECEIVER

In the discussion on page 131 it was assumed that the temperature of the hot junction of the thermocouple, T_0, did not vary much from that of the room. This assumption is approximately true in the case of some pyrometers; for example, the Thwing, but with certain Féry instruments, T_0, may rise very considerably.

The value of T_0 for any value of T may be computed under certain assumptions. Thus, in the case of the Féry mirror pyrometer, let the concave mirror be considered a perfect reflector and suppose no heat is lost from the black receiver by either conduction or convection. Let the source sighted upon be a black body of temperature T. As far as the receiver is concerned the mirror itself acts as the source, subtending a solid angle at the receiver of about 0.2 π. If the source subtended a solid angle of 4 π, i. e., a spherical shell inclosing the receiver, T_0 would equal T. It follows then from the Stefan-Boltzmann law that

$$\frac{T^4_0}{T^4}=\frac{0.2\pi}{4\pi}$$

Hence,

$$T_0=0.47\ T \text{ approximately.}$$

Thus, for the given case cited, the absolute temperature of the receiver would be about half that of the radiator. Actually, T_0 never attains such high values on account of convection and conduction losses.

The temperature of the cold junction of the thermocouple receiver is, in general, closely that of the room, and should change very little for different values of T. Hence, if the temperature-emf relation of the thermocouple is known, the temperature of the hot junction or receiver, T_0, for any furnace temperature, T, may be determined from the emf developed.

As an example of the actual variation of T_0 with the temperature of the furnace, the following table is given for Féry pyrometer

No. 109. This instrument has an iron-constantan couple which develops an emf of about 0.054 millivolts per degree difference in temperature between its hot and cold junction. The temperature of the cold junction is considered that of the pyrometer case, 30° C.

Temperature of furnace	Emf	Temperature of receiver	T_0	T
°C	Millivolts	°C	° abs.	° abs.
700	1.004	49	322	973
800	1.462	57	330	1073
900	2.058	68	341	1173
1000	2.811	82	355	1273
1100	3.759	100	373	1373
1200	4.925	121	394	1473
1300	6.310	147	420	1573
1400	8.026	178	451	1673
1500	10.03	216	489	1773

Thus, when the pyrometer is sighted upon a source at a temperature 1500° C the receiver is at 216° C, the cold junction of the thermocouple being 30° C.

The temperature of the receiver of a Thwing pyrometer does not rise to this high value, as is seen from the following table for pyrometer No. 120. This pyrometer has a thermocouple of special composition, the temperature-emf relation of which was determined at the Bureau, and found to be very nearly linear, approximately 0.060 millivolts per degree difference in temperature of the hot and cold junctions. The cold junction temperature is taken as 30° C.

Temperature of furnace	Emf	Temperature of receiver	T_0	T
°C	Millivolts	°C	° abs.	° abs.
700	0.17	33	306	973
800	.24	34	307	1073
900	.34	36	309	1173
1000	.45	37	310	1273
1100	.60	40	313	1373
1200	.78	43	316	1473
1300	.99	46	319	1573
1400	1.25	50	323	1673
1500	1.54	54	327	1773

The preceding two examples can not be taken to represent the receiver temperatures of all the Thwing and Féry pyrometers. Many instruments of apparently the same construction have been found to give quite different values. The statement, however, is generally made that the receiver of the Féry pyrometer does not rise above 80° C. A value of 200° C would be nearer the receiver temperature, when the pyrometer is sighted on a source at 1500° C, for the average run of the Féry instruments.

It is quite apparent if T_0 increases fast enough with increasing T in the expression $e = a\ (T^b - T_0^b)$, that T_0^b may have nearly as great weight at high values of T (say 2000° C) as at low values (700° C). If T_0 remains nearly constant at room temperature, the curve drawn through the observations, plotted log e as abscissas and log T as ordinates, is a straight line at higher values of T with a slight bending away from the line at lower temperatures, the values lying above the line in this region. For such an instrument the correction, p (see p. 131), decreases with increasing values of log T. Frequently, however, T_0 increases so fast with increasing T, that the correction p is approximately constant, and the best curve through the observations on the log e vs. log T plot is a straight line from 600° to 1500° C. Fig. 16, curve B, represents the correction p for Féry pyrometer No. 109 discussed in this section. Values of p are just beginning to increase at the higher temperatures. From log $T = 3.10$ to 3.24 (i. e., $t°$ C = 1000 to 1500), p is nearly constant.

The errors in a calibration of a pyrometer as a result of neglecting the T_0 term entirely are small, only a few degrees at the most in the range 500° to 1500° C. The question has been considered here mainly because of the general impression that the effect of the T_0 term must necessarily rapidly decrease with increasing T. Actually, in some cases the T_0 term is just about as important at $t = 1500°$ C as at $t = 700°$ C.

VII. APPLICATIONS

1. DETERMINATION OF TOTAL EMISSIVITY OF NONBLACK MATERIALS

The total emissivity, E, of a nonblack material is defined as the ratio of the total energy (J'), emitted by the material at a given temperature to that (J) emitted by a black body at the same temperature. The apparent absolute temperature, S, of the nonblack radiator at a true absolute temperature, T, has a value such that the total energy emission is equivalent to that of a black body at an absolute temperature S. Hence, from the Stefan-Boltzmann relation, one obtains the following, σ being an empirical constant:

$$J = \sigma\ (T^4 - T_0^4)$$

$$J' = \sigma\ (S^4 - T_0^4)$$

$$\text{(1)} \qquad \frac{J'}{J} = E = \frac{S^4 - T_0^4}{T^4 - T_0^4} = \text{total emissivity}$$

If the energies J and J' are measured indirectly by the emf (e) developed in a thermoelectric circuit, as in the case of the thermoelectric radiation pyrometer, the following equations are convenient to use:

$$e = a\ (T^b - T_0^b)$$

$$e' = a\ (S^b - T_0^b)$$

$$\text{(2)} \qquad E = \frac{\left[\frac{e'}{a} + T_0^b\right]^{4/b} - T_0^4}{\left[\frac{e}{a} + T_0^b\right]^{4/b} - T_0^4} = \text{total emissivity}$$

In general, since the T_0 term is small, and since it occurs alike in numerator and denominator, the equations (1) and (2) may be written as follows:

$$\text{(1)}' \qquad E = \left(\frac{S}{T}\right)^4$$

$$\text{(2)}' \qquad E = \left(\frac{e'}{e}\right)^{4/b}.$$

The small error in E, caused by neglecting the T_0 term (equation 1), is illustrated by the following table. The value of T_0 is assumed constant at 300° abs.

Apparent temperature	S^4/T^4	Correction to subtract from $\frac{S^4}{T^4}$
°C		
500	0.9 or 0.1	0.0020
	.7 or .3	.0045
	.5	.0056
800	.9 or .1	.0008
	.7 or .3	.0010
	.5	.0015
>1100	.05 to 1.0	<.0005

The above derivation is on the assumption that the pyrometer employed may be properly considered as a black or gray receiver. (See p. 99.) This assumption is usually justifiable over a moderate range of temperature.

2. THE DETERMINATION OF TEMPERATURES

As follows from equation (1)′ above, the relation between the observed absolute temperature S, measured by the radiation pyrometer, and the true absolute temperature T, of the material sighted upon, is:

$S = \sqrt[4]{E} \cdot T$, or expressed in terms of the centigrade scale:

$$s°C = \sqrt[4]{E} \cdot (t°C + 273) - 273$$

where E is the emissivity or characteristic constant of the material. For substances in the open E is always less than 1, but frequently, in the case of the measurement of the temperature of furnaces, E becomes sufficiently near unity that s equals t for all practical purposes—that is, the temperature measured by the pyrometer is the correct temperature of the furnace, since the furnace is emitting black-body radiation. Thus, if the pyrometer is sighted upon a small hole in the side of a kiln, the walls of which are at a uniform temperature, correct temperature values will be indicated by the pyrometer.

It is frequently desired to measure the temperature of a certain part of the furnace which is not uniformly heated or to obtain the temperature of the interior of a furnace when there are present, around the walls, dense clouds of cooler smoke, gases, or fumes. This may be done by sighting the pyrometer into a long porcelain or metal tube hemispherically closed at one end, the closed end being inserted in the furnace at the required position, as illustrated in Fig. 31.

If a considerably greater area of the tube than that radiating to the pyrometer receiver is uniformly heated, the temperatures measured will be approximately correct, since black-body conditions are then approximately obtained. In general, the tube must have a rather large diameter, larger than that of the pyrometer. This is quite evident in the use of a fixed focus pyrometer such as the Thwing, Foster, or Brown. The cone of rays falling upon the receiver is diverging toward the source, measuring from a point within the pyrometer. Hence, if the tube were of the same diameter as the front opening of these pyrometers, the greater part of the area of the tube capable of radiating to the pyrometer receiver would be that included in the front part of the tube (i. e., the cold portion located in the wall of the kiln, Fig. 31). Hence, the temperature measured would be by no means that of the end of the tube *A*. The tube must, therefore, be large enough in diameter, so that the cone of rays drawn from the pyrometer will intersect its walls at a position where the temperature distribution from this point, at least, to the end *A* of the tube is uniform. If a long tube is required, it frequently happens that the diameter necessary to fulfill this condition is so great that the method becomes unsatisfactory. Moreover, in the use of large tubes, especially if the tube is of metal, the heat conduction away from the part of the furnace of which the temperature is desired becomes so high that correct temperature values can not be obtained. There appears to be no satisfactory method of readily meeting this difficulty, although reducing the aperture of the pyrometer will help, at a sacrifice, of course, of sensibility.

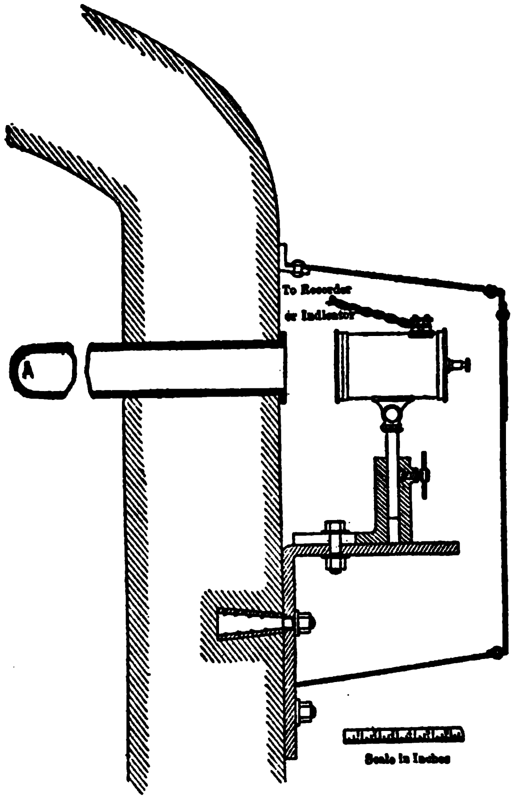


Fig. 31.—*Method of sighting a pyrometer into a closed tube inserted in the walls of a furnace*

It is not always necessary to have the tube of at least the diameter of the pyrometer in the case of focusing instruments such as the Féry, if the pyrometer is located some distance from the opening of the tube. If the tube is of smaller diameter than that of the mirror of the Féry, care must be taken that the cone of rays represented by lines drawn from the edges of the mirror to the limits of the required area of the source (of which the receiver is the image, i. e., the source of sufficient size to cover the hole in the focusing mirrors) lies wholly within the tube and does not cut the front end. Thus, if the pyrometer were sighted at such a distance that a source, say, 2 cm in diameter, were required, lines drawn from the top, bottom, and sides of this source to the corresponding points of the mirror must not cut the surface of the front part of the tube. The pyrometer, however, may be located at too great a distance from the front of the tube, thus requiring such a large source that lines drawn in the above manner will still intersect the tube. In this case the pyrometer must be focused at a closer distance, and if the cone still intersects the tube, the tube must be enlarged. Of course, it is not absolutely essential that these correct geometrical relations be realized, if a permanent installation is made and merely temperature control desired. In order to obtain true temperatures, however, with an incorrectly mounted system, the pyrometer would have to be calibrated in the exact position in which it is used, since in the above cases where the cone of rays intersects the tube, some part of the tube acts as a limiting diaphragm or entrance pupil and really becomes an essential unit of the system.

Whipple has slightly modified the Féry pyrometer by permanently mounting on the instrument a fire-clay, metallic, or graphite tube closed at one end, the pyrometer being focused on the bottom of the tube. This instrument has been employed for the purpose, among others, of measuring the temperature of molten metals, the tube being plunged directly into the metal.

When the radiation pyrometer is sighted directly upon objects in the open, it will read too low by amounts depending upon the emissivity of the material in question. This is especially true in the case of sighting upon molten metals which, when issuing from the furnace, generally present a clear surface, comparatively free from oxide. A slight film of oxide, in general, greatly increases the emissivity of the substance, and the pyrometer then reads more nearly correct. Thus, the corrections necessary to apply to the observed readings to convert them to true temperatures are usually

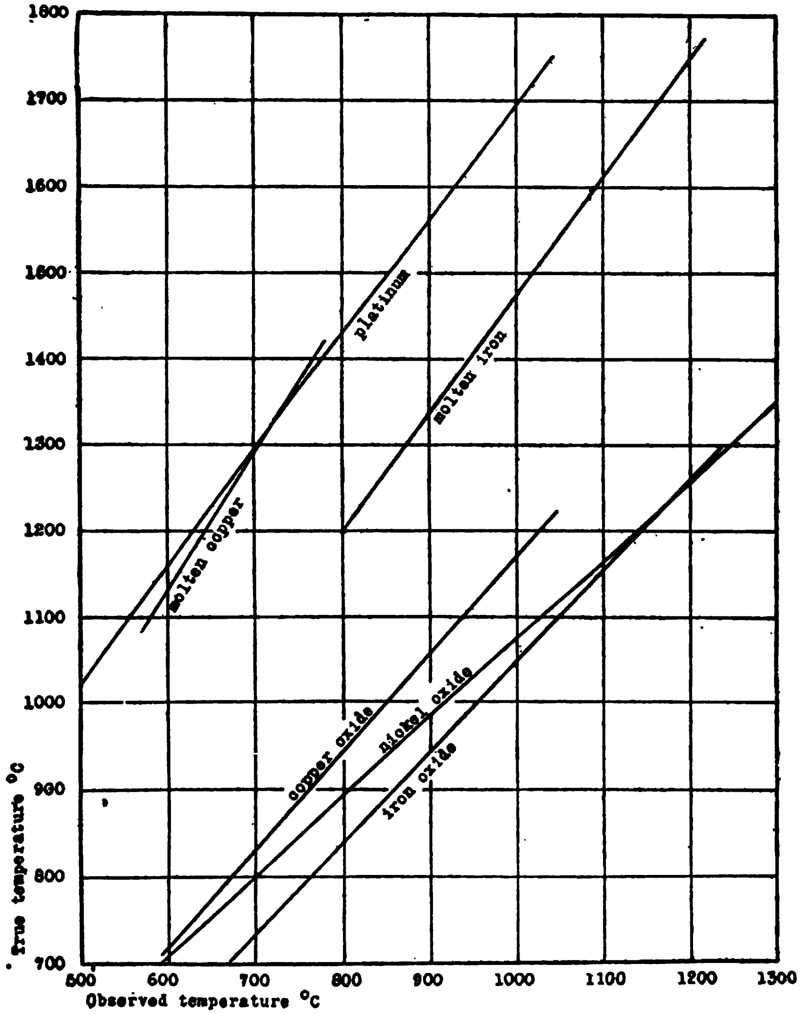


FIG. 32.—*True temperatures vs. temperatures observed with radiation pyrometer sighted upon materials in the open*

small in the case of solid iron or nickel at high temperatures in air, since the surface upon exposure to air almost immediately becomes heavily oxidized. If a molten surface is exposed for a few seconds to the air, it also becomes coated with an oxide slag, having a higher emissivity than the pure metal. The great increase in emissivity of a surface after oxidation has taken place is a phenomenon well known to the workmen in a foundry, who find that they can approach quite close to the stream of metal when poured from the furnace, but that as soon as the metal strikes the molds and comes in contact with the air, although it is necessarily cooler at this stage, the heat radiated becomes so intense that one can not longer stand near the metal.[24]

The experimental investigation of the total radiation of non-black materials or metals in the open, at high temperatures, has been confined to an extremely small number of materials. The following table shows the true temperatures corresponding to apparent temperatures measured by a radiation pyrometer when sighted upon molten copper, copper oxide, iron oxide, nickel oxide, and platinum. Determinations for other metals and oxides are in progress. This table is represented graphically in Fig. 32.

Observed temperature	True temperature ° C					
	Molten iron[25]	Molten copper[26]	Copper oxide[26]	Iron oxide[27]	Nickel oxide[28]	Platinum[29]
° C						
600	....	1130	720	630	710	1160
700	....	1290	830	735	800	1295
800	1200	....	945	835	895	1430
900	1340	....	1060	940	985	1560
1000	1475	....	1170	1040	1075	1695
1100	1610	....	....	1145	1165	....
1200	1750	....	....	1250	1255	....
1300	....	....	....	....	1350	....

[24] Cases have been noted for some of the rarer metals where the oxide possessed the smaller emissivity. However, this condition is very exceptional and is never met in practice.

[25] Thwing, Phys. Rev., 26, pp. 190–192; 1908.

[26] Burgess, Bureau of Standards Scientific Paper No. 121.

[27] Burgess and Foote, Bureau of Standards Scientific Paper No. 249.

[28] Burgess and Foote, Bureau of Standards Scientific Paper No. 224.

[29] Foote, Bureau of Standards Scientific Paper No. 243.

VIII. CONCLUSION

In the foregoing pages there have been given an account of the principles which form the basis for the operation of total radiation pyrometers, descriptions in some detail of representative types of this instrument, together with the results of an experimental study of their calibration and behavior under various conditions of use, and as modified by changing the several factors which may influence the readings of such pyrometers. A considerable portion of the text is devoted to the examination of the sources of error and their elimination or correction.

In all, there were carefully examined some 20 instruments, including all the ordinary types commonly met with in practice, such as the Féry, Foster, Thwing, and Brown pyrometers.

Although this investigation, which is largely concerned with instrumental details, does not readily lend itself to a brief summary, yet the following conclusions of a general nature may be mentioned:

The Stefan-Boltzmann law $E = a\ (T^4 - T_0^4)$ is not, in general, except by accident, obeyed in its exactness by any of the pyrometers examined. The similar equation $E = a\ T^4\ .\ T^{b-4}$, in which b is slightly different from 4 (usually neglecting the T_0 term) is, however, obeyed with sufficient exactness by all total-radiation pyrometers.

The main factors influencing the value of the exponent b are the geometry and mechanical construction of the instrument; the value of b for 20 thermoelectric pyrometers ranged from about 3.5 to 4.5. The same instrument of the Féry type may have a different exponent, according to its use with or without the sectored diaphragm for increasing the temperature range.

In general, a radiation pyrometer behaves not as a "black" or total receiver of energy but as a "gray" receiver for the ordinary range of temperatures. For the Féry pyrometer with a gold mirror, for example, the effect of selective reflection of the gold mirror does not become practically appreciable until above 2500° C but may cause an error of 500° C at the temperature of the sun.

The auxiliary apparatus, galvanometer, or potentiometer, and recording devices, can be constructed so that their errors will be practically negligible. For the work of the highest accuracy the potentiometric method of measurement is to be preferred with thermoelectric radiation pyrometers.

The principal errors to which the several types of radiation pyrometers are subject are shown to lie in the design and mechanical construction of these instruments; and certain of these inherent errors, such as lag or slowness in reaching an equilibrium reading, require for satisfactory results that the pyrometer be calibrated and used under similar conditions of time of exposure, distance from, and aperture of source. Wide variations in the lag effect exist among apparently similar instruments, ranging from a few seconds to an hour or more. These and other errors of appreciable magnitude, such as stray reflection, convection currents, intervening atmosphere, size of source, tarnishing of receiving mirror, etc., are discussed at length in the text.

It is shown that errors greater than 100° C may readily be caused by dirt on or oxidation of mirror.

The magnitude of errors due to varying the focusing distance may amount to several hundred degrees, if suitable precautions are not taken.

A convenient method for the rapid comparison of different types of pyrometer, and for determining the effects of size of aperture and focusing distance, was devised, consisting of a wide nickel (oxide) strip heated electrically, a series of circular, water-cooled diaphragms, and an optical bench.

The methods in use at this Bureau for the calibration of radiation pyrometers by means of especially designed experimental "black bodies" are described in detail and methods of extrapolation outlined, as well as methods of use of radiation pyrometers, including methods of obtaining approximately correct temperatures for the case in which a source of insufficient size is sighted upon.

Finally, there is considered the application of the radiation pyrometer to the determination of the total emissivity of non-black substances and to the measurement of temperatures.

In conclusion, the writers desire to thank the Taylor Instrument companies, of Rochester, N. Y., and the Thwing and the Brown companies, of Philadelphia, for very kindly placing at their disposal several radiation pyrometers.

WASHINGTON, January 2, 1915.

PUBLICATIONS ISSUED BY THE BUREAU OF STANDARDS, DEPARTMENT OF COMMERCE

SCIENTIFIC PAPERS

1. Recomparison of the United States Prototype Meter . . . . *L. A. Fischer*
2. A Study of the Silver Voltameter . . . . *K. E. Guthe*
3. The So-called International Electrical Units . . . . . . *Frank A. Wolff*
4. The Spectra of Mixed Gases . . . . *P. G. Nutting*
5. On Secondary Spectra and the Conditions under which They May Be Produced . . . . *P. G. Nutting*
6. Some New Rectifying Effects in Conducting Gases . . . . . *P. G. Nutting*
7. On Fibers Resembling Quartz in Their Elastic Properties . . . *K. E. Guthe*
8. On the Temperature of the Arc . . . . *C. W. Waidner and G. K. Burgess*
9. The Absolute Measurement of Inductance . . *E. B. Rosa and F. W. Grover*
10. The Absolute Measurement of Capacity . . . *E. B. Rosa and F. W. Grover*
11. Optical Pyrometry . . . . *C. W. Waidner and G. K. Burgess*
12. On the Theory of the Matthews and the Russell-Leonard Photometers for the Measurement of Mean Spherical and Mean Hemispherical Intensities . . . . *E. P. Hyde*
13. The Testing of Clinical Thermometers . . . *C. W. Waidner and L. A. Fischer*
14. Measurement of Inductance by Anderson's Method, Using Alternating Currents and a Vibration Galvanometer . *E. B. Rosa and F. W. Grover*
15. Use of Serpentine in Standards of Inductance . . *E. B. Rosa and F. W. Grover*
16. The Silver Coulometer . . . . *K. E. Guthe*
17. History of Standard Weights and Measures of the United States . . *L. A. Fischer*
18. Wattmeter Methods of Measuring Power Expended upon Condensers and Circuits of Low Power Factor . . . . *E. B. Rosa*
19. The Relative Intensities of Metal and Gas Spectra from Electrically Conducting Gases . . . . *P. G. Nutting*
20. The Use of White Walls in a Photometric Laboratory . . . . *E. P. Hyde*
21. Influence of Wave Form on the Rate of Integrating Induction Wattmeters . . . . *E. B. Rosa, M. G. Lloyd, and C. E. Reid*
22. Detector for Small Alternating Currents and Electrical Waves . . *L. W. Austin*
23. The Positive Charges Carried by the Canal Rays . . . . . *L. W. Austin*
24. Radiation from Platinum at High Temperatures . . . . . . *G. K. Burgess*
25. A Five-Thousand-Volt Generator Set . . . . *P. G. Nutting*
26. Talbot's Law as Applied to the Rotating Sectored Disk . . . *E. P. Hyde*
27. A New Determination of the Electromotive Force of Weston and Clark Standard Cells by an Absolute Electrodynamometer . . *K. E. Guthe*
28. The Gray Absolute Electrodynamometer . . . . *Edward B. Rosa*
29. Construction and Calculation of Absolute Standards of Inductance . *J. G. Coffin*
30. An Efficiency Meter for Electric Incandescent Lamps . . . . *E. P. Hyde and H. B. Brooks*
31. Calculation of the Self-Inductance of Single-Layer Coils . . . *Edward B. Rosa*
32. Heat Treatment of High-Temperature Mercurial Thermometers . . . . *Hobert C. Dickinson*
33. A New Potentiometer for the Measurement of Electromotive Force and Current . . . . *H. B. Brooks*
34. Spectrum Lines as Light Sources in Polariscopic Measurements . *Frederick Bates*
35. Polarimetric Sensibility and Accuracy . . . . *P. G. Nutting*
36. On the Platinum-Point Electrolytic Detector for Electrical Waves . *L. W. Austin*
37. The Influence of Frequency upon the Self-Inductance of Coils . . *J. G. Coffin*
38. Experiments on the Heusler Magnetic Alloys . . *K. E. Guthe and L. W. Austin*
39. A Pocket Spectrophotometer . . . . *P. G. Nutting*

40. Preliminary Measurements on Temperature and Selective Radiation of Incandescent Lamps . . . . . . *C. W. Waidner and G. K. Burgess*
41. Revision of the Formulæ of Weinstein and Stefan for the Mutual Inductance of Coaxial Coils . . . . *Edward B. Rosa*
42. The Mutual Inductance of Two Circular Coaxial Coils of Rectangular Section . . . . *Edward B. Rosa and L. Cohen*
43. On the Determination of the Mean Horizontal Intensity of Incandescent Lamps by the Rotating Lamp Method . . . *E. P. Hyde and F. E. Cady*
44. Purity and Intensity of Monochromatic Light Sources . . . . *P. G. Nutting*
45. Radiometric Investigations of Infra-Red Absorption and Reflection Spectra . . . . *W. W. Coblentz*
46. A Vacuum Radiomicrometer . . . . *W. W. Coblentz*
47. On the Geometrical Mean Distances of Rectangular Areas and the Calculation of Self-Inductance . . . . *E. B. Rosa*
48. The Compensated Two-Circuit Electrodynamometer . . . . *E. B. Rosa*
49. Complete Form of Fechner's Law . . . . *P. G. Nutting*
50. A Comparison of the Unit of Luminous Intensity of the United States with Those of Germany, England, and France . . . . . . *E. P. Hyde*
51. Geometrical Theory of Radiating Surfaces, with Discussion of Light Tubes . . . . *E. P. Hyde*
52. The Influence of Basic Lead Acetate on the Optical Rotation of Sucrose in Water Solution . . . . *F. J. Bates and J. C. Blake*
53. On the Calorimetric Determination of Iron, with Special Reference to Chemical Reagents . . . . *H. N. Stokes and J. R. Cain*
54. On Sulphocyanic Acid . . . . *H. N. Stokes and J. R. Cain*
55. Radiation from and Melting Points of Palladium and Platinum . . . . *C. W. Waidner and G. K. Burgess*
56. The Mutual Inductance of a Circle and a Coaxial Single-Layer Coil—The Lorenz Apparatus and the Ayrton-Jones Absolute Electrodynamometer . . . . *E. B. Rosa*
57. On the Establishment of the Thermodynamic Scale of Temperature by Means of the Constant-Pressure Thermometer . . . *Edgar Buckingham*
58. An Exact Formula for the Mutual Inductance of Coaxial Solenoids . . . . *Louis Cohen*
59. The Mutual Inductance of Coaxial Solenoids . . . *E. B. Rosa and L. Cohen*
60. The Production of High-Frequency Oscillations from the Electric Arc . . . . *L. W. Austin*
61. An Explanation of the Short Life of Frosted Lamps . . . . *E. P. Hyde*
62. Melting Points of the Iron Group Elements by a New Radiation Method . . . . *G. K. Burgess*
63. On the Determination of the Mean Horizontal Intensity of Incandescent Lamps . . . . *Edward P. Hyde and F. E. Cady*
64. Simultaneous Measurement of the Capacity and Power Factor of Condensers . . . . *F. W. Grover*
65. A New Determination of the Ratio of the Electromagnetic to the Electrostatic Unit of Electricity . . . . . *E. B. Rosa and N. E. Dorsey*
66. A Comparison of the Various Methods of Determining the Ratio of the Electromagnetic to the Electrostatic Unit of Electricity . . . . *E. B. Rosa and N. E. Dorsey*
67. Preliminary Specifications for Clark and Weston Standard Cells . . . . *F. A. Wolff and C. E. Waters*
68. Calorimetric Resistance Thermometers and the Transition Temperature of Sodium Sulphate . . . . . *H. C. Dickinson and E. F. Mueller*
69. On the Standard Scale of Temperature in the Interval 0° to 100° C. . . . . *C. W. Waidner and H. C. Dickinson*
70. Clark and Weston Standard Cells . . . . . *F. A. Wolff and C. E. Waters*
71. The Electrode Equilibrium of the Standard Cell . *F. A. Wolff and C. E. Waters*
72. A Comparative Study of Plain and Frosted Lamps . *E. P. Hyde and F. E. Cady*
73. The Variation of Resistances with Atmospheric Humidity . . . . *E. B. Rosa and H. D. Babcock*
74. On the Self-Inductance of a Toroidal Coil of Rectangular Section . . . . *Edward B. Rosa*

75. On the Self-Inductance of Circles . . . . *Edward B. Rosa and Louis Cohen*
76. The Influence of Frequency on the Resistance and Inductance of Solenoidal Coils . . . . *Louis Cohen*
77. The Atomic Weight of Hydrogen . . . . *W. A. Noyes*
78. The Best Method of Demagnetizing Iron in Magnetic Testing . *C. W. Burrows*
79. A Deflection Potentiometer for Voltmeter Testing . . . . . *H. B. Brooks*
80. The Self and Mutual Inductance of Linear Conductors . . . *Edward B. Rosa*
81. The Atomic Weight of Chlorine . . . . *W. A. Noyes and H. C. P. Weber*
82. The Preparation of Chloroplatinic Acid by Electrolysis of Platinum Black . . . . *H. C. P. Weber*
83. The Self-Inductance of a Coil of any Length and any Number of Layers of Wire . . . . *Edward B. Rosa*
84. Self-Inductance of a Solenoid of any Number of Layers . . . *Louis Cohen*
85. Instruments and Methods Used in Radiometry . . . . . *W. W. Coblentz*
86. A Quartz Compensating Polariscope with Adjustable Sensibility . . *F. J. Bates*
87. An Apparatus for Determining the Form of a Wave of Magnetic Flux . . . . *M. G. Lloyd and J. V. S. Fisher*
88. Effect of Wave Form upon the Iron Losses in Transformers . *Morton G. Lloyd*
89. The Luminous Properties of Electrically Conducting Helium Gas . . . . *P. G. Nutting*
90. Function of a Periodic Variable Given by the Steady Reading of an Instrument, with a Note on the Use of the Capillary Electrometer with Alternating Voltages . . . . *Morton G. Lloyd*
91. Selective Radiation from the Nernst Glower . . . . . . *W. W. Coblentz*
92. The Testing of Glass Volumetric Apparatus . . *N. S. Osborne and B. H. Veazey*
93. Formulæ and Tables for the Calculation of Mutual and Self-Inductance . . . . *Edward B. Rosa and Louis Cohen*
94. Some Contact Rectifiers of Electric Currents . . . . *L. W. Austin*
95. A Method for Producing Feebly Damped High-Frequency Electrical Oscillations for Laboratory Measurements . . . . *L. W. Austin*
96. On the Advantages of a High Spark Frequency in Radio-Telegraphy . . . . *L. W. Austin*
97. Selective Radiation from Various Solids . . . . *W. W. Coblentz*
98. Remarks on the Quartz Compensating Polariscope with Adjustable Sensibility . . . . *Frederick Bates*
99. Methods of Obtaining Cooling Curves . . . . *George K. Burgess*
100. Note on the Approximate Values of Bessel's Functions for Large Arguments . . . . *Louis Cohen*
101. The Influence of Terminal Apparatus on Telephonic Transmission . *Louis Cohen*
102. The Principles Involved in the Selection and Definition of the Fundamental Electrical Units to be Proposed for International Adoption . . . . *F. A. Wolff*
103. The Luminous Equivalent of Radiation . . . . *P. G. Nutting*
104. The Temperature Formula of the Weston Standard Cell . . . *F. A. Wolff*
105. Radiation, Constants of Metals . . . . *W. W. Coblentz*
106. Dependence of Magnetic Hysteresis upon Wave Form . . . *Morton G. Lloyd*
107. A New Form of Standard Resistance . . . . *Edward B. Rosa*
108. Errors in Magnetic Testing with Ring Specimens . . . . *Morton G. Lloyd*
109. The Testing of Transformer Steel . . . . *M. G. Lloyd and J. V. S. Fisher*
110. A New Method of Determining the Focal Length of a Converging Lens . . . . *Irwin G. Priest*
111. A New Method for the Absolute Measurement of Resistance . *Edward B. Rosa*
112. The Theory of Coupled Circuits . . . . *Louis Cohen*
113. A Volt Scale for a Watts-per-candle Meter . . . . . *Herbert E. Ives*
114. The Coefficient of Reflection of Electrical Waves at a Transition Point . . . . *Louis Cohen*
115. A Tungsten Comparison Lamp in the Photometry of Carbon Lamps . . . . *Herbert E. Ives and L. R. Woodhull*
116. The Determination of the Ratio of Transformation and of the Phase Relations in Transformers . . . . *E. B. Rosa and M. G. Lloyd*
117. The Determination of the Magnetic Induction in Straight Bars . . . . *Charles W Burrows*

118. A Method for Constructing the Natural Scale of Pure Color . . *P. G. Nutting*
119. An Approximate Method for the Analysis of EMF. Waves . . *P. G. Agnew*
120. The Thermoelectric Properties of Tantalum and Tungsten . *W. W. Coblentz*
121. The Estimation of the Temperature of Copper by Means of Optical Pyrometers . . . . . . *George K. Burgess, assisted by J. F. Crowe*
122. The Resolving Power of Objectives . . . . *P. G. Nutting*
123. The Theory of the Hampson Liquefier . . . . . . *Edgar Buckingham*
124. Platinum Resistance Thermometry at High Temperatures . . . . *C. W. Waidner and G. K. Burgess*
125. The Daylight Efficiency of Artificial Illuminants . . . . . *H. E. Ives*
126. Coupled Currents in which the Secondary has Distributed Inductance and Capacity . . . . *Louis Cohen*
127. Effect of Phase of Harmonics upon Acoustic Quality . . . . *M. G. Lloyd and P. G. Agnew*
128. White Light from the Mercury Arc and its Complementary . *Herbert E. Ives*
129. The Regulation of Potential Transformers and the Magnetizing Current . . . . *M. G. Lloyd and P. G. Agnew*
130. The Determination of the Constants of Instrument Transformers . . . . *P. G. Agnew and T. T. Fitch*
131. Selective Radiation from Various Solids, II . . . . . . *W. W. Coblentz*
132. Luminous Efficiency of the Firefly . . *Herbert E. Ives and W. W. Coblentz*
133. Luminosity and Temperature . . . . *P. G. Nutting*
134. A Theoretical and Experimental Study of the Vibration Galvanometer . . . . *F. Wenner*
135. Specific Heat of Some Calcium Chloride Solutions between −35° C. and +20° C . . . . *H. C. Dickinson, E. F. Mueller, and E. B. George*
136. On the Definition of the Ideal Gas . . . . *Edgar Buckingham*
137. Mica Condensers as Standards of Capacity . . . . . . *Harvey L. Curtis*
138. The Mutual Induction of Two Parallel Coaxial Circles in Terms of Hypergeometrical Series . . . . *Frederick W. Grover*
139. A New Method for the Absolute Measurement of Electric Quantity . . . . *Burton McCollum*
140. The Comparative Sensitiveness of Some Common Detectors of Electrical Oscillations . . . . *Louis W. Austin*
141. Photometric Units and Nomenclature . . . . *E. B. Rosa*
142. A Modified Method for the Determination of Relative Wave Lengths, Especially Adapted to the Establishment of Secondary Standards . . . . *Irwin G. Priest*
143. Note on the Temperature Scale between 100 and 500° C. . . . . *C. W. Waidner and G. K. Burgess*
144. A New Form of Direct-Reading Candlepower Scale and Recording Device for Precision Photometers . . . . . . *George W. Middlekauff*
145. A Device for Measuring the Torque of Electrical Instruments . . . . *F. G. Agnew*
146. The Intensities of Some Hydrogen, Argon, and Helium Lines in Relation to Current and Pressure . . . *P. G. Nutting and Orin Tugman*
147. The Temperature Coefficient of Resistance of Copper . . . *J. H. Dellinger*
148. The Electrical Conductivity of Commercial Copper . . . . *F. A. Wolff and J. H. Dellinger*
149. On the Constancy of the Sulphur Boiling Point . . . . *C. W. Waidner and G. K. Burgess*
150. Note on Oscillatory Interference Bands and Some of their Practical Applications . . . . *G. O. Squier and A. C. Crehore*
151. The Effect of Preliminary Heat Treatment upon the Drying of Clays . . . . *A. V. Bleininger*
152. The Reflecting Power of Various Metals . . . . *W. W. Coblentz*
153. The Action of Sunlight and Air upon Some Lubricating Oils . *C. E. Waters*
154. The Visibility of Radiation. A Recalculation of Konig's Data . *P. G. Nutting*
155. A Photometric Attachment for Spectroscopes . . . . . . *P. G. Nutting*
156. Selective Radiation from Various Substances, III . . . . *W. W. Coblentz*
157. The Measurement of Electrical Oscillations in the Receiving Antenna . . . . *L. W. Austin*

158. Some Experiments with Coupled High-Frequency Circuits *L. W. Austin*
159. Some Quantitative Experiments in Long Distance Radiotelegraphy *L. W. Austin*
160. The Behavior of High-Boiling Oils on Heating in the Air *C. E. Waters*
161. The Determination of Vanadium in Vanadium and Chrome-Vanadium Steels *J. R. Cain*
162. On the Computation of the Constant C_2 of Planck's Equation by an Extension of Paschen's Method of Equal Ordinates *Edgar Buckingham and J. H. Dellinger*
163. A Comparison of American Direct Current Switchboard Voltmeters and Ammeters *T. T. Fitch and C. J. Huber*
164. Study of the Current Transformer with Particular Reference to Iron Loss *P. G. Agnew*
165. Thermodynamics of Concentration Cells *Henry S. Carhart*
166. The Capacity and Phase Difference of Paraffined Paper Condensers as Functions of Temperature and Frequency *Frederick W. Grover*
167. The Steam Expansion Line on the Mollier Diagram and a Short Method of Finding the Reheat Factor *Edgar Buckingham*
168. Radiometric Investigation of Water of Crystallization, Light Filters, and Standard Absorption Bands *W. W. Coblentz*
169. Formulas and Tables for the Calculation of Mutual and Self Induction (2d edition, revised and enlarged) *E. B. Rosa and Frederick W. Grover*
170. The Correction for Emergent Stem of a Mercurial Thermometer *Edgar Buckingham*
171. Thermometric Lag *D. R. Harper*
172. Deflection Potentiometers for Current and Voltage Measurements *H. B. Brooks*
173. Outline of Design of Deflection Potentiometers with Notes on the Design of Moving-Coil Galvanometers *H. B. Brooks*
174. The Determination of Total Sulphur in India Rubber *C. E. Waters and J. B. Tuttle*
175. The Measurement of the Inductances of Resistance Coils *Frederick W. Grover and Harvey L. Curtis*
176. Luminous Properties of Electrically Conducting Helium Gas. II. Reproducibility *P. G. Nutting*
177. Resistance Coils for Alternating Current Work *H. L. Curtis and F. W. Grover*
178. The Hydrolysis of Sodium Oxalate and Its Influence Upon the Tests for Neutrality *William Blum*
179. Wave-Lengths of Neon *Irwin G. Priest*
180. On the Deduction of Wien's Displacement Law *Edgar Buckingham*
181. The Four-Terminal Conductor and the Thomson Bridge *Frank Wenner*
182. Standardization of Potassium Permanganate Solution by Sodium Oxalate *R. S. McBride*
183. Benzoic Acid as an Acidimetric Standard *G. W. Morey*
184. A Tubular Electrodynamometer for Heavy Currents *P. G. Agnew*
185. Thermometric Lag *D. R. Harper*
186. Determination of Manganese as Sulphate and by the Sodium Bismuthate Method *William Blum*
187. A New Precision Colorimeter *P. G. Nutting*
188. Instruments and Methods Used in Radiometry, II *W. W. Coblentz*
189. Antenna Resistance *L. W. Austin*
190. Energy Losses in Some Condensers Used in High-Frequency Circuits *L. W. Austin*
191. Selective Radiation from Various Substances, IV *W. W. Coblentz*
192. On a Modified Form of Stability Test for Smokeless Powder and Similar Materials *H. C. P. Weber*
193. Atomic Weight of Bromine *H. C. P. Weber*
194. The Silver Voltameter.—Part I. First Series of Quantitative Experiments *E. B. Rosa and G. W. Vinal*
195. The Silver Voltameter.—Part II. The Chemistry of the Filter Paper Voltameter and the Explanation of Striations *E. B. Rosa and G. W. Vinal*
196. The Diffuse Reflecting Power of Various Substances *W. W. Coblentz*

197. Density and Thermal Expansion of Ethyl Alcohol and of Its Mixtures with Water . . . *N. S. Osborne, E. C. McKelvy, and H. W. Bearce*
198. A Micropyrometer . . . . *G. K. Burgess*
199. A Simplified Formula for the Change in Order of Interference Due to Changes in Temperature and Pressure of Air . . . . . . *I. G. Priest*
200. New Calorimetric Resistance Thermometers . *H. C. Dickinson and E. F. Mueller*
201. The Silver Voltameter.—Part III. Second Series of Quantitative Experiments and the Preparation and Testing of Silver Nitrate . . . . *E. B. Rosa, G. W. Vinal, and A. S. McDaniel*
202. Note on Cold-Junction Corrections for Thermocouples . . . . *P. D. Foote*
203. The Analysis of Alternating Current Waves by the Method of Fourier, with Special Reference to Methods of Facilitating the Computations . . . . *F. W. Grover*
204. The Constants of Spectral Radiation of a Uniformly Heated Inclosure or So-Called Black Body, Part I . . . . . . *W. W. Coblentz*
205. Melting Points of the Refractory Elements.—I. Elements of Atomic Weight from 48 to 59 . . . . . . *G. K. Burgess and R. G. Waltenberg*
206. High-Frequency Ammeters . . . . *J. H. Dellinger*
207. A Comparative Study of American Direct-Current Watthour Meters . . . . *T. T. Fitch and C. J. Huber*
208. Windage Resistance of Steam Turbine Wheels . . . . . . *E. Buckingham*
209. Latent Heat of Fusion of Ice . *H. C. Dickinson, D. R. Harper 3d, and N. S. Osborne*
210. Observations on Ocean Temperatures in the Vicinity of Icebergs and in other Parts of the Ocean . *C. W. Waidner, H. C. Dickinson, and J. J. Crowe*
211. Accuracy of the Formulas for the Ratio, Regulation, and Phase Angle of Transformers . . . . *P. G. Agnew and F. B. Silsbee*
212. Melting Points of Some Refractory Oxides . . . . *C. W. Kanolt*
213. The Critical Ranges A_2 and A_3 of Pure Iron . . *G. K. Burgess and J. J. Crowe*
214. Note on the Setting of a Mercury Surface to a Required Height . *M. H. Stillman*
215. Micrometer Microscopes . . . . *A. W. Gray*
216. The Pentane Lamp as a Working Standard . . *E. C. Crittenden and A. H. Taylor*
217. Testing Potential Transformers . . . . *H. B. Brooks*
218. Comparison of the Silver and Iodine Voltameters and the Determination of the Faraday . . . . *G. W. Vinal and S. J. Bates*
219. Production of Temperature Uniformity in an Electric Furnace . *A. W. Gray*
220. The Silver Voltameter.—Part IV. Third Series of Quantitative Experiments and Special Investigations . *E. B. Rosa, G. W. Vinal, and A. S. McDaniel*
221. Influence of Atmospheric Conditions in the Testing of Sugars . . . . *F. J. Bates and F. P. Phelps*
222. Flame Standards in Photometry . . . . . *E. B. Rosa and E. C. Crittenden*
223. The Testing of Potentiometers . . . . . . *Frank Wenner and Ernest Weibel*
224. The Emissivity of Metals and Oxides.—I. Nickel Oxide (NiO) in the Range 600 to 1300° C . . . . *G. K. Burgess and P. D. Foote*
225. Adjustments of the Thomson Bridge in the Measurement of Very Low Resistances . . . . *Frank Wenner and Ernest Weibel*
226. Quantitative Experiments in Radiotelegraphic Transmission . *L. W. Austin*
227. Measurements on Standards of Radiation in Absolute Value . *W. W. Coblentz*
228. An Experimental Study on the Koepsel Permeameter . . *Charles W. Burrows*
229. Various Modifications of Bismuth-Silver Thermopiles Having a Continuous Absorbing Surface . . . . *W. W. Coblentz*
230. Combustion Calorimetry and the Heats of Combustion of Cane Sugar, Benzoic Acid, and Naphthalene . . . . *H. C. Dickinson*
231. Specific Heat of Copper in the Interval 0° to 50° C.; with a Note on Vacuum-Jacketed Calorimeters . . . . *D. R. Harper 3d*
232. Equilibrium in the System: Lead Acetate, Lead Oxide, and Water, at 25° . . . . *Richard F. Jackson*
233. A Watthour Meter Method of Testing Instrument Transformers . *P. G. Agnew*
234. Insulating Properties of Solid Dielectrics . . . . *Harvey L. Curtis*
235. A Direct-Reading Instrument for Measuring the Logarithmic Decrement and Wave Length of Electromagnetic Waves . . *Frederick A. Kolster*
236. Electrical Resistance and Critical Ranges of Pure Iron . . . . *G. K. Burgess and I. N. Kellberg*
237. Absorption, Reflection, and Dispersion Constants of Quartz . *W. W. Coblentz*

238. Characteristic Equations of Tungsten Filament Lamps and Their Application in Heterochromatic Photometry . . . *G. W. Middlekauff and J. F. Skogland*
239. A Vibration Electrometer . . . *W. W. Coblentz*
240. Studies on the Silver Voltameter . . . *G. A. Hulett and G. W. Vinal*
241. A Wheatstone Bridge for Resistance Thermometry . . . *C. W. Waidner, H. C. Dickinson, E. F. Mueller, and D. R. Harper 3d*
242. The Emissivity of Metals and Oxides. II. Measurements with the Micropyrometer . . . *G. K. Burgess and R. G. Waltenberg*
243. The Emissivity of Metals and Oxides. III. The Total Emissivity of Platinum and the Relation Between Total Emissivity and Resistivity . . . *Paul D. Foote*
244. A Comparison of Stellar Radiometers and Radiometric Measurements on 110 Stars . . . *W. W. Coblentz*
245. Temperature Coefficient of Magnetic Permeability Within the Working Range . . . *R. L. Sanford*
246. Methods of Measuring the Inductances of Low Resistance Standards . . . *Frank Wenner, Ernest Weibel, and F. B. Silsbee*
247. An Aneroid Calorimeter . . . *H. C. Dickinson and N. S. Osborne*
248. Specific Heat and Heat of Fusion of Ice . *H. C. Dickinson and N. S. Osborne*
249. The Emissivity of Metals and Oxides. IV. Iron Oxide . . . *George K. Burgess and Paul D. Foote*
250. Characteristics of Radiation Pyrometers . *George K. Burgess and Paul D. Foote*

CIRCULARS

1. Verification of Standards and Measuring Instruments.
2. Measurements of Length and Area, Including Thermal Expansion.
3. Verification of Standards of Mass.
4. Verification of Standards of Capacity.
5. Testing of Clinical Thermometers.
6. Fees for Electric, Magnetic, and Photometric Testing.
7. Pyrometer Testing and Heat Measurements.
8. Testing of Thermometers.
9. Testing of Glass Volumetric Apparatus.
10. Legal Weights (in pounds) per Bushel of Various Commodities.
11. The Standardization of Bomb Calorimeters.
12. Verification of Polariscopic Apparatus.
13. United States Government Standard Specifications for Incandescent Electric Lamps.
14. Analyzed Irons and Steels—Methods of Analysis.
15. The International Unit of Light.
16. The Testing of Hydrometers.
17. Magnetic Testing.
18. Standard Gauge for Sheet and Plate Iron and Steel.
19. Standard Density and Volumetric Tables.
20. Testing of Electrical Measuring Instruments.
21. Precision Measurements of Resistance and Electromotive Force.
22. Standard Specifications for Transformers, Oil-immersed, Self-cooled, 60-cycle, 2200 Volts.
23. Standardization of Electrical Practice in Mines.
24. Publications of the Bureau of Standards.
25. Standard Analyzed Samples—General Information.
26. Analyzed Iron and Manganese Ores—Methods of Analysis.
27. The Testing and Properties of Optical Instruments.
28. The Determination of the Optical Properties of Materials.
29. Announcement of a Change in the Value of the International Volt.
30. Lime: Its Properties and Uses.
31. Copper Wire Tables.
32. Standards of Gas Service.
33. United States Government Specification for Portland Cement.
34. The Relation of the Horsepower to the Kilowatt.
35. Melting Points of Chemical Elements.

36. The Testing and Properties of Electrical Condensers.
37. Electric Wire and Cable Terminology.
38. The Testing of Mechanical Rubber Goods.
39. Specifications for and Measurement of Standard Sieves.
40. Sodium Oxalate as a Standard in Volumetric Analysis.
41. Testing and Properties of Textile Materials.
42. Metallographic Testing.
43. The Metric Carat.
44. Polarimetry.
45. The Testing of Materials.
46. The Testing of Barometers.
47. Units of Weight and Measure; Definitions and Tables of Equivalents.
48. Standard Methods of Gas Testing.
49. Safety Rules to be Observed in the Operation of Electrical Equipment and Lines.
50. National Standard Hose Couplings and Fittings for Public Fire Service.
51. Measurement of Time and Tests of Timepieces.
52. Regulation of Electrotyping Solutions.
53. The Composition, Properties, and Testing of Printing Inks.
54. Proposed National Electrical Safety Code.
55. Measurements for the Household.

TECHNOLOGIC PAPERS

1. The Effect of Preliminary Heat Treatment upon the Drying of Clays (53 pp.) . . . . *A. V. Bleininger*
2. The Strength of Reinforced Concrete Beams. Results of Tests of 333 Beams (first series) (200 pp.) . . . . . *R. L. Humphrey and L. H. Losse*
3. Tests of Absorptive and Permeable Properties of Portland Cement Mortars and Concretes, Together with Tests of Damp Proofing and Waterproofing Compounds and Materials (127 pp.) . . . . *Rudolph J. Wig*
4. The Effect of Added Fatty and Other Oils upon the Carbonization of Mineral Lubricating Oils (14 pp.) . . . . *C. E. Waters*
5. The Effect of High-Pressure Steam on the Crushing Strength of Portland Cement Mortar and Concrete (25 pp.) . . . . *R. J. Wig*
6. The Determination of Chromium and Its Separation from Vanadium, in Steels (6 pp.) . . . . *J. R. Cain*
7. The Testing of Clay Refractories, With Special Reference to Their Load Carrying Capacity at Furnace Temperatures (78 pp.) . . . . *A. V. Bleininger and G. H. Brown*
8. A Rapid Method for the Determination of Vanadium in Steels, Ores, etc., Based on Its Quantitative Inclusion by the Phosphomolybdate Precipitate (20 pp.) . . . . . . *J. R. Cain and J. C. Hostetter*
9. The Density and Thermal Expansion of Linseed Oil and Turpentine (27 pp.) . . . . *H. W. Bearce*
10. The Melting Points of Fire Bricks (17 pp.) . . . . *C. W. Kanolt*
11. Comparison of Five Methods Used to Measure Hardness (27 pp.) . . . . *Ralph P. Devries*
12. Action of the Salts in Alkali Water and Sea Water on Cements (157 pp.) . . . . *P. H. Bates, A. J. Phillips, and R. J. Wig*
13. The Evaporation Test for Mineral Lubricating and Transformer Oils (13 pp.) . . . . *C. E. Waters*
14. Legal Specifications for Illuminating Gas (31 pp.) . *E. B. Rosa and R. S. McBride*
15. Surface Insulation of Pipes (44 pp.) . . . *Burton McCollum and O. S. Peters*
16. The Manufacture of Lime (130 pp.) . . . . *W. E. Emley*
17. The Function of Time in the Vitrification of Clays (26 pp.) . . . . *G. H. Brown and G. A. Murray*
18. Electrolysis in Concrete (137 pp.) . *E. B. Rosa, Burton McCollum, and O. S. Peters*
19. Physical Testing of Cotton Yarns (31 pp.) . . . . *W. S. Lewis*
20. Determination of Sulphur in Illuminating Gas (46 pp.) . . . . *R. S. McBride and E. R. Weaver*
21. The Dehydration of Clays (23 pp.) . . . *G. H. Brown and E. T. Montgomery*
22. The Effect of Overfiring Upon the Structure of Clays (23 pp.) . . . . *A. V. Bleininger and E. T. Montgomery*

23. The Technical Control of the Colloidal Matter of Clays (118 pp.) . *H. E. Ashley*
24. The Determination of Phosphorus in Steels Containing Vanadium (11 pp.) . . . . *J. R. Cain and F. H. Tucker*
25. Electrolytic Corrosion of Iron in Soils (69 pp.) . . . . *Burton McCollum and K. H. Logan*
26. Earth Resistance and Its Relation to Electrolysis . . . . *Burton McCollum and K. H. Logan*
27. Special Studies in Electrolysis Mitigation.—I. A Preliminary Study of Conditions in Springfield, Ohio, with Recommendations for Mitigation and Control (55 pp.) . . . . *E. B. Rosa and Burton McCollum*
28. Methods of Making Electrolysis Surveys . *Burton McCollum and A. H. Ahlborn*
29. The Variation in Results of Sieving with Standard Cement Sieves (16 pp.) . . . . *R. J. Wig and J. C. Pearson*
30. The Viscosity of Porcelain Bodies (11 pp.) . *A. V. Bleininger and Paul Teetor*
31. Some Leadless Boro-Silicate Glazes Maturing at about 1100° C. (22 pp.) . . . . *E. T. Montgomery*
32. Special Studies in Electrolysis Mitigation, No. 2. Electrolysis from Electric Railway Currents and Its Prevention—Experimental Test on a System of Insulated Negative Feeders in St. Louis (34 pp.) . . . . *E. B. Rosa, Burton McCollum, and K. H. Logan*
33. Determination of Carbon in Steel and Iron by the Barium Carbonate Titration Method (12 pp.) . . . . *J. R. Cain*
34. Determination of Ammonia in Illuminating Gas (23 pp.) . . . . *J. D. Edwards*
35. Combustion Method for the Direct Determination of Rubber (11 pp.) . *L. G. Wesson*
36. Industrial Gas Calorimetry (150 pp.) . . . *C. W. Waidner and E. F. Mueller*
37. Iodine Number of Linseed and Petroleum Oils (17 pp.) . . . . *W. H. Smith and J. B. Tuttle*
38. Observations on Finishing Temperatures and Properties of Rails (63 pp.) . . *G. K. Burgess, J. J. Crowe, H. S. Rawdon, and R. G. Waltenberg*
39. Analysis of Printing Inks (20 pp.) . . . . . *J. B. Tuttle and W. H. Smith*
40. The Veritas Firing Rings (10 pp.) . . . . *A. V. Bleininger and G. H. Brown*
41. Lead Acetate Test for Hydrogen Sulphide in Gas (46 pp.) . . . . *R. S. McBride and J. D. Edwards*
42. Standardization of No. 200 Cement Sieves (51 pp.) *R. J. Wig and J. C. Pearson*
43. Hydration of Portland Cement (71 pp.) . . . *A. A. Klein and A. J. Phillips*
44. Investigation of the Durability of Cement Drain Tile and Alkali Soils (56 pp.) . . *R. J. Wig and G. M. Williams, in cooperation with S. H. McCrory, E. C. Bebb, and L. R. Ferguson*
45. A Study of Some Recent Methods for the Determination of Total Sulphur in Rubber (16 pp.) . . . . *J. B. Tuttle and A. Isaacs*
46. A Study of the Atterberg Plasticity Method (18 pp.) . . . *Charles S. Kinnison*
47. Value of the High-Pressure Steam Test of Portland Cements (34 pp.) . . . . *R. J. Wig and H. A. Davis*
48. An Air-Analyzer for Determining the Fineness of Portland Cement (74 pp.) . . . . *H. C. Pearson and W. H. Sligh*
49. Emergent Stem Correction for Thermometers in Creosote Oil Distillation Flasks (21 pp.) . . . . *R. M. Wilhelm*
50. Viscosity of Porcelain Bodies High in Feldspar (7 pp.) . . . . *A. V. Bleininger and C. S. Kinnison*
51. Use of Sodium Salts in the Purification of Clays and in the Casting Process (40 pp.) . . . . *A. V. Bleininger*
52. Electrolysis and Its Mitigation . . . . . *E. B. Rosa and Burton McCollum*
53. An Investigation of Fusible Tin Boiler Plugs . . . . *George K. Burgess and P. D. Merica*
54. Special Studies in Electrolysis Mitigation. III. A Report on Conditions in Springfield, Ohio, with Insulated Feeder System Installed . . . . *Burton McCollum and G. H. Ahlborn*
55. Special Studies in Electrolysis Mitigation. IV. A Preliminary Study of Electrolysis Conditions in Elyria, Ohio, with Recommendations for Mitigation . . . . *Burton McCollum and K. H. Logan*
56. Protection of Life and Property Against Lightning . . . . . . *O. S. Peters*
57. Difference in Weight Between Raw and Clean Wools . . . . *Walter S. Lewis*

MISCELLANEOUS

International Metric System. (Chart.)
Tables of Equivalents of United States Customary and Metric Weights and Measures. (See Circular 47.)
The International Metric System of Weights and Measures. (Pamphlet.)
First Conference on the Weights and Measures of the United States.
Second Annual Conference on the Weights and Measures of the United States.
Third Annual Conference on the Weights and Measures of the United States.
Fourth Annual Conference on the Weights and Measures of the United States.
Fifth Annual Conference on the Weights and Measures of the United States.
Sixth Annual Conference on the Weights and Measures of the United States.
Seventh Annual Conference on the Weights and Measures of the United States.
Eighth Annual Conference on the Weights and Measures of the United States.
Ninth Annual Conference on the Weights and Measures of the United States.
State and National Laws Concerning the Weights and Measures of the United States (second edition).
The National Bureau of Standards. (Descriptive pamphlet.)
Report of the International Committee on Electrical Units and Standards.

⁂

WASHINGTON : GOVERNMENT PRINTING OFFICE : 1915

DEPARTMENT OF COMMERCE

Vol. 12 **No. 2**

BULLETIN

OF THE

BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

NOVEMBER 5, 1915

CONTENTS

Page

Interference Measurements of Wave Lengths in the Iron Spectrum (2851-3701), with Notes on Comparisons of Lengths of Light Waves by Interference Methods, and Some Wave Lengths in the Spectrum of Neon Gas . . . *Keivin Burns and W. F. Meggers* 179

Effective Resistance and Inductance of Iron and Bimetallic Wires . . . *John M. Miller* 207

A Direct-Reading Device for use in Computing Characteristics of Vacuum Tungsten Lamps . . . *J. F. Skogland* 269

A Study of the Quality of Platinum Ware . . . *George K. Burgess and P. D. Sale* 289

WASHINGTON
GOVERNMENT PRINTING OFFICE
1915

DEPARTMENT OF COMMERCE

Vol. 12 No. 2

BULLETIN

OF THE

BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

NOVEMBER 8, 1915

CONTENTS

Page

Interference Measurements of Wave Lengths in the Iron Spectrum (2851-3701), with Notes on Comparisons of Lengths of Light Waves by Interference Methods, and Some Wave Lengths in the Spectrum of Neon Gas . . . . *Keivin Burns and W. F. Meggers* 179

Effective Resistance and Inductance of Iron and Bimetallic Wires . . . . *John M. Miller* 207

A Direct-Reading Device for use in Computing Characteristics of Vacuum Tungsten Lamps . . . . *J. F. Skogland* 269

A Study of the Quality of Platinum Ware . . *George K. Burgess and P. D. Sale* 289

WASHINGTON
GOVERNMENT PRINTING OFFICE
1915

Harvard College Library
Dec. 15, 1915
From the
United States Government.

ADDITIONAL COPIES
OF THIS PUBLICATION MAY BE PROCURED FROM
THE SUPERINTENDENT OF DOCUMENTS
GOVERNMENT PRINTING OFFICE
WASHINGTON, D. C.
AT
25 CENTS PER COPY
SUBSCRIPTION PRICE, $1 PER VOLUME OF 4 NUMBERS.

INTERFERENCE MEASUREMENTS OF WAVE LENGTHS IN THE IRON SPECTRUM (2851-3701)

By Keivin Burns

With Notes on Comparisons of Lengths of Light Waves by Interference Methods, and Some Wave Lengths in the Spectrum of Neon Gas

By W. F. Meggers

CONTENTS

	Page
Introduction	179
Purpose	181
Apparatus	181
Sources	183
Method	184
Order	185
Reduction to normal conditions	185
Correction for phase change	185
(*a*) Wedge-shaped interferometer	185
(*b*) Etalons of different thickness	185
Reduction	186
Sources of error	189
Procedure in making a plate	189
Results	191
Comparison with former measurements	193
Summary	196

NOTES ON COMPARISONS OF LENGTHS OF LIGHT WAVES, ETC.

Introduction	198
Determination of the phase-change correction	199
Wave lengths in the spectrum of neon	202
Determination of the order of interference	203
Summary	205

INTRODUCTION

The determination of accurate wave lengths in the spectra of iron and of other elements has recently been undertaken at the Bureau of Standards. The need for wave-length determinations of great accuracy has long been felt by physicists, chemists, and astronomers. As an example, it is impossible to make a complete spectral analysis of iron, owing to our lack of exact knowledge of the spectra of some elements of the ferrous group. Before any reliable wave lengths can be obtained there must necessarily be a consistent fundamental system. On account of the irregularities existing in the Rowland system of wave lengths, an international conference of eminent spectroscopists in 1907 recommended the adoption of a new system of wave lengths based on the length of the red cadmium line as determined by Benoit, Fabry, and Perot by direct comparison with the meter. In this system, by means of the interferometer, secondary standards were to be determined, each by three independent observers, in the iron spectrum at intervals of 50 Å (0.005 μ); that is, the wave length of each standard was to differ by 50 Å from the wave lengths of its nearest neighbors to the violet and red. All wave lengths are to be reduced to 760 mm and 15°. Wave lengths measured on this system are designated by I. Å. (international Ångström). This same group of spectroscopists recommended in 1913 that the standards be observed, not at intervals of 50 Å, but of 10 Å, thus greatly increasing the work of determining a fundamental system. The Bureau of Standards has undertaken to do the part of this work for which the present need seems to be the greatest, and this paper presents the first installment of wave lengths to be measured here in keeping with the international plan. As soon as a satisfactory system of standards has been obtained we can proceed to the accurate measurement of wave lengths in the spectra of all the elements.

The spectrograms upon which this work is based were obtained at Marseille, in the laboratories of Messrs. Buisson and Fabry. The interferometer plates which were used there were afterwards very kindly loaned to the Bureau of Standards, and the phase change was redetermined here by W. F. Meggers. His results are found in the article which follows directly after this paper. All of the measuring and the greater part of the computing upon which the wave lengths in the iron spectrum are based was done by Mr. Meggers.

It is a pleasure to acknowledge indebtedness to Messrs. Buisson and Fabry for their interest and help; and to Director Campbell, who, by appointing me Martin Kellog Fellow of the Lick Observatory, University of California, made it possible to take the plates necessary for this investigation.

PURPOSE

At the time this investigation was undertaken (May, 1913) no international standards of wave length had been decided upon in the region to the violet of 3700 Å, and one object was to make measurements which might be combined with the results of other observers to furnish international standards. While this work was in progress international standards[1] were published for the region to the red of 3370 Å; but as yet only one series[2] of interferometer measurements of wave lengths shorter than 3370 Å has been published. Another object which was kept in view in undertaking this work was the determination of standards at intervals of 10 Å. Since this was recommended afterward by the wave-length committee of the Solar Union,[1] its importance will not be insisted upon here.

APPARATUS

It was necessary to use rather high dispersion and resolution in the spectograph in order to obtain a sufficiently large number of isolated lines. The apparatus was essentially the same as that used by former observers for the purpose of measuring secondary standards of wave lengths. The light of the iron arc was focused upon the etalon by the simple quartz lens, L_1, Fig. 1. The etalon, made of invar, was 3.75 mm thick. The interference plates were quartz disks, whose inner surfaces were coated with a film of nickel, cathodically deposited. These plates were just as they had been used by M. M. Buisson and Fabry some seven years before, a fact that is of interest in the discussion of the phase change.

---

[1] Kayser et alii, Astroph. J., 39, p. 93; 1914.

[2] Buisson and Fabry, Astroph. J., 28, p. 169; 1908.

The reflecting power of the films is satisfactory throughout the region under discussion. The rings are projected upon the slit by the quartz-fluorite lens, L_2. In the present instance the slit is mounted as indicated in Fig. 1, otherwise the spectograph is as described by Buisson and Fabry.[8] The light from the arc after passing through the slit falls upon the concave mirror *M*, which sends a parallel beam of light to the grating *G*. The slit and grating are so close together, as seen from the mirror, that the aberration due to the mirror is small, and in any case the effect is very small along the diameter of the ring system which is measured. The grating was ruled by Rowland. It has a radius of 635 cm and it is ruled about 580 lines per millimeter, having some 80 000 lines in all. The grating was used exclusively in the first order. In the present mounting the dispersion in this order is about 5 Å per millimeter. Using a slit 0.04 mm wide, lines 0.2 Å apart are easily measurable. The spectrum is focused by the grating upon the photographic plate at C_2. A flat plate was used, which gave practically perfect definition from 2900 Å to 3500 Å. Measurements extending to 3700 Å show that the slight falling off in definition had no appreciable effect upon the resulting wave lengths.

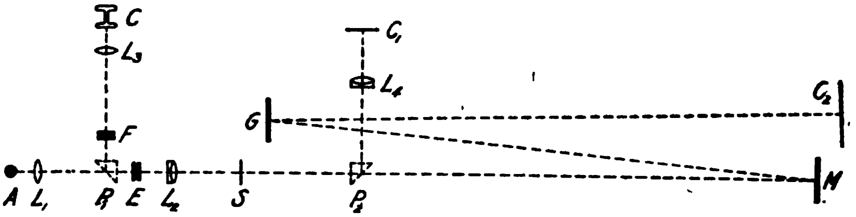


Fig. 1.—*Plan of apparatus*

Two holes cut in a brass plate served as a guage to show the ratio of the size of the ring system on the slit to the diameter of each ring on the photographic plate. This plate was slid into position along a groove in the front of the slit head. The cadmium light is focused upon the etalon by the simple quartz lens, L_3. The filter *F* cuts off all the light excepting that from the red line. When this cadmium line is to be photographed, the totally reflecting prisms P_1 and P_2 are put into position, as shown in the figure.

---

[8] Fabry and Buisson, Jour. de Phys., 9, p. 229; 1910.

P_1 throws the light upon the etalon and the rings are focused upon the slit by the quartz-fluorite lens, L_2. After passing through the slit the cadmium light is reflected by the prism P_2 into the objective of a small camera, L_4. The rings are photographed at C_1 without having passed through any dispersive element. In both of the cameras, C_1 and C_2, it is possible to slide the plate holder up and down, which provides for the making of several exposures on the same plate.

SOURCES

The standard wave length was furnished by a Michelson H tube, which was kept at a temperature of about 315°. It was necessary to bathe plates in order to photograph rings from this rather faint source in a reasonably short exposure. Fast plates, Lumière sigma or Seed 27, were bathed with dicyanin and ammonia. The iron arc was fed by the city current at 220 volts, and the arc burned between round electrodes 8 mm in diameter. In Table 1 are found some further data relating to the conditions of the arc and the length of the exposures. The amperage is seen to vary from 3.3 to 4, the mean being 3.6. While the length of the arc varies from 1.5 to 5 mm, only 5 out of 25 exposures were made with the arc shorter than 4 mm. The arc was stationary with the positive above, excepting on 6 exposures. In the case of 4a and 7c the arc was raised and lowered continuously, so that light from the positive pole might fall in turn upon all parts of the ring system.

On plate 8 the poles were interchanged by means of a switch several times during each exposure. It may be said that the polarity appears to have no effect upon the wave length. Excepting for the two lines, 3497.8 Å and 3610 A, the length of the arc within the limits used seems to be without effect. In the case of these two lines there may be a displacement, amounting to less than 0.005 Å, due to change in the length of the arc, but the reality of this displacement is quite uncertain. It is well known that many ultra-violet lines are broad when heavy current is used, and undoubtedly some of these lines would be shifted by increasing the current. But the lines are very sharp for the most part under the conditions used in making these plates, and no shift is to be expected from the use of a current less than 4 amperes.

In Table 1 the column headed "Temperature" contains the mean temperature at the etalon. Under "Arc" is given the length of the arc; under "Cadmium" the number of measurable exposures on the cadmium line in connection with each plate. Other column headings are self-explanatory.

TABLE 1

Plate	Exposure	Temperature	Bar	Amperes	Arc	Time of exposure	Cadmium	Slit
		°C	mm		mm	min.		mm
1	a	16.4	760	3.5	5.0	1	....	0.04
	b	16.4	760	3.5	5.0	2	....	.04
2	a	16.6	758	3.5	5.0	⅙	2	.04
	b	16.6	758	3.5	5.0	1	2	.04
	c	16.6	758	3.5	5.0	2	2	.04
	d	16.6	758	3.5	5.0	4	2	.04
3	a	17.2	759	3.3	4.5	8	4	.08
	b	17.2	759	3.3	4.5	4	4	.08
	c	17.2	759	3.3	4.5	2	4	.08
	d	17.2	759	3.3	4.5	1	4	.08
4	a[4]	17.2	759	3.3	2.0	5	4	.08
	b	17.2	759	3.3	4.0	30	4	.08
6	....	17.8	758	3.5	5.0	20	4	.12
7	a	17.6	756	3.5	5.0	10	4	.12
	b	17.6	756	4.0	1.5	11	4	.12
	c[4]	17.6	756	4.0	1.5	5	4	.12
	d	17.6	756	3.7	5.0	6	4	.12
	e	17.6	756	3.7	5.0	1	4	.12
8	a[5]	17.6	761	3.9	4.0	5	3	.12
	b[5]	17.6	761	3.9	4.0	10	3	.12
	c[5]	17.6	761	3.9	4.0	1	3	.12
	d[5]	17.6	761	3.9	4.0	⅓	3	.12
9	a[6]	18.0	761	3.4	2.0	10	....	.12
	b[7]	18.0	761	3.4	2.0	10	....	.12
10	a[8]	18.4	761	3.9	4.0	60	....	.12

[4] Displaced continuously.
[5] Poles interchanged at intervals.
[6] Positive above.
[7] Positive below.
[8] Poor.

METHOD

The method used is fully discussed by Buisson and Fabry.[9] The formulas are given below, but it seems unnecessary to repeat their derivation.

---

[9] See Note 2, p. 181.

ORDER

p = order of the center of the ring system.

P = order of a given ring (whole number).

θ = angular distance between the reference marks as seen from the Gauss point of the ring projection lens.

r = the distance between these marks on the plate.

d = the diameter of a ring.

$$p = P + P\, d^2 \frac{\theta^2}{8\, r^2}$$

$$\lambda_1 = \lambda_2 \frac{p_2}{p_1}$$

REDUCTION TO NORMAL CONDITIONS

Let γ be the correction in wave length necessary to reduce the observed value to what it would be under normal conditions.

λ_0 = wave length under normal conditions.

η_0 = refractive index of air corresponding to λ_0.

η'_0 = refractive index of air corresponding to λ 6438 under normal conditions.

M = density of the air under the conditions of observation.

M_0 = = density of the air under normal conditions. Then

$$\gamma = \lambda_0\, (\eta_0 - \eta'_0)\, \frac{M - M_0}{M_0}$$

CORRECTION FOR PHASE CHANGE

(*a*) WEDGE-SHAPED INTERFEROMETER.—Let η be the correction for phase change, expressed in wave length.

Let λ and λ' be the wave lengths, respectively, of the line in question and of the standard (cadmium red); q and q' the corresponding orders at a point in the interferometer marked by a cross; and e and e' the corresponding thickness. Then $2\,(e - e') =$

$$\epsilon = q\,\lambda - q'\,\lambda', \text{ and } \eta = \frac{\epsilon\lambda}{2e}$$

(*b*) ETALONS OF DIFFERENT THICKNESS.—Let T_1 be the thickness of one etalon; T_2 the thickness of another; and T that of the etalon in connection with which the phase change is required.

Let λ_1, λ_2, and λ be the corresponding wave lengths obtained by means of these etalons without correction for phase. Then

$$\eta = \frac{T_1(\lambda_2 - \lambda_1)}{T_2 - T_1} \times \frac{T_2}{T}$$

Messrs. Buisson and Fabry measured the phase change by means of a wedge-shaped interferometer. Mr. Meggers used etalons of different thicknesses. In this case the wave lengths of the neon lines were observed by means of an etalon of 2 mm thickness and by one of 15 mm thickness. The correction due to phase change as determined by Messrs. Buisson and Fabry and seven years later by Mr. Meggers differs by about 0.003 Å (3.75 mm etalon) for the interval 6400 Å to 3400 Å. (The former determination was made over five years before my work was done, and the latter determination followed my work by a year and a half.) This difference may be real; however, we have treated it as partly accidental and we have used the weighted mean of the two determinations. Since Mr. Meggers's result was obtained closer in time to the observations of wave length, his value was given double the weight of the other. The values at 3400 Å are: B & F+ 0.0084 Å; M., +0.0117 Å.

The correction for dispersion of the atmosphere amounted to only 0.0005 Å at most. The mean reduction for the six plates was added to the correction for phase change and applied to the mean wave length of each line.

REDUCTION

The cadmium line was measurable in connection with six plates (20 exposures). If every line had been measured an equal number of times on each plate, the mean of all measurements could be taken directly; but each plate differed systematically from the mean (due largely to accidental errors in measurement of the cadmium line), and some lines were measured almost entirely on plates that were above the mean. Other lines were measured mainly on plates which were below the mean. To correct these lines for this systematic difference, which might amount to 0.005 Å in an extreme case, there was formed what shall be called the fundamental mean. The mean of all exposures on a plate was taken as the value for each line on that plate, and the mean of the values thus obtained was formed for all lines occurring on four or more plates. In this way the cadmium line in connection with each plate affects the mean in about the same degree, regardless of the number of exposures on the plate.

Forty-eight lines distributed through the whole region were found on four or more plates, and many of them on all six plates. Differences between this series of means and the observed value were now plotted for each exposure and corrections were made by means of a straight line, which represented the observations. This line was inclined slightly in some instances with respect to the axis of wave length. The largest correction was 0.006 Å. From these corrections the probable value of a systematic error was found to be ±0.0011 Å. In other words, if we were interested in determining the wave length of the red cadmium line, using these iron lines as standards, the probable error of the determination would be just over ±0.002 Å. This is fairly satisfactory, considering the interval between these lines and the red cadmium line and the small thickness of etalon that had to be used.

TABLE 2

Mean Wave Lengths from Individual Plates

Plate \ λ	3370.776		3380.104		3383.978	
	Obsd.	Corr.	Obsd.	Corr.	Obsd.	Corr.
2..........	776	776	....	....	974	975
3..........	774	776	102	104	....	....
4..........	770	776	100	106	973	979
6..........	780	777	107	104	....	....
7..........	779	776	108	105	981	979
8..........	779	776	106	102	....	....
Mean....	0.7763	0.7762	0.1046	0.1042	0.9760	0.9777
Mean by exposures	....	.7761	....	.1041	....	.9777

In Table 2 are given the individual values of the wave lengths of three lines chosen arbitrarily in the middle of the series. The first column under the wave length gives the measured value, the second column gives the value corrected for the systematic difference of the plate from the mean. It is readily seen that when a line has been measured on several plates it is immaterial whether the mean of the corrected or uncorrected values be taken. If the line occurs on only three plates, the corrected mean may differ by a few thousandths of an Ångström from the uncorrected mean. In these means each plate was given equal weight, although one had four exposures. Giving each exposure equal weight the corrected mean is written at the bottom, and a comparison of this with the mean by plates shows that there is no appreciable difference between the two.

TABLE 3

Order at λ3370 as Determined from Separate Rings

Plate	Exposure	Order		2−1
		First ring	Second ring	
2	c	22321.125	123	− 2
	d	124	127	+ 3
3	a	138	138	± 0
	b	147	147	± 0
	c	121	127	+ 6
4		127	149	+22
6		098	118	+20
7	a	114	123	+ 9
	b	120	132	+12
	c	109	132	+23
	d	136	138	+ 2
8	a	131	136	+ 5
	b	146	172	+26
	c	22321.146	143	− 3
Mean				+ 9

In Table 3 are given the values of the order as determined from the first and second ring for each exposure on the line 3370. Only two rings were measured on each line. The tendency for the second ring to give a larger value of the order is common to all lines, but the reason for this could not be found. If one is right and the other wrong, the wave lengths are one part in four million in error for this reason alone.

SOURCES OF ERROR

The angle θ was measured in two different ways and the agreement of the results from the two measurements was not perfect. It is possible that the value of θ which was used may have caused a systematic error of 0.001 Å, but that is the extreme limit. It is probable that the error arising from this source is of no importance.

The measurements were examined for an effect of varying exposure. Interference rings are sharper on the outside than on the inside and it was feared that increasing exposure might tend to shift the center of gravity of the image toward the center of the system, resulting in increased wave length. No such effect was found, although very weak and very strong exposures of the same line were measured in several instances. This asymmetrical form of the interference rings may cause apparent shift towardt he red if the line is broadened. Such an effect is not apparent in the present work in case of lines marked "h" when these values are compared with grating measurements. The shifts due to instrumental causes in case of broad lines measured by interference are probably smaller than the displacements to which grating measurements of poor lines are liable, when the resolution and dispersion of the grating are varied.

PROCEDURE IN MAKING A PLATE

The usual procedure in making a plate is illustrated by the work on May 30, 1913. Plates are bathed in order to sensitize them to the red and put to dry. Fresh plates had to be bathed every day since the red sensitive plates do not keep very well unless held at a low temperature. While they are drying the gas is lighted under the cadmium lamp. By the time this is heated to the proper degree the grating, which is kept overnight in a drying chamber, has been put into place and a focus plate has been taken and developed. Also the parallelism of the interference plates has been adjusted and the etalon has been put into position and the center of the rings brought to the middle of the slit. For adjusting the parallelism of the plates I observed the rings due to the mercury lines; the light was furnished by a Cooper Hewitt lamp. The stained plates are now ready for use, and the cadmium is at the proper temperature. After testing the position of the cadmium lamp the totally reflecting prisms are put into place and two exposures of three minutes each are made on the cadmium rings. The cadmium camera must now be closed and the spectrograph opened and the prisms removed. The arc is adjusted on the etalon and the spectrograph plate holder opened. A 30-minute and a 10-minute exposure is made on the iron. This plate holder must now be closed, the spectrograph opened, and the prisms put into place. The cadmium camera is opened and two exposures of 3 minutes each are made on the cadmium rings. The etalon is now removed and two exposures of 20 seconds made on the gauge, which has to be slid into place. This finishes the cadmium plate and the holder is closed and removed. The spectrograph must be opened again and the prisms removed. Then the spectrograph plate holder is opened and a 7-second exposure made on the gauge. This completes the observation.

The temperature of the etalon is read at the first and the third cadmium exposure. The temperature of the cadmium lamp is read at each exposure. On account of using two cameras in different parts of the room and of the necessity for opening the spectrograph, there are many accidents which happen to plates. However, flaws in the red sensitive plates caused the greater part of the actual loss of exposures. On account of the location of the Marseille laboratory in the center of the city between two of the busiest streets, and due to the somewhat temporary nature of the mounting, it was not possible to make long exposures with satisfactory results. All attempts to get the faint lines of wave length shorter than 3000 Å failed through displacement of the apparatus during the long exposures. Six satisfactory plates were secured, and three or four other plates have one or two measurable exposures of the iron, but through accident these lack exposures of cadmium rings or reference marks. These latter plates, and in fact all of the plates, might be reduced by means of the I. A. Standards, of which there are six that can be measured. The wave lengths would be about 0.001 Å greater if reduced by these standards.

RESULTS

In Table 4 the column headed "Notes" contains the intensity and in some cases a letter indicating the character of the line at 6 amperes. It is to be noted that these lines were not reversed when the current strength was 4 amperes. The letters have significance as follows: b = broad, d = double, h = hazy, l = shaded to red, r = narrow reversal, R = broad reversal, and V = shaded to violet. The column headed "p. e." contains letters to indicate the dependence which may be put upon the given value. The best lines are indicated by A, those marked by B are fairly good, but C means that the line in question should be used with caution.

TABLE 4

Wave Lengths in the Iron Spectrum

(760 mm and 15°)

λ	Notes	p. e.	λ	Notes	p. e.	λ	Notes	p. e.
2851. 802	8	C	3116. 638	5	A	39. 440	8	B
2899. 422	4	C	25. 665	6	A	44. 189	8	B
2918. 031	5	C	29. 340	4	C	54. 367	4	A
41. 348	8	B	34. 115	—Ni[1]	A	57. 598	4	C
59. 998	4	C	51. 349	6b[2]	C	65. 622	6	A
87. 298	5	A	57. 043	4	B	71. 005	6b	A
90. 397	4	B	60. 660	6	B	80. 264	5	A
2999. 518	5r	B	75. 450	6	A	84. 593	4	C
3003. 036	4	B	78. 014	6	C	86. 760	8	A
11. 487	4	B	80. 229	8b	C	90. 992	4	B
17. 634	5r	B	84. 900	4	A	3298. 136	5	A
18. 989	5r	B	91. 664	5d	A	3305. 977	8	A
24. 038	5r	B	3199. 527	6b	C	06. 358	8	A
30. 156	4	A	3200. 478	6b	A	14. 746	6	A
40. 435	4	A	05. 401	7b	A	23. 741	4	A
45. 086	4	C	15. 944	5b	B	28. 870	4	A
55. 268	4	A	17. 385	4	A	37. 670	4	A
68. 180	4	A	22. 072	6b	A	47. 930	4	A
75. 726	5r	A	25. 792	8h	A	55. 232	4	A
83. 747	4r	A	30. 972	6b	B	69. 553	—Ni[1]	A
91. 582	4r	A	33. 056	5	A	70. 787	6	A
3098. 194	3	C	36. 227	6	A	80. 115	5	A

TABLE 4—Continued

Wave Lengths in the Iron Spectrum—Continued

λ	Notes	p. e.	λ	Notes	p. e.	λ	Notes	p. e.
83.988	5	C	76.707	5r[3]	A	3603.207	5	A
92.659	5	B	85.343	6	A	06.682	5	A
94.588	4	A	95.292	4V	A	10.159	5h	B
96.981	3	B	97.111	4	B	12.081	4	C
3399.339	6	A	3497.847	5r	B	17.789	6	A
3401.523	4	A	3506.500	5	A	21.462	6	B
02.262	4	B	13.822	5	A	22.004	6	B
07.464	71, d	A	21.266	5r	B	23.188	5	A
13.136	7	A	27.796	4	B	25.150	6	B
15.540	4	C	29.819	4	B	38.298	6	B
17.844	6	A	36.558	6	A	40.392	6	B
18.513	5	A	41.089	6h	A	45.826	4	C
24.289	6	A	42.080	6	A	51.469	6	B
27.122	6	B	45.643	5	A	59.520	5	B
28.197	6	A	54.928	8h	B	76.312	4	B
43.882	6R	C	56.878	6h	A	77.636	6[4]	B
45.152	4	A	58.519	5r	C	83.057	4	C
47.282	6	A	71.999	7h	B	84.112	5	C
50.332	6	A	78.689	—Cr[1]	A	89.457	6	C
58.306	3	B	89.108	4	A	3595.054	3	C
59.916	4	A	93.488	—Cr[1]	B	3701.082	6	C
68.850	4	B	3594.632	5	A			

NOTES TO TABLE 4

1. Lines of impurities are not as yet recommended for standards. In case the impurity is present only in small quantities, it is probable that the wave length of the impurity lines will be constant. Until the matter has been thoroughly investigated I prefer to use no such line.

2. This line is included because it shortens the gap between 3129 Å and 3157 Å. The measurements are discordant.

3. A line of intensity 2 at 3476.86 Å.

4. This line has been measured by Buisson and Fabry, by Pfund, and by Eversheim.[10] It is not reliable on my plates on account of a close companion of too great intensity.

TABLE 5

Comparison with Other Interference Measurements

B & F	B	P	I Å	B—B & F	B—P	B—I. Å.
2851.800	802	....	....	+2	....	....
2941.347	348	....	....	+1	....	....
2987.293	298	....	....	+5	....	....
3030.152	156	153	....	+4	+3	....
3075.725	726	726	....	+1	±0	....
3125.661	665	661	....	+4	+4	....
3175.447	450	....	....	+3	....	....
3225.790	792	....	....	+2	....	....
3233	056	055	....	....	+1	....
3271.008	005	008	....	+2	−3	....
3323.739	741	738	....	+2	+3	....
3370.789	787	788	789	−2	−1	−2
3399.337	339	338	337	+2	+1	+2
3445.155	152	154	154	−3	−2	−2
3485.344	343	346	345	−1	−3	−2
3515.820	822	821	821	+2	+1	+1
3556.879	878	883	881	−1	−5	−3
3606.681	682	....	682	+1	....	±0
3640.391	392	392	392	+1	±0	±0
76.312	312	....	313	±0	....	−1
Mean to 3300	....	....	....	+2.7	+1.0	−0.8
Mean after 3300	....	....	....	+0.1	−0.8	....

COMPARISON WITH FORMER MEASUREMENTS

Comparison with such I. Å. (Table 5) standards as occur in the region shows a mean difference of ±0.0015 Å and a systematic difference of half as much. The data are insufficient to establish the reality of so small a systematic difference. Table 5 also gives a comparison of my values with those of Buisson and Fabry[11] and of Pfund. Prof. Pfund kindly furnished me with a list of some of his lines which have not appeared in print. The systematic difference, Burns—Fabry and Buisson, is zero in the region of 3300 Å to 3676 Å. This difference in the region 2851 Å to 3300 Å is +0.0027, which is too large to be accidental. This systematic change at 3300 Å is supported to a large extent by the differences Burns − Pfund, yet it is difficult to see how anything systematic could enter into my measurements at this point, as all lines are measured on plates which covered the whole region at once.

---

[10] See Note 1, p. 181.

[11] See note 2, p. 181.

Between 3513 Å and 3622 Å there are 21 lines common to the present table and the measurements of Miss Howell.[12] This investigator used two gratings. One of these was ruled by Rowland and the other by Anderson. Both of these were 6-inch gratings of 21 feet radius; the former was ruled 20 000 lines per inch, the latter 15 000. Miss Howell used a 4-mm arc fed by 110 volts and running at 5.5 to 6.5 amperes. The I. Å. standards 5266, 5371, 5405, and 5434 were measured in the second order, and the lines to be determined were measured in the third order. Three I. Å. standards occur in this list, and the mean difference Howell − I. Å. is +0.0023 Å. For the same lines, B. I. (Burns interference) − I. Å. = −0.0007, so H − B. I. = +0.0030 Å. Three lines are not enough to furnish a basis for any conclusions, but it is to be noted that the mean difference H − B. I. for 21 lines is +0.0027 Å, and for 9 lines B. I. − I. Å. = −0.0008 Å.

It seems, then, reasonably safe to assume that the mean of Miss Howell's measurements is about 0.002 Å above the I. Å. scale at this point. This may be due to a discrepancy between the green and violet I. Å. standards, which in turn may be due to the variation of the wave lengths of the iron lines with changed conditions of the source. Aside from the systematic difference the mean value of H − B. I. is ±0.0015, which is remarkably small. Both the systematic and accidental difference between the two series is slightly reduced by using only the values which were determined by the 20 000-line grating instead of the means of the results which Miss Howell measured by each grating.

There are two series of measurements made by means of gratings, which extend throughout the region under discussion, the one by Burns,[13] the other by Viefhaus.[14] The former used the second order of a 6-inch Rowland grating of 21 feet radius, which was ruled 20 000 lines to the inch. He used the center of a long arc which was fed by a storage battery of 90 volts and burned at 6.5 amperes. Viefhaus used the center of a long arc, which was fed by 220 volts and burned at 7 amperes. His grating was by Rowland; it was ruled 10 000 lines to the inch and had over 45 000 lines. In the second order the scale was about 3 Å per millimeter. Comparing the present values with the grating measurements of Burns a mean difference of ±0.0028 Å is found. For the measurements of Viefhaus the mean difference is ±0.0056 Å.

---

[12] Howell, Astroph. J., 39, p. 230; 1914.

[13] Burns, Zs. f. wiss. Phot., 12, p. 207, 1913; and Lick Obs. Bulletin, 8, p. 27; 1913.

[14] Viefhaus, Zs. f. wiss. Phot., 13, pp. 209, 245; 1914.

Both series show the systematic difference due to the discrepancy in the region 2851 Å to 3300 Å between Buisson and Fabry values and those found in this article. In the region where I, Å. standards exist there is no systematic difference in V. – B. I. and B – B. I., which is large enough to reduce the residuals materially. In both series the need of more numerous standards is quite apparent, for large differences of like sign persist in some cases for 100 Å or more. As an extreme case of these departures consider the values of Burns between 3225 Å and 3323 Å. The standards in this region did not agree on his plates and as 3271 was broad and hazy it was not used, the reduction curve being passed through the other standards. This proves to have been a mistake, as shown by the residuals B – B. I. From 3225 to 3323 the residuals are: –2, +4, +5, +4, +9, +7, +5, +5, +7, +8, +4, +4, +3, +3, +1, +3, –1, ±0, ±0. The presence of two or three more standards between 3240 Å and 3300 Å would undoubtedly have made this large error impossible.

Goos [15] has recently published a series of wave-length determinations which extends from 3370 Å to the end of my list, the two series having 58 lines in common. This investigator used the standard conditions of the arc, which were recommended by the international wave-length committee. These conditions are for this region: Arc 6 mm long, of which only the central 2 mm are to be used; 4 amperes; 220 volts; 7 mm rods of iron; arc vertical, with positive pole above. The grating was used in parallel light, giving a dispersion in the second order of 2.5 Å per millimeter. This grating was ruled by Anderson and had some 78 000 lines.

---

[15] Goos, Astron. Nach., 199, p. 31; 1914.

The mean difference with regard to sign between the measurements by Goos and Burns's interference values is −0.0001 Å; in other words, the systematic difference is nil.

The tendency for residuals to be either positive or negative over a large interval of wave length is not so pronounced as in the case of the other grating results, which have been cited. The mean difference Goos − B. I. without regard to sign is ±0.0019 Å. This excellent agreement leads one to suppose that both series are nearly free from systematic error, and that the accidental errors seldom exceed one part in a million. No doubt the lines which show large residuals will not prove to be satisfactory as standards, but there is not sufficient homogeneous material at hand to warrant the discussion of this matter at present. A study of the lines by means of etalons of different thickness will eventually be undertaken. Considering only the lines which have been measured by interference, each series of grating measurements agrees slightly better with the interference values than with any other grating series.

While for the greater part of the region 2987 Å to 3701 Å satisfactory lines have been found at intervals of 10 Å or less, yet there are a few gaps of 15 to 20 Å where suitable lines are lacking. Both faint and strong lines were measured where that was possible. The advantage of having standards differing greatly in intensity has been discussed in previous papers.[16] The investigation will be extended as soon as possible to shorter and longer wave lengths.

SUMMARY

Between the limits 2851 Å and 3701 Å, 131 wave lengths have been measured in conformity with the recommendations of the international wave-length committee. Where it was found possible, faint lines as well as strong ones were measured. Throughout the greater part of the regions, 2987 Å to 3701 Å lines were measured at intervals of 10 Å or less.

The spectroscope used was a 21-foot grating, mounted in parallel light, and the etalon was 3.75 mm thick. The method was that of Buisson and Fabry.

---

[16] Burns, Zs. f. wiss. Phot., 12, p. 207, 1913; and 13, p. 235, 1915.

The mean difference between the present observations and the I. Å. standards is one part in two million; the systematic difference is half of that. Between 2851 Å and 3300 Å, where there are no I. Å. standards, a comparison with the work of Buisson and Fabry shows a systematic difference of one part in a million; allowing for the systematic difference, there is a mean difference of one part in three million. The systematic difference Burns − Pfund is +0.001 Å.; the mean difference is ±0.0018 Å in the region of wave length shorter than 3300.

Comparisons with different series of grating measurements confirms the belief, which is gaining ground, that secondary standards are needed at much closer intervals than was at first supposed.

WASHINGTON, February 26, 1915.

NOTES ON COMPARISONS OF LENGTHS OF LIGHT WAVES BY INTERFERENCE METHODS, AND SOME WAVE LENGTHS IN THE SPECTRUM OF NEON GAS

By W. F. Meggers

INTRODUCTION

The determination of secondary light-wave length standards in the arc spectrum of iron from $\lambda = 2900$ Å to $\lambda = 3700$ Å was recently completed at this Bureau, the results of which are published in the preceding article. To obtain these, it was necessary to redetermine the corrections due to change of phase at reflection from the nickeled-quartz interferometer plates. Messrs. Fabry and Buisson used these identical plates in similar work eight years ago[17] and found the corrections for change of phase at reflection to be an inverse linear function of the wave length.[18] Their method of obtaining this correction was again applied to these films recently, but an iron arc served as a source of light instead of a mercury lamp. The results confirmed the linear nature of the phase change.

Then the phase-change correction was found by another method, which will be described here. For this new determination neon in a fused-quartz vacuum tube was used as a source of light, and some experiences in this connection may be of interest and value to those engaged in this kind of work. Some wave lengths of neon lines, a new type of neon lamp, and a simple way of finding the order of interference in an etalon are incidental results of this phase-change determination, and notes on these points are contained in this article.

---

[17] Journal de Physique, 7, p. 169; 1908.

[18] Journal de Physique, 7, p. 417; 1908.

DETERMINATION OF THE PHASE-CHANGE CORRECTION

At reflection from metallic films light apparently penetrates the films a short distance, and the amount of this penetration varies with the wave length. This phenomenon is considered as a change of phase at the reflecting surface.

The correction due to change of phase at reflection which must be applied to wave lengths obtained by interference methods is usually made for each wave length either as a correction to the thickness of the etalon[19] e or to the order of interference[20] p. The purpose of this paragraph is to show how the correction can be made directly to the wave lengths from their uncorrected values as determined from a thin etalon e_1 and a thick etalon e_2. This method was used by Dr. Burns in his interference measurements of wave lengths in the spectrum of the iron arc $\lambda = 8824$ to $\lambda = 5434$.[21] The values from e_1 will be smaller or larger than those from e_2, depending on their position in the spectrum and on the nature of the metallic film. Let a point λ' in the spectrum be chosen as a standard of wave length and represent the measurements of another wave length λ as obtained from e_1 and e_2 by λ_{e_1} and λ_{e_2}, respectively. Let the difference $\lambda_{e_2} - \lambda_{e_1} = c$. If the effect of phase change be regarded zero at λ', the total correction to λ can be shown to be proportional to c.

Let $\delta\lambda_{e_1}$ and $\delta\lambda_{e_2}$ represent the phase-change error in wave length from e_1 and e_2, respectively. If the penetration of light into the films at reflection be regarded as an increase ϵ in the optical distance e between the interferometer plates, then

$$\lambda = \lambda_{e_1} + \delta\lambda_{e_1} = \frac{2\,(e_1+\epsilon)}{p_1} = \frac{2\,e_1}{p_1} + \frac{2\,\epsilon}{p_1}$$

and

$$\lambda = \lambda_{e_2} + \delta\lambda_{e_2} = \frac{2\,(e_2+\epsilon)}{p_2} = \frac{2\,e_2}{p_2} + \frac{2\,\epsilon}{p_2} \quad .$$

But $\lambda_{e_1} = \frac{2\,e_1}{p_1}$ and $\lambda_{e_2} = \frac{2\,e_2}{p_2}$ since the measured values of λ are always expressed in terms of the etalon thicknesses which result from the standard wave length λ' and the orders of interference which are obtained from measurements on the rings. Consequently,

---

[19] Journal de Physique, 7, p. 169; 1908.

[20] Astro-Physical Journal, 28, p. 197; 1908.

[21] Journal de Physique, (5) 3, p. 457; 1913.

$$\delta\lambda_{e_1} = \frac{2\,\epsilon}{p_1} \text{ and } \delta\lambda_{e_2} = \frac{2\,\epsilon}{p_2}$$

Then $\delta\lambda_{e_1}/\delta\lambda_{e_2} = p_2/p_1$, and since p is directly proportional to e for a given value of λ we have

$$\delta\lambda_{e_1}/\delta\lambda_{e_2} = e_2/e_1 \text{ or } \delta\lambda_{e_1} - \frac{e_2}{e_1}\delta\lambda_{e_2} = 0 \qquad (1)$$

Also

$$\delta\lambda_{e_1} - \delta\lambda_{e_2} = \lambda_{e_2} - \lambda_{e_1} = c \qquad (2)$$

Subtracting (2) from (1) gives

$$\delta\lambda_{e_2}\left(\frac{e_2}{e_1} - 1\right) = c \text{ or } \delta\lambda_{e_2} = \frac{e_1\,c}{e_2 - e_1} \qquad (3)$$

The phase-change correction from λ' to λ for any other thickness e is then

$$\frac{e_2}{e}\,\delta\lambda_{e_2} \text{ or } \frac{e_2}{e}\cdot\frac{e_1\,c}{e_2 - e_1}$$

In this way this error can be corrected from point to point in the spectrum.

For example, several wave lengths in the ultra-violet portion of the spectrum of neon were measured with 2-mm and 15-mm etalons, using $\lambda' = 5852.4862$ Å as a standard.

λ_{e_1}	λ_{e_2}	$\lambda_{e_2} - \lambda_{e_1} = c$
3417.888	3417.903	+.015 Å
3447.685	3447.703	.018
3520.457	3520.472	.015

The mean correction to λ in this region is therefore proportional to 0.016 Å and the amount which must be added to λ_{e_2} is

$$\delta\lambda_{e_2} = \frac{e_1\,c}{e_2 - e_1} = \frac{2 \times 0.016}{15 - 2} = 0.0025 \text{ Å}$$

Thus, the mean of three measures of $\lambda = 3417+$ with 15 mm etalons gave $\lambda_{e_2} = 3417.9031$, whence the true wave length is

3417.906 Å to three decimal places. Similarly, several lines near $\lambda = 6500$ gave $\lambda_{e_2} - \lambda_{e_1} = c = -0.004$ from which $\delta\lambda_{e_2} = \frac{2 \times -0.004}{15-2}$ $= -0.0005$ Å. The mean of three measures of $\lambda = 6532+$ with 15 mm etalons gave $\lambda_{e_2} = 6532.8834$ Å. The correct wave length is therefore 6532.883 Å.

The method used by Fabry and Buisson to determine the phase change correction consists in finding the difference in optical distances between the interferometer plates for different wave lengths. This can be done by finding the relatively small orders of interference p and p' for two wave lengths λ and λ' at the same point of an interferometer arranged with a thin wedge-shaped air space so as to give localized straight-line fringes. Then the difference in double optical thicknesses e and e' at this point for λ and λ' is $\epsilon = 2\ (e - e') = p\lambda - p'\ \lambda'$. From their curve ϵ between $\lambda = 6500$ and $\lambda = 3460$ was equal to 21 $\mu\mu$ or 210 Å. If the correction to wave length be regarded zero at $\lambda = 6500$ the correction to $\lambda' = 3460$ from a 15 mm etalon becomes $\frac{210 \times 3460}{30 \times 10^7} = 0.0024$ Å. The above measures on the neon lines give the correction over this interval as $0.0025 + 0.0005 = 0.0030$ Å. This difference may represent a real change in the film with time.

Whenever ϵ varies linearly with λ as in the case of these films, the correction to any wave length λ is $\frac{\delta\lambda_{e_2}}{\lambda - \lambda'}$ angstrom for each angstrom, which separates λ from the standard wave length used in the measurement. Thus the mean of three measures of $\lambda = 7032.+$ gave $\lambda_{e_2} = 7032.4143$ Å. The correction is $\frac{0.0030}{6500 - 3460} \cdot (5852 - 7032) = -0.0012$ and the true wave length is 7031.413 Å.

The difference $\lambda_{e_2} - \lambda_{e_1} = c$, upon which the correction depends, is larger the greater e_2 is and the smaller e_1 is. The accuracy of its determination is thus increased as the order of interference in e_2 becomes larger. The use of a source of light which will give a high order of interference is therefore to be recommended. Since it is always advisable to use as large as possible an etalon in interferometer measures of wave length, sufficient data is immediately at hand from which to make the phase change correction to the wave lengths measured if an additional set of measures is made with a thinner etalon.

WAVE LENGTHS IN THE SPECTRUM OF NEON

For measurements of wave lengths in the spectrum of neon the wave length of the neon yellow line $\lambda = 5852.4862$ Å, measured by Priest,[21] in terms of the cadmium red line $\lambda = 6438.4696$ was used as a standard. Eight wave lengths $\lambda = 5852$ to $\lambda = 6304$, which Priest measured, were again compared with this standard and found to agree very satisfactorily. The largest deviations were ±0.002 Å except for $\lambda = 6304.789$, where Priest's value is 0.004 Å larger. In addition, 7 longer wave lengths and 10 in the ultraviolet were measured and are here presented:

3369.903 c
3417.906 a
3447.706 a
3454.158 a
3460.533 c
3466.547 a
3472.573 a
3501.220 c
3515.196 c
3520.474 a

6506.532 b
6532.883 a
6598.951 a
6678.274 a
6717.042 a
6929.468 a
7032.413 a

The letters a, b, and c indicate probable relative accuracy in the determinations. The values marked "a" were obtained with the 15-mm etalon and should be quite reliable, while those marked "b" and "c" come from 7.5 mm and 2 mm etalons, respectively, and probably contain larger errors. These wave lengths are only provisional and will be verified by a direct comparison with the cadmium red standard as soon as possible.

Attention is called to the neon tubes which were used in this work. Their design is shown in Fig. 2. They are made of fused quartz and contain flat circular aluminum electrodes sealed into the ends. We take this occasion to thank the Cooper-Hewitt Electric Co., of Hoboken, for the interest and kindness shown in making these excellent tubes for us. They are extremely efficient and can be made to emit light of great intensity without appreciable heating or electrical evaporation of the electrodes. One of these tubes has seen over 25 hours' service at high intensity and shows no signs of electrode disintegration or gas occlusion. The tubes were filled at the Bureau of Standards with gas which was obtained from M. Claude. They were thoroughly exhausted, then filled with neon and a discharge passed through for several hours while they were alternately exhausted and filled three or four times. The electrodes probably became saturated with the gas in this way. The tubes were finally sealed off at a gaseous pressure corresponding to about 1 mm dark space.

---

[21] This Bulletin, 8, p. 539; 1911-12.

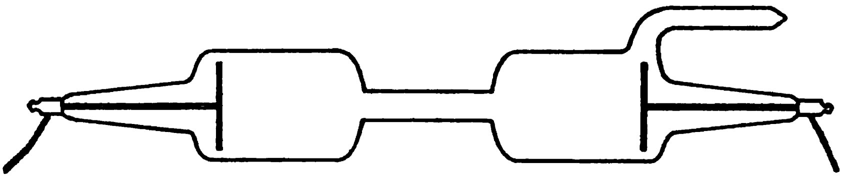


FIG. 2.—*Scale: Half size.*

DETERMINATION OF THE ORDER OF INTERFERENCE

The thickness of the layer of air between two interferometer plates is $e = \frac{\lambda p}{2} = \frac{\lambda}{2}(P + p')$ where p is the order of interference at the center of the system of circular fringes. P is an ordinal number representing the waves in the path difference producing any ring, and p' is the fractional portion calculated from observations on the rings with the aid of $p' = P\theta^2 d^2/8r^2$. The linear diameters of the ring and gauge images are represented by d and r, respectively, and θ is the angle at the lens subtended by the gauge. A roughly approximate value of P is sufficient for the first calculation of p'. If P is the order of the first ring from the center, p' will lie between 0 and 1.

To get the exact value of P quickly and easily the following modification of the methods used by Messrs. Perot and Fabry[22] and by Lord Rayleigh[23] is suggested. The use of 4 or 5 wave lengths from the spectrum of neon is recommended because of their particular applicability. Find the ratios R of any wave length to each of the others, e. g., $\frac{5852.4862}{\lambda}$. This, of course, gives the rate at which the order for each λ changes when that for $\lambda = 5852.4862$ changes one wave length. These values of λ and R can subsequently be used with any etalon. The following example will illustrate their use. This particular etalon is assumed to possess the length it was ordered to have, i. e., 15 mm. The approximate order of interference, P_{5852}, assuming the double thickness 30 mm is then $\frac{30}{0.0005852486} = 51260$. The fractional order p', however, is found to be 0.777, so that the tentative order p_λ may be called 51260.777 and the corresponding double thickness $51260.777 \times 0.0005852486 = 30.0002990$ mm. Dividing this thickness by the other wave lengths gives their tentative orders p_λ.

---

[22] Annales de Chimie et de Physique, (7) 17, p. 289; 1899.

[23] Phil. Mag., 11, p. 685; 1906.

λ	$R = \frac{5852.4862}{\lambda}$	$p' = P\frac{\theta^2 d^2}{8r^2}$	$p_\lambda = \frac{30.0002990}{\lambda(\text{in mm})}$	$p = \frac{29.9891793}{\lambda(\text{in mm})}$
3417.903	1.7123	0.482	87773.992	87741.458
5852.4862	1.0000	.777	51260.777	51241.777
5881.894	.9950	.578	51004.487	50985.582
6096.161	.9600	.554	49211.789	49193.549
6304.789	.9283	.715	47583.351	47565.714

The fractional order for $\lambda = 5881$ computed on this basis is $(0.578 - 0.487 =)$ 0.091 smaller than p' calculated from observations on the rings. Since $R = 0.9950$ coincidence should occur if P_{5852} is reduced by $\frac{0.091}{1.0000 - 0.9950} = 18$. This gives the order of magnitude of the change necessary in P_{5852} to get the true whole number.[24] This required change can be obtained more accurately from the other wave lengths. Thus,

$$\lambda = 6096 \text{ gives } \frac{1.554 - 0.789}{1.0000 - 0.9600} = \frac{0.765}{0.040} \doteq 19.12$$

Similarly, $\lambda = 6304$ gives 19.02 and $\lambda = 3417$ gives 18.94. This shows that P_{5852} is 19 too large and must be reduced from

---

[24] The next coincidence would require a change of just 200 waves either above or below this one and such a large change is excluded by the preliminary knowledge of the approximate length of the etalon.

51260.777 to 51241.777 which gives the true thickness $2e = 51241.777 \times 0.0005852486 = 29.9891793$ mm. The correctness of this can be tested very readily by simply subtracting proportional amounts from the provisional p_λ previously found for the other wave lengths and observing how well the resulting fraction agrees with the p' calculated from the rings. For example, p for $\lambda = 6096$ is equal to $49211.789 - (0.9600 \times 19) = 49193.549$, while the observed p' was 554. The remaining differences are very small considering that experimental errors and the change of phase at reflection are present. After the correction for phase change at reflection from these plates is once determined it can be applied to the wave lengths used, so that the resulting agreement between fraction orders is even more perfect. This procedure is not necessary but it is advisable in the case of larger etalons. The particular advantage which lies in the spacing of the neon lines used in this method is apparent. They enable one to find the exact length of an etalon quickly and without difficulty when only a roughly approximate value is known. In the above example 15 mm was used because this was the length ordered and we had faith in its approximate correctness. A Brown & Sharpe micrometer caliper may be used to give a sufficiently accurate value with which to begin on any etalon.

SUMMARY

A convenient way to apply the corrections which are necessary to interference measurements of wave lengths because of the change of phase at reflection is described. The error can be corrected for each wave length directly from its measured values obtained from thick and thin etalons. The determination of this correction is illustrated by some measurements of wave lengths in the spectrum of neon. The values of ten lines from $\lambda = 3369$ to $\lambda = 3520$ and seven from $\lambda = 6506$ to $\lambda = 7032$ are given. An example is given to show how the determination of the exact order of interference, or the optical measurement of length can be made easily and rapidly with the aid of certain wave lengths of neon radiation.

WASHINGTON, February 26, 1915.

EFFECTIVE RESISTANCE AND INDUCTANCE OF IRON AND BIMETALLIC WIRES

By John M. Miller

CONTENTS

Page

I. INTRODUCTION 208

II. HISTORICAL 210

III. THEORETICAL. SELF-INDUCTANCE AND CIRCULAR MAGNETIZATION 212

  1. Intensity of the electric field due to induction inside a straight cylindrical conductor 212
  2. Extra current due to induction 212
  3. Definition of self-inductance for a circuit which contains iron 213
  4. External and internal inductance 214
  5. Field energy 214
  6. A comparison of electrical and magnetic quantities 216
  7. Method of obtaining an exact permeability curve for circular magnetization of an iron wire 216

IV. EXPERIMENTAL. CIRCULAR AND AXIAL MAGNETIZATION 218

  1. Determination of the permeability for circular magnetization 218
  2. Determination of the permeability for axial magnetization 221
  3. Results and comparison of results 222

V. THEORETICAL. EFFECTIVE RESISTANCE AND INDUCTANCE 229

  1. Effective resistance and inductance of straight cylindrical wires 229
  2. Equivalent permeability of iron wires 230
  3. Effective resistance and inductance of bimetallic wires 231

VI. EXPERIMENTAL. METHODS 237

  1. The bridge 237
  2. Range of the measurements 238
  3. Method used for 50 and 100 cycles 239
  4. Method used for 500, 1000, and 3000 cycles 240
  5. A test measurement 241

VII. THE EFFECTIVE RESISTANCE AND INDUCTANCE OF IRON WIRES 242

  1. Experimental results 242
  2. Comparison of observed and computed values for zero current 247
  3. Equivalent permeability—Effects of hysteresis 248

VIII. THE EFFECTIVE RESISTANCE AND INDUCTANCE OF COPPER-STEEL BIMETALLIC WIRES 250

  1. Experimental results 250
  2. A comparison of observed and computed values 254
  3. Effect of noncircularity of the core 257

IX. WIRE TABLES. EFFECTIVE RESISTANCE AND INDUCTANCE OF COPPER-STEEL BIMETALLIC WIRES FOR SMALL CURRENTS 260

X. SUMMARY 266

I. INTRODUCTION

When alternating current is flowing in a conductor the density and phase of the current will not be uniform over the cross section of the conductor. This is due to differences in impedance of the different paths of current-flow within the conductor, which become more pronounced the higher the frequency of the alternations. The effective resistance and inductance of the conductor as a whole will be different from the direct-current values and will depend upon the frequency. The effective resistance will increase and the inductance decrease with increasing frequency. This effect, frequently called the skin effect, is of interest from a purely scientific standpoint and is sometimes of importance in engineering practice.

When the conductor is of simple form and the permeability of the material is known and may be considered a constant, the effective resistance and inductance can be calculated in a number of cases. Formulas have been given for a straight cylindrical conductor,[1] a concentric main,[2] a tubular conductor,[2] parallel wires,[3] a circle,[4] a single-layer coil,[5] a stranded wire,[6] a strip which is part of an infinite plane plate,[7] and perhaps for other cases. When, however, the conductor is made up entirely or in part of ferromagnetic material and the magnetizing forces in that material are such that the permeability can not be considered a constant, the problem becomes so complex that a solution is apparently impossible. In such cases the changes in inductance and resistance are very large and depend not only upon the frequency but also upon the current strength.

These two phases of the problem correspond to two uses of iron or bimetallic wires in practice—telephone and power transmission work. Conductors of iron or steel are used to some extent in telephone work where the currents are feeble and the magnetizing forces within the conductor are very small. It has been shown experimentally that for very small magnetizing forces the permeability of iron and steel is approximately constant. If we know this value of the permeability for the material and for the circular magnetization which results when a current flows through a wire, the values of the effective resistance and inductance for any frequency can be calculated by formulas which have been developed, provided the material is the same at all parts of the conductor. In this connection it is of interest to compare the permeability of iron wires when magnetized circularly, as when carrying a current, with the permeability for magnetization along the axis of the wire or axial permeability such as is ordinarily obtained in a magnetic measurement. In this paper the results of a number of such comparisons are given, and the method permits the exact permeability curves for circular magnetization to be obtained when the material of the wire is homogeneous throughout.

---

[1] Maxwell, Elect. and Mag., II, 690. Heaviside, Elect. Papers, II, p. 64. Rayleigh, Phil. Mag., 21, p. 381; 1886. Kelvin, Math. and Phys. Papers, III, p. 491.

[2] Russell, Phil. Mag., 17, p. 524; 1909.

[3] Nicholson, Phil. Mag., 17, p. 255; and 18, p. 417; 1909.

[4] Rosa and Grover, this Bulletin, 8, p. 181; 1911.

[5] M. Wien, Ann. d. Phys., 14, p. 1; 1904. Sommerfeld, Ann. d. Phys., 15, p. 673; 1904. Picciati, Il Nuovo Cimento (5), 2, p. 351. Coffin, this Bulletin, 2, p. 275. Cohen, this Bulletin, 4, p. 161.

[6] Lindemann, Jahrb. d. Drahtl. Tel., 4, p. 561; 1910–11.

[7] Bethenod, Jahrb. d. Drahtl. Tel., 2, p. 397; 1909.

Conductors which are made up of two metals are used to some extent for telephone work. We will consider mainly the type of bimetallic wire which has a more or less circular core of steel surrounded by a shell of copper. The present paper deals with experimental determinations of the electrical properties of these wires up to and including telephone frequencies and develops formulas which, with some limitations, make it possible to compute the effective resistance and inductance of bimetallic wires at different frequencies.

Steel and bimetallic wires are also used for power-transmission purposes. Here the magnetizing forces in the conductor may be considerable and the resistance and inductance may be quite different from the values for feeble currents. In this work the effect of current strength upon the effective resistance and inductance of the conductors included in the investigation has also been studied experimentally and values of effective permeabilities of the materials have been computed and plotted in some cases.

II. HISTORICAL

The amount of experimental work upon conductors made up of or containing iron is very great. Of these papers we shall review two which have more or less direct bearing upon the work in hand.

Klemenčič[8] made measurements of the self-inductance of an annealed soft iron, a hard iron, and a Bessemer steel wire with direct current of different strengths. The wires were 1 m long and 2 mm in diameter. Essentially the method of measurement was to balance a resistance bridge which contained the iron wire in one arm and then read the ballistic deflection of the galvanometer when the current in the bridge was reversed. From the deflections the values of the self-inductance were determined and values of the susceptibility for circular magnetization computed, using a formula which holds only for constant susceptibility. Thus were obtained what may be called effective values of the susceptibility. Then arose the difficulty as to what values of the magnetizing force these values of the susceptibility corresponded. An average value was taken, which was obtained by summing up all of the values of the magnetizing force over the total cross section of the wire and then dividing by the cross section. Susceptibility curves were also determined for the same wires by the ordinary ballistic method, in which the magnetization was along the axis of the wire. The results of the two experiments were compared and the author concluded that in general the susceptibility is different in the circular and axial directions. The results of Klemenčič have been recomputed in part on the basis of developments outlined in the present paper and will be presented later.

Fowle,[9] in a series of three papers, treats very thoroughly the electrical properties of compound or bimetallic wires.

The first paper gives a number of expressions for the ratios of the weight and conductance of a bimetallic wire to a solid copper conductor of the same size. It contains tables and curves of these ratios for copper-clad and aluminum-clad steel wires with different ratios of copper and aluminum content by cross section and weight.

---

[8] Klemenčič, Wied. Annal., 53, p. 1053; 1894.

[9] Fowle, Elect. World, 56, pp. 1471, 1521, 1910; and 57, p. 108, 1911.

The second paper is concerned with the distribution of current between core and shell in the above-mentioned wires as a function of the shell cross section and develops a somewhat complicated formula for the internal self-inductance per unit length of a compound wire. Now, the expression for the internal inductance per centimeter of length of a solid wire of one metal is very simply given by $\frac{\mu}{2}$ where μ is the permeability (constant). This inductance does not depend upon the size of the wire. It is interesting to note that also in the bimetallic wire the internal inductance is independent of the wire size. If the characteristics of the materials are the same from wire to wire, the inductance varies with the conductivity of the wire alone.

The author discusses the probable conductivity of the steel in bimetallic wires and gives simple formulas to calculate the inductance when the conductance ratio of steel to copper is 12 per cent and steel to aluminum is 19.35 per cent.

In the third paper the results of experiments upon Nos. 6, 8, 9, 10, 12, and 14 copper-clad wires are given. The resistances and temperature coefficients of resistance were measured, and the per cent conductivities relative to solid copper were computed. An attempt to determine the conductance ratio of the steel in the core was somewhat unsatisfactory, on account of the difficulty in measuring the cross section of the core accurately. The results seem to show a decreasing conductivity of the steel with decreasing size of wire.

The alternating current measurements gave the ratio of apparent to ohmic resistances and the inductive reactances for currents ranging from 3 to 35 amperes and frequencies of 25, 40, and 60 cycles. The wires were strung in rectangular loops and with separations of 12, 18, and 24 inches between the long sides.

From the inductance measurements average values of the permeability of the core were computed for different separations, current strengths, and frequencies. The accuracy of these values is reduced by the fact that the internal inductance of the wire was a small part of the whole.

III. THEORETICAL. SELF-INDUCTANCE AND CIRCULAR MAGNETIZATION

1. INTENSITY OF THE ELECTRIC FIELD DUE TO INDUCTION INSIDE A STRAIGHT CYLINDRICAL CONDUCTOR

Suppose a current of uniform distribution to flow through a homogeneous cylindrical conductor and return along the surface so that there is no field external to the conductor. If the strength of the current in the conductor should change, we can determine the intensity of the electric field due to induction at any point P (Fig. 1) internal to the wire as follows: We have

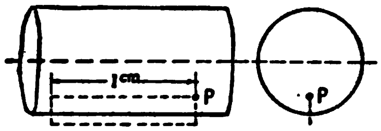


FIG. 1

$$\int E\,ds = -\frac{d}{dt}\int B_n d\,S$$

that is, the line integral of the field intensity around a closed path is equal to the rate of decrease with the time of the surface integral of the induction normal to a surface bounded by the closed curve. Integrating around the rectangular path shown in the figure, we get $\int E\ ds = E_P \times 1$ where E_P is the field intensity at the point P. This is equal by the equation to the rate of change of the flux of induction through the rectangle, or since the induction is zero outside is equal to the rate of change of the induction between P and the outside of the conductor. Hence the field intensity depends upon the rate of change of the flux of induction outside of the point.

2. EXTRA CURRENT DUE TO INDUCTION

Assume a conductor of radius ρ and resistivity σ and consider the magnetic induction only in the concentric cylindrical shell of radius x and thickness dx and that the total flux of induction per unit length is φ. Suppose this induction to be reduced to zero. The field intensity at all points inside of x would be

$$E=-\frac{d\varphi}{dt}$$

the current density

$$i=\frac{E}{\sigma}=-\frac{1}{\sigma}\frac{d\varphi}{dt};$$

the total current

$$I=\frac{\pi x^2}{\sigma}E=-\frac{\pi x^2}{\sigma}\frac{d\varphi}{dt}$$

and the total quantity

$$\int_0^\infty I\,dt=\frac{\pi x^2}{\sigma}\varphi$$

If this same flux of induction were outside of the wire the quantity $\frac{\pi \rho^2}{\sigma}\varphi$ would flow. Hence the effect of induction inside the wire in contributing to the extra current of induction must be weighted in the ratio of the area of the cross section cut by the induction in collapsing upon the axis to the total area of cross section of the conductor.

3. DEFINITION OF SELF-INDUCTANCE FOR A CIRCUIT WHICH CONTAINS IRON

If there is a mutual inductance M between two circuits (1) and (2) and a current I_1 in the first is interrupted the quantity of electricity which will flow in the circuit (2) due to induction will be $q=\frac{M\,I_1}{R_2}$ where R_2 is the resistance of the second circuit. We could define M in terms of the quantity q. In a similar manner the self-inductance of a circuit can be defined in terms of the quantity of electricity which would flow, due to the induction linked with the circuit, if the current in the circuit were interrupted or reversed. If the circuit contains iron the inductance will depend upon the current.

Suppose the circuit to be carrying a current I and by repeated reversals of this current the iron has been brought into a cyclic state. Then the self-inductance L corresponding to the current I is given by $q = \frac{IL}{R}$ where R is the resistance of the circuit and q is one-half of the quantity of electricity which flows due to induction when I is reversed.

4. EXTERNAL AND INTERNAL INDUCTANCE

If the flux of induction is outside of the wire

$$q = \frac{1}{R}\int B\,dS = \frac{\varphi}{R}$$

and $L = \frac{\varphi}{I}$, the flux per unit current.

If the induction is inside the wire we have for the flux of induction per unit length $\int_0^\rho B_x\,dx$. This is equivalent in its contribution to the extra current of induction to φ tubes of induction outside the wires where

$$\varphi = \int_0^\rho B_x \frac{x^2}{\rho^2}\,dx$$

if the wire is homogeneous in respect to its conductivity.

Hence the internal inductance or that portion of the self-inductance which is due to the internal flux

$$L_i = \frac{\varphi}{I} = \frac{1}{I}\int_0^\rho B_x \frac{x^2}{\rho^2}\,dx.$$

5. FIELD ENERGY

If dv is the element of volume of the wire, we have $dv = 2\pi x\,dx$ for a unit of length

and $$2\pi x H_x = 4\pi I \frac{x^2}{\rho^2}; \quad H_x = \frac{2xI}{\rho^2}.$$

Substituting these values in the above expression for L_i, we have

$$\frac{1}{2} L_i I^2 = \int \frac{B_x H_x}{8\pi}\,dv = \frac{1}{8\pi}\int \mu_x H_x^2\,dv \tag{1}$$

where the integral is taken throughout the volume of the conductor. This is in the usual form of the expression for the electromagnetic energy in the field, but though the equation can be used to compute the inductance, neither side of the equation is energy unless L_1 and μ are constants.

To show this, let us consider the case of a long solenoid of radius r filled with perfectly soft homogeneous iron, and let the solenoid have one turn of conductor per centimeter of length. Let some source of energy start up a current in the solenoid. We will calculate the energy which must be supplied to a single turn to build up its share of the field, which evidently is for 1 cm of length of the solenoid. The emf of induction in this single turn is

$$E = -\frac{d}{dt}\int B\,d\,S$$

Since the magnetic intensity H is constant over the cross section of the solenoid, B is also constant and

$$E = -\frac{d}{dt}\left[\pi r^2\,B\right] = -\pi r^2\frac{dB}{dt}$$

The energy which goes into the field is

$W = -\int_o^\infty E\,I\,dt = \pi r^2\int_o^{B_f} I\,d\,B$, writing the subscript f for final values, but $H = 4\,\pi\,I$ and $B = \mu\,H$

Hence $W = \frac{r^2}{4}\int_o^{B_f} H\,d\,B = \frac{r^2}{4}\left[\int_o^{H_f}\mu\,H\,d\,H + \int_{\mu_o}^{\mu_f} H^2\,d\,\mu\right]$

The values of the integrals depend upon μ as a function of H, hence the energy is not determined unless this function is specified.

If $\mu =$ constant, then $W = \frac{\mu_f\,H^2{}_f\,r^2}{8} = \frac{1}{8\pi}\int\mu_f\,H^2{}_f\,dv$, and we have the ordinary expression for the energy.

In any case

$$L = \frac{\int B\,d\,S}{I} = \frac{\mu_f\,H_f\,\pi r^2}{I}$$

and since $H = 4\pi I$; $\frac{1}{2}\,L\,I^2 = \frac{\mu_f\,H^2{}_f\,r^2}{8} = \frac{1}{8\pi}\int\mu_f\,H^2{}_f\,dv$.

The latter is only an energy equation when μ is a constant. We may apply this equation when the iron shows hysteresis if we consider it to be in a cyclic state and μ to be the ratio of B to H, corresponding to the tip of the hysteresis loop.

6. A COMPARISON OF ELECTRICAL AND MAGNETIC QUANTITIES

The current I is related to the magnetic intensity H by means of a line integral. Our definition of the self-inductance L in terms of quantity of electricity has as a consequence that IL is related to the magnetic induction B through a surface integral. Therefore L per unit length is related to μ. Also LI^2 is related to μH^2 through a volume integral, as has been shown in the preceding section.

7. METHOD OF OBTAINING AN EXACT PERMEABILITY CURVE FOR CIRCULAR MAGNETIZATION OF AN IRON WIRE

Suppose a current I to be flowing in a homogeneous iron wire of radius ρ and that the iron is in a cyclic state. The self-inductance of the wire is given by $\frac{1}{2} LI^2 = \frac{1}{8\pi}\int \mu H^2\, dv$. Let us consider only the internal inductance per unit length and write it L_i.

Then $\frac{1}{2} L_i I^2 = \frac{1}{8\pi}\int_0^\rho \mu H^2\, 2\pi x\, dx$ where dx is an element of the radius at a distance x from the axis.

Since $2\pi x H = 4\pi \frac{Ix^2}{\rho^2}$; $H = \frac{2Ix}{\rho^2}$; $x = \frac{\rho^2 H}{2I}$; $dx = \frac{\rho^2\, dH}{2I}$

and $L_i = \frac{2}{I^2}\,\frac{1}{8\pi}\int_0^{\frac{2I}{\rho}} \mu H^2 \cdot 2\pi \frac{\rho^2 H}{2I} \cdot \rho^2 \frac{dH}{2I} = \frac{\rho^4}{8I^4}\int_0^{\frac{2I}{\rho}} \mu H^3\, dH$

If the current in the wire is increased to $I + \Delta I$ and the iron again brought into a cyclic state, the self-inductance will then be

$L_i + \Delta L_i = \frac{\rho^4}{8(I+\Delta I)^4}\int_0^{\frac{2(I+\Delta I)}{\rho}} \mu H^3\, dH$, which, since the iron is homogeneous

$$= \frac{\rho^4}{8(I+\Delta I)^4}\int_0^{\frac{2I}{\rho}} \mu H^3\, dH + \frac{\rho^4}{8(I+\Delta I)^4}\int_{\frac{2I}{\rho}}^{\frac{2(I+\Delta I)}{\rho}} \mu H^3\, dH$$

$$= L_i\left(\frac{I}{I+\Delta I}\right)^4 + \frac{\rho^4}{8(I+\Delta I)^4}\int_{\frac{2I}{\rho}}^{\frac{2(I+\Delta I)}{\rho}} \mu H^3\, dH$$

This may be interpreted as follows: That by increasing the current those tubes of induction present in the earlier case have been crowded in toward the axis tending to diminish the inductance in the ratio of $1 : \left(\frac{I}{I+\Delta I}\right)^4$ and a new flux of induction has been added in the outside shell where the value of H varies from $\frac{2\,I}{\rho}$ to $\frac{2\,(I+\Delta I)}{\rho}$ and which tends to increase the inductance by the amount given in the second term on the right.

Now let us assume μ to have the uniform value μ_p in the outside shell over which the integral is taken. Then

$$L_i+\Delta L_i=L_i\left(\frac{I}{I+\Delta I}\right)^4+\frac{\mu_p\rho^4}{8(I+\Delta I)^4}\cdot\frac{16}{4\rho^4}[(I+\Delta I)^4-I^4]$$

and

$$\Delta L_i=\left[L_i-\frac{\mu_p}{2}\right]\left[\frac{I^4}{(I+\Delta I)^4}-1\right]=\left[L_i-\frac{\mu_p}{2}\right]\left[-4\,\frac{\Delta I}{I}\right]$$

neglecting terms in higher powers of $\frac{\Delta I}{I}$

Hence $\mu_p=2L_i+\frac{I}{2}\,\frac{\Delta L_i}{\Delta I}$

and in the limit when ΔI and ΔL_i become dI and dL_i

$$(2)\quad \mu_p=2\;L_i+\frac{1}{2}\,I\,\frac{dL_i}{dI}$$

Where μ_p is the value of μ corresponding to the magnetizing force $H=\frac{2I}{\rho}$. For $\mu=\mu_p$ a constant, $\frac{dL_i}{dI}=0$ and we have $L_i=\frac{\mu}{2}$ which is the ordinary expression for the internal inductance per centimeter of length when the permeability is constant. If we have determined a curve by direct current which gives L_i as a function of I, we can, from the values of L_i and $\frac{dL_i}{dI}$ at different points of the curve, determine a curve for μ as a function of H.

IV. EXPERIMENTAL. CIRCULAR AND AXIAL MAGNETIZATION

1. DETERMINATION OF THE PERMEABILITY FOR CIRCULAR MAGNETIZATION

In determining L_i as a function of I an Anderson bridge was used, as shown in Fig. 2, with a secohmeter to alternately reverse the battery and galvanometer connections to the bridge. Referring to the figure, P and R were 10-ohm coils in oil; C, mica condensers of about 3 microfarads capacity; r was made up of two resistance boxes, one of which was variable from tenths to thousandths of an ohm and the other contained larger valued coils; S was a special box of current carrying coils in oil in which the resistance could be varied in steps of one-thirtieth of an ohm from 0.033 to 0.767 ohm. The Q arm contained the iron wire to be measured or a substitute copper wire of the same length, a small inductance for ballast, several manganin or copper links of the same length and same size wire and a mercury slider.[10] The latter was used in getting a resistance balance and has the advantage that the change in inductance with the setting is extremely small—less than 20 cm for the whole range, which is about 0.1 ohm. To reduce the heating it was, in general, partially submerged in water. The total current in the bridge was read by an ammeter in series with the battery. Currents as high as 10 amperes were used in the measurements. The bridge is balanced for inductance when the total quantity of electricity which flows through the galvanometer due to a reversal of the battery current is zero. The condition for balance is $L = CS\ (R + 2r)$ when $P = R$ and $Q = S$.

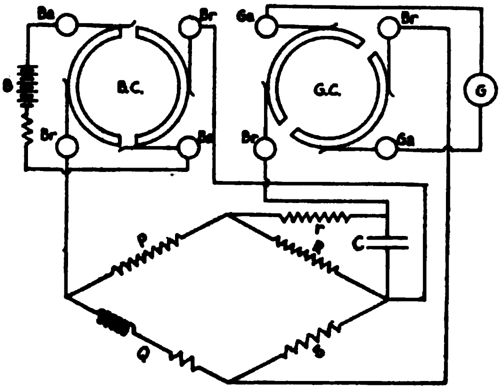


FIG. 2.—*Secohmeter and bridge*

---

[10] Dellinger, Phys. Rev., 32, p. 614, 1911; and, 33, p. 215, 1911. Grover and Curtis, this Bulletin, 8, p. 466; 1912.

A method of differences was used in making the measurements. First a copper wire was put in the Q arm and a resistance balance obtained by means of the manganin links and mercury slider; then the secohmeter was run and an inductance balance obtained by means of the resistance r. This was in effect a zero balance for the bridge. The iron wire was then substituted for the copper and a resistance balance obtained by the mercury slider alone or by subsituting copper links for manganin links to bring the balance within the range of the mercury slider. Then a series of inductance balances were obtained by means of r with increasing currents in the bridge and between these balances the resistance balance was adjusted. The zero balance for the bridge was then again obtained. The difference in inductance of the iron and copper wires was then given by $\Delta L = 2\ CS\Delta r$ where Δr is the difference in the setting of r with the iron wire in and out. The difference between the external inductance for the iron and copper wires was computed and applied as a correction, which then gave the internal inductance of the iron wire less one-half a centimeter of inductance per centimeter of length.

In the earlier work the bridge was so arranged that it was necessary to bend the wire into a rectangle in order to introduce it into the bridge. In order to eliminate the working of the iron caused by the bending, the bridge was modified so that straight wire up to 140 cm in length could be used. The wires were soldered into copper terminals which dipped into mercury cups. A heavy copper wire of the same length and with similar terminals was used in determining the zero balance of the bridge. While being measured the wires were immersed in oil in a long wooden tank, in order to reduce the drift due to temperature changes. The leads which connected to the bridge were run perpendicularly to the wire under measurement for a distance of 20 cm. Then one of these leads ran parallel to meet the other, and the two were run side by side about 2 m to the bridge.

In order to get accurate results and a steady balance of the bridge, considerable care was necessary in the adjustment and manipulation of the secohmeter. In the first place it was found necessary to keep the commutator segments and brushes very clean and polished and generously oiled with a light oil, such as sewing-machine or clock oil. On account of the high currents used a frequent overhauling was essential to good operation. Secondly, it was found to be very important that the brushes should be so adjusted that the reversal of the galvanometer connections came very exactly midway between the reversals of the battery connections. The result of this adjustment is to eliminate to a great extent effects upon the inductance balance due to a lack of balance of the bridge for resistance. It was of special importance in this work, for when high-current strengths were used the bridge was extremely sensitive with respect to a resistance balance and the heating in the Q arm, which was made up in good part of metals with high-temperature coefficients, made it practically impossible to preserve a balance. The adjustment of the secohmeter was accomplished by substituting a dry cell for the battery, a less sensitive galvanometer for the galvanometer used in the measurements, and, in place of the bridge, connecting wires across from the Br terminals on one side to those on the other. The brush holder on the galvanometer commutator was then adjusted, so that when the secohmeter was running the galvanometer showed no deflection in either direction. With the secohmeter adjusted in this manner the resistance balance of the bridge could be disturbed so as to deflect the galvanometer off the scale without producing an appreciable effect upon the inductance balance.

The secohmeter was run by a small shunt motor and the speed could be regulated by a variable resistance in the armature circuit. The effect of the speed upon the balance was investigated and it was kept so low that a considerable increase produced no appreciable change in the balance. At different times from 13 to 30 reversals per second were used.

In order to obtain consistent results for circular magnetization, it was found to be necessary to demagnetize the iron wire carefully just as is necessary in measurements of axial magnetization. This is illustrated in the curves in Fig. 3, which show two sets of determinations of L_1 as a function of I for a soft-iron wire (No. 43 Steel Wire Gage). The first set was taken without any attempt to demagnetize the wire, and the results are plotted by closed circles. Before taking the next set the wire was demagnetized by reducing the current in the wire slowly from 4 amperes to 0.1 ampere with the secohmeter running. The results of the second set are plotted by open circles. The curves show the same characteristics as the induction curves obtained by Burrows [11] in investigating the effect of imperfect demagnetization in ordinary magnetic measurements; i. e., considerably higher values of the magnetic induction for the demagnetized wire in the region where the curve is steep, but practically no difference in the values after the knee of the curve is reached.

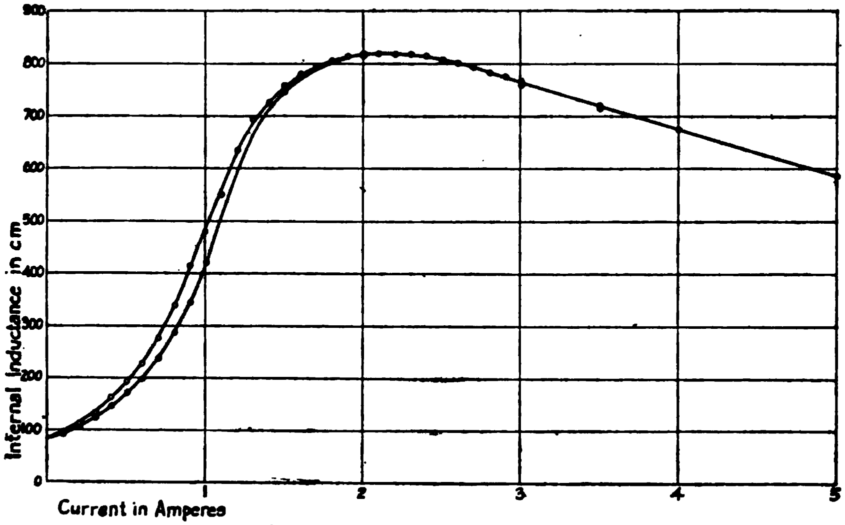


FIG. 3.—*Effect of demagnetization*

Having obtained a curve for L_i, tangents to the curve were drawn at a number of points and values of $\frac{dL_i}{dI}$ determined. Then μ was calculated from equation (2) and plotted as a function of H.

2. DETERMINATION OF THE PERMEABILITY FOR AXIAL MAGNETIZATION

In order to compare the permeability for circular magnetization with that obtained when the wires were magnetized axially, measurements were also made upon the wires by the ordinary ballistic method.[12] The wires were a meter or more in length and the magnetizing coil was 115 cm long and 2.6 cm in diameter, with eight turns to the centimeter. The test coil had a single layer of 1000 turns of No. 32 enameled wire wound on a glass tube and was about 22 cm long. The galvanometer was standardized by means of a variable mutual inductance, whose secondary was always kept in the circuit. Deflections were obtained with the iron wire in the test coil and with the test coil alone, and this data, together with the diameter of the wire and the sensitivity of the galvanometer, sufficed to compute μ. In the circular measurements an average value over the whole length of wire is obtained, and in the axial an average over about 22 cm. The effect of having different portions of the wire inside the test coil was tried without showing a marked lack of homogeneity in any case.

---

[11] This Bulletin, 4, p. 205; 1907 (Scientific Paper No. 78).

3. RESULTS AND COMPARISON OF RESULTS

In the earlier work measurements were made upon a soft-iron wire (No. 43 Steel Wire Gage) and drill rod 1½ mm in diameter. These measurements were not satisfactory. At this time it was necessary to bend the wire into a rectangle in order to make the inductance measurements. The determinations shown in Fig. 3 were made before a method was determined upon for the axial measurements, and it was not considered desirable to bend the wire straight again for the axial measurements. The other measurements upon the No. 43 wire were made upon annealed specimens; the process of annealing was faulty and the wires showed considerable scale. The measurements upon the drill rod and the recomputed results of Klemenčič [12a] for an annealed soft-iron wire are shown in Figs. 4 and 5. Klemenčič gives two series of axial measurements taken before and after the circular determination. It was necessary in this work to bend the wire to introduce it into the bridge, and the differences between the two series of axial measurements may possibly be due to the bending. These curves show marked differences between the permeabilities for circular and axial magnetization. The circular permeability is lower, and as is more definite in Fig. 4, the maximum comes at a higher magnetizing force. These characteristics are repeated in some of the later curves.

---

[12] The author is indebted to Dr. C. W. Burrows for suggestions with respect to this work.

[12a] See p. 210.

After this the bridge was so arranged that straight pieces of wire could be used. Samples of ungalvanized commercial iron telegraph and telephone wires were then obtained.[13] Axial permeability curves were determined for a No. 4, a No. 8, and a No. 10 of each of the three grades known as E. B. B. (Extra Best Best), B. B. (Best Best), and S. (Steel). (The diameters of the wires accorded with the Birmingham Wire Gage, excepting for the No. 4 B. B. and No. 4 Steel, which agreed better with the Steel Wire Gage.) These curves are shown in Fig. 6. The wires of the same grade show similar magnetic characteristics, with the exception of No. 10 B. B. Another piece from the same coil of this latter wire gave a similar curve. In the later work the six samples of the E. B. B. and S. grades were used, since they represented the extremes in magnetic characteristics. In Figs. 7, 8, and 9 are given the curves for both axial and circular permeabilities. Two sets of curves are given for No. 4 E. B. B. (Fig. 7). The set marked (1) shows the characteristics of the wire in its original condition. It will be noted that there is a very great difference

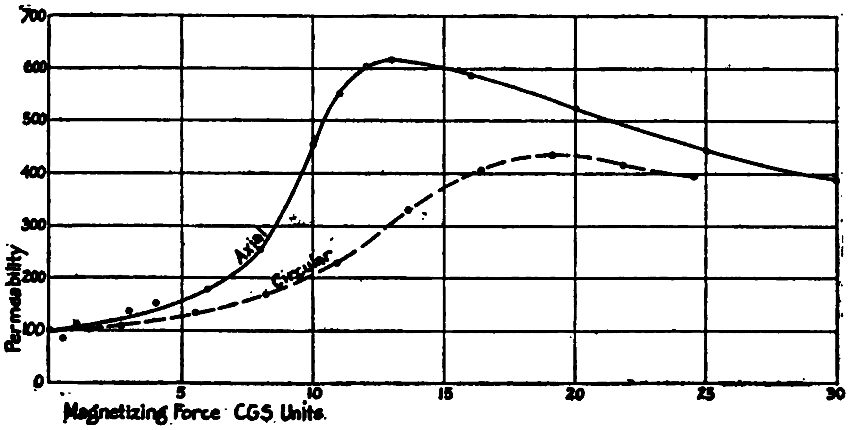


FIG. 4.—*Permeability curves for drill rod*

between the axial and circular curves for No. 4 E. B. B. and No. 4 S., while No. 8 E. B. B. and No. 8 S. and also No. 10 S. show a fair agreement between the two permeabilities. No. 10 E. B. B. reaches a somewhat higher maximum for the circular magnetization and it comes at a higher magnetizing force. With the exception of No. 4 E. B. B., the values for minimum permeability show a close agreement.

---

[13] Kindly furnished by C. L. Miller, of the American Steel & Wire Co.

The differences are undoubtedly due to nonhomogeneity caused either by drawing the wire or heat treatment. In the axial measurement an average value of the magnetic flux over the whole cross section is obtained and each portion is subjected to the same magnetizing force. In the circular measurement the flux is weighted in its effect upon the self-inductance in proportion to the area of the cross section which it surrounds; for this reason the flux at the outside of the wire is more important. Also the magnetizing force is highest at the outside of the wire. If a wire

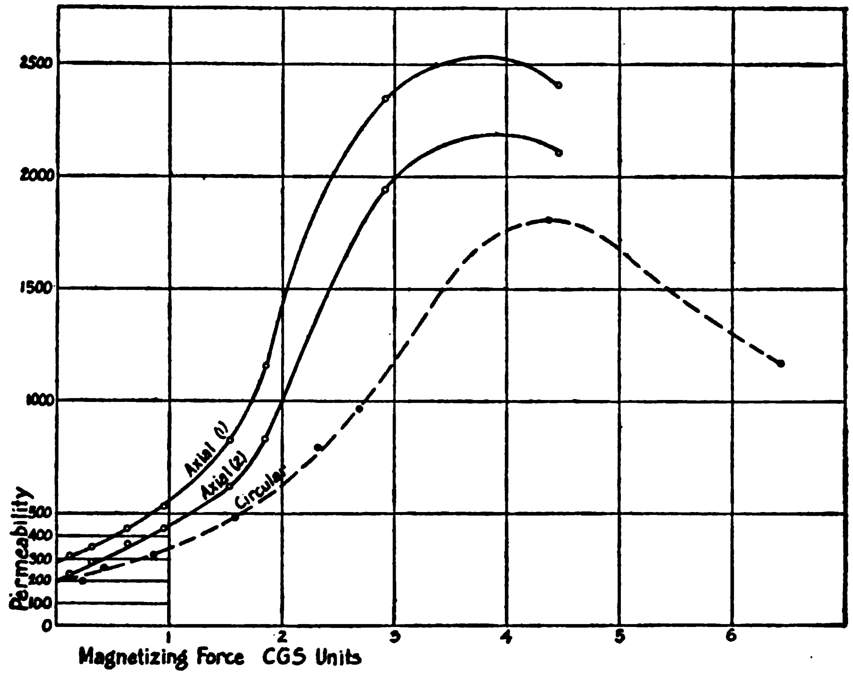


Fig. 5.—*Annealed soft iron wire. Klemenčič*

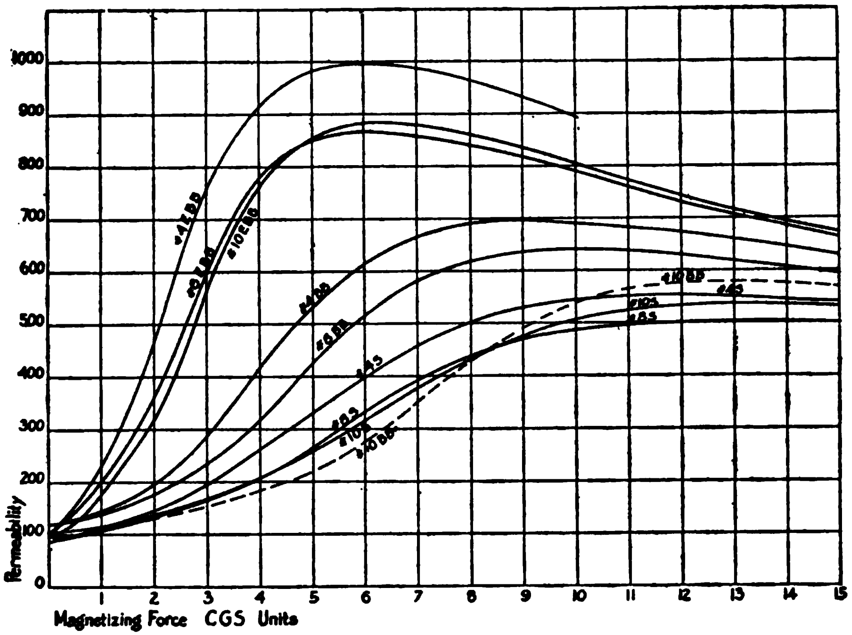


FIG. 6.—*Axial permeability. Iron telephone wires*

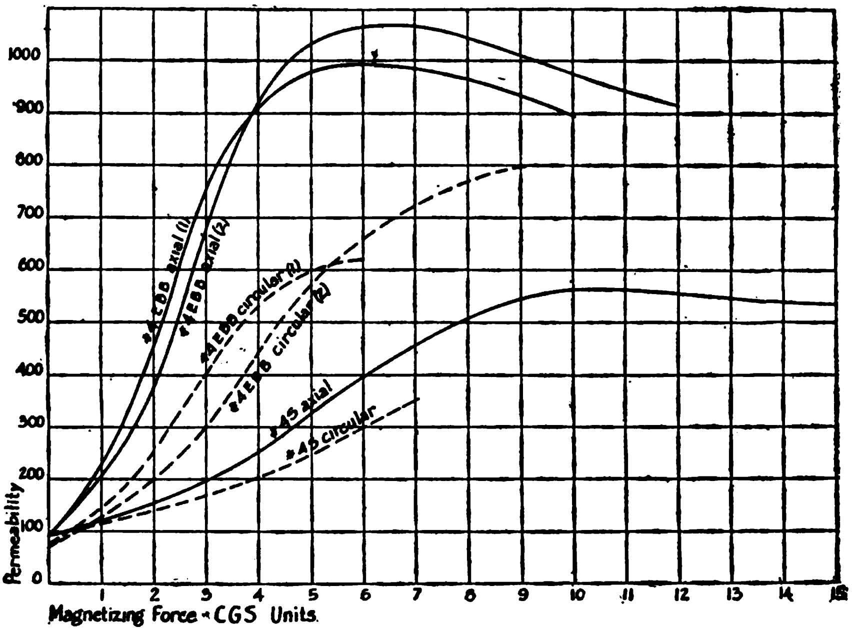


FIG. 7.—*Circular and axial permeabilities. No. 4 E. B. B. and No. 4 Steel*

had an outside shell of somewhat harder (magnetically) material we might expect a behavior such as is shown by No. 8 E. B. B. The axial curve would be lowered somewhat throughout the whole range on account of the harder portion, the circular still more influenced in the region of lower magnetizing forces, and the maximum might come at a higher magnetizing force; for the magnetizing forces as plotted are those at the outside of the wire and the softer material within would reach its maximum later.

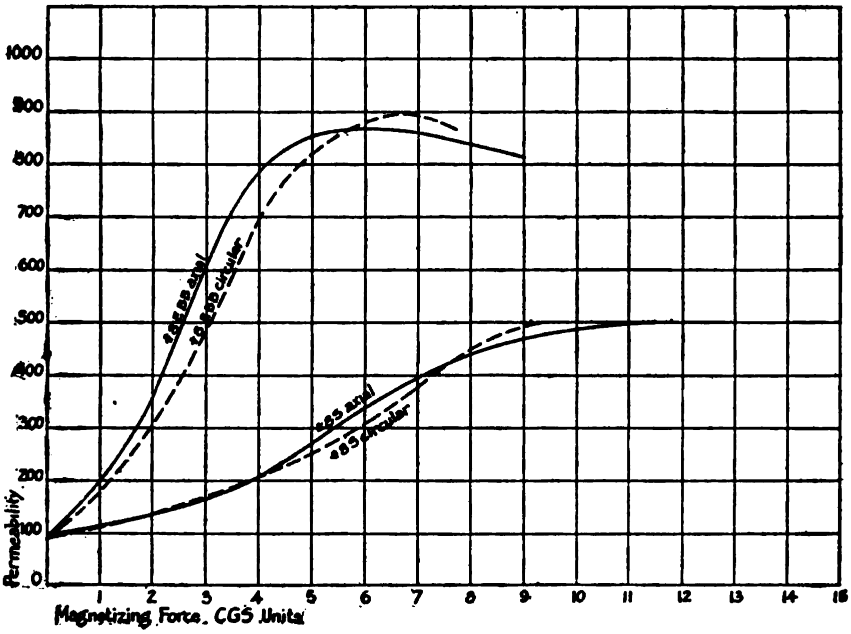


FIG. 8.—*Circular and axial permeabilities. No. 8 E. B. B. and No. 8 Steel*

The drawing of the wire would probably tend to lower the circular values relative to the axial. It is found, even in annealed transformer iron, that the permeability of the material is considerably higher in the direction of rolling than in the transverse direction.

In order to obtain some further light upon the difference between the two permeabilities, the No. 4 E. B. B. wire, which showed a very great difference, was turned down from about 6 mm to 4 mm in diameter. The curves then obtained are those numbered (2) in Fig. 7. The differences were still great. Lower permeabilities were obtained in the steep portion of the curve for both magnetizations; both maxima came later and gave considerably higher values. No explanation for this behavior is at hand and further work in the direction of obtaining an explanation was not attempted.

For the purpose of obtaining information with respect to the permeability characteristics of the core of a copper-steel bimetallic

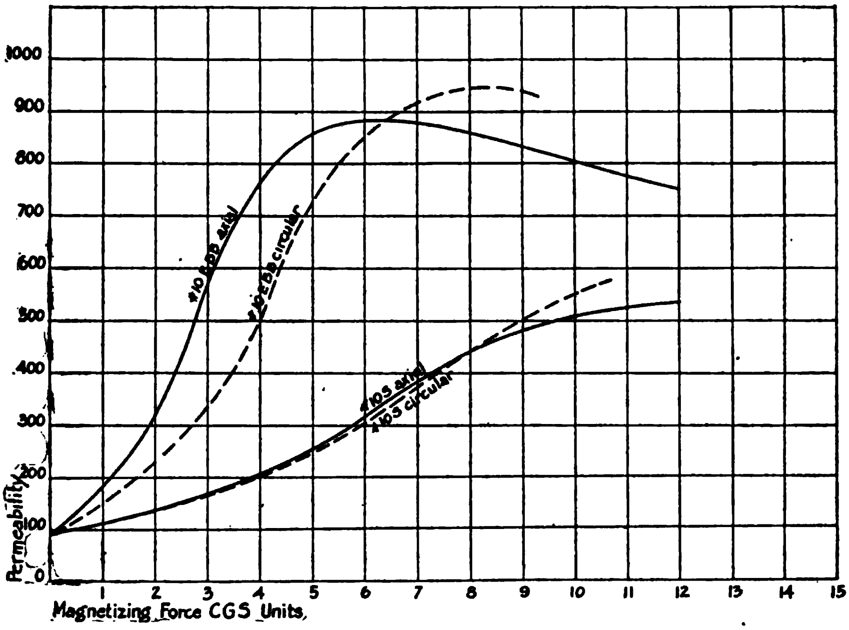


FIG. 9.—*Circular and axial permeabilities. No. 10 E. B. B. and No. 10 Steel*

wire, a No. 4 wire of this kind was turned down so as to give a steel wire of 2.5 mm diameter. It was expected that on account of the protecting coat of copper, the steel within might be more homogeneous. The curves for the circular and axial permeabilities (Fig. 10) do show a fair agreement of the two quantities. The working due to turning down the wire must, of course, produce some effect. In Fig. 11 are given curves of the permeability for axial magnetization of the cores of No. 4, No. 8, and No. 10 (A. W. G.) copper-steel wires.[14] The curve for the No. 4 wire is the same as that in Fig. 10; the values for the No. 8 and No. 10 were obtained from measurements upon the whole wire, the cross section of the steel being computed from electrical data. In each of the latter determinations two samples, which were taken from the ends of a 30-meter stretch of wire, gave close agreement. As might be expected, the permeabilities were lower for the smaller wires.

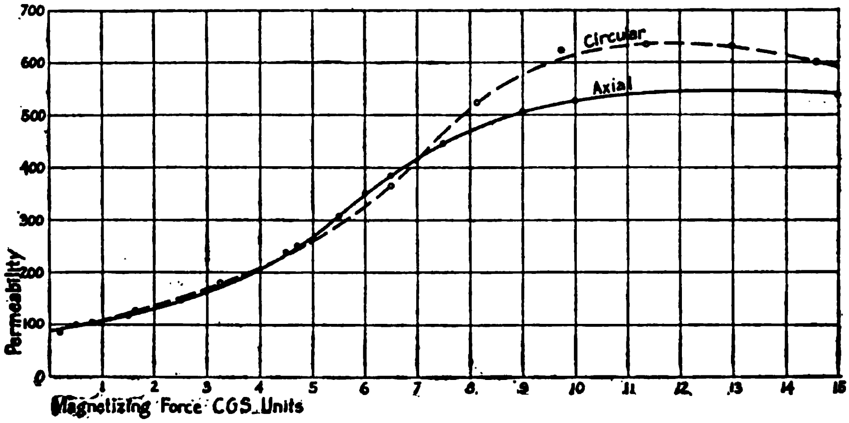


Fig. 10.—*Permeability curves. Steel core of bimetallic wire*

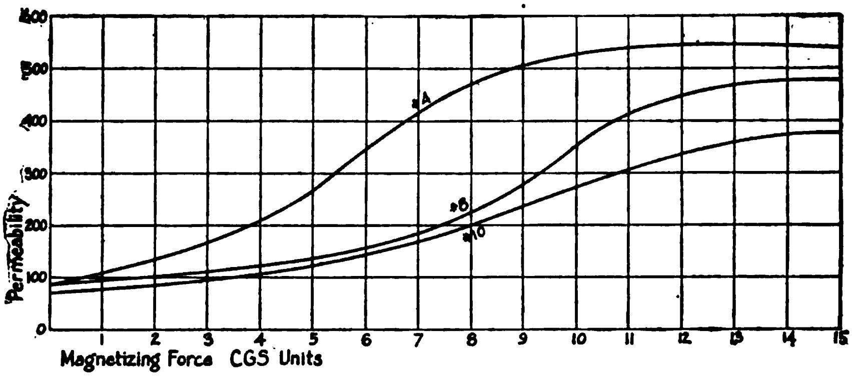


Fig. 11.—*Axial permeability. Steel cores of bimetallic wires*

---

[14] The samples of wire were kindly furnished by the Duplex Metals Co.

V. THEORETICAL. EFFECTIVE RESISTANCE AND INDUCTANCE

1. EFFECTIVE RESISTANCE AND INDUCTANCE OF STRAIGHT CYLINDRICAL WIRES

As has been mentioned before, the problem of the effective resistance and inductance of straight cylindrical wires has been worked out for the case of constant permeability. The formulas generally used are those given by Kelvin, which make use of the ber and bei functions and their derivatives ber′ and bei′. These functions introduced by Kelvin are given by the following series:

$$\text{ber}\ x = 1 - \frac{x^4}{2^2\,4^2} + \frac{x^8}{2^2\,4^2\,6^2\,8^2} - \cdots\cdots$$

$$\text{bei}\ x = \frac{x^2}{2^2} - \frac{x^6}{2^2\,4^2\,6^2} + \frac{x^{10}}{2^2\,4^2\,6^2\,8^2\,10^2} - \cdots\cdots$$

where $x = 2\rho\sqrt{\frac{\pi p \mu}{\sigma}}$

and ρ = radius of conductor in cm.

σ = resistivity of the material in absolute electromagnetic units.

μ = permeability.

$p = 2\pi$ times the frequency.

The formulas for the effective resistance and inductance can be put in simple forms by the use of the auxiliary quantities

$$W = \text{ber}\ x\ \text{bei}'x - \text{bei}\ x\ \text{ber}'x$$
$$Y = (\text{ber}'x)^2 + (\text{bei}'x)^2$$
$$Z = \text{ber}\ x\ \text{ber}'x + \text{bei}\ x\ \text{bei}'x$$

We have then for the resistance R' at a frequency $f = \frac{p}{2\pi}$ in terms of the direct current resistance

$$R' = R\left(\frac{x}{2}\,\frac{W}{Y}\right) \tag{3}$$

and similarly for the *internal inductance*

$$L' = L\left(\frac{4}{x}\,\frac{Z}{Y}\right) \tag{4}$$

in which L, the internal inductance for direct current and length l, is given by

$$L=\frac{l\mu}{2}$$

The tables of $\frac{x}{2}\frac{W}{Y}$ and $\frac{4}{x}\frac{Z}{Y}$ as given by Rosa and Grover[15] have been of very great use in the present work.

2. EQUIVALENT PERMEABILITY OF IRON WIRES

If the material of the wire is homogeneous and the value of the initial permeability is known, we can calculate the effective resistance and inductance of solid iron wires by means of equations (3) and (4) for the case when the current in the wire is very small.

The same formulas may be used to determine what may be called the equivalent permeability of the wire from experimental determinations of the effective resistance and inductance with considerable currents. By equivalent permeability is meant that value of the permeability which, if constant, would result for a given wire and frequency in the same value of effective resistance or inductance as that observed.

The equivalent permeability is quite readily computed from the resistance observations; for the ratio of the effective resistance to the direct current resistance gives the quantity $\frac{x}{2}\frac{W}{Y}$ in (3) directly. From the above-mentioned tables we can then obtain the corresponding value of x and then the equivalent permeability μ'. The computation from the internal inductance values is not as direct, for here the direct current inductance as well as the quantity $\frac{4}{x}\frac{Z}{Y}$ in (4) are functions of μ'. The following method was used to obtain μ' from the inductance values: A series of values of μ' was assumed and a table of the corresponding values of the inductance for a given frequency was computed together with the second and third differences. Then from the measured values of the inductance at that frequency the corresponding value of μ' was obtained by interpolation.

---

[15] Formulas and Tables for the Calculation of Mutual and Self-Inductance (Revised) this Bulletin, 8, p. 226; 1911 (Scientific Paper No. 169).

3. EFFECTIVE RESISTANCE AND INDUCTANCE OF BIMETALLIC WIRES

A simple method of computing the effective resistance and inductance of bimetallic wires over a considerable range of frequencies was developed from the consideration of two circuits in parallel. We will consider the copper-steel wire and assume the steel core to be circular in cross section and concentric with the shell. The magnetizing forces in the steel will be assumed so low that the permeability of the material is practically constant and equal to the initial value. Even for direct current most of the current flows in the high conductivity material of the shell and the magnetizing forces in the steel are relatively low. As the frequency increases the concentration of current in the shell also increases, and the effective resistance and inductance of the conductor become independent of the current strength. The assumption is also made that the distribution of the current in the shell is uniform at all frequencies. This would introduce appreciable error at high frequencies, but since the changes in distribution are small for a thin tube, the formulas are quite accurate in general up to telephone frequencies and over the range of frequencies wherein the changes are characteristic of the bimetallic wire. A check calculation upon a solid copper conductor will be given later. This serves to furnish an estimate of the error consequent upon the above assumption.

If we consider the magnetic fields of the core and shell separately, the total internal inductance of a bimetallic wire can be resolved into the following elements:

$$\begin{array}{c}\text{Inductive}\\ \text{effect of}\end{array}\left\{\begin{array}{l}\left.\begin{array}{l}\text{(a) the flux in core due to core current}\\ \text{(b) the flux in shell due to core current}\\ \text{(c) the flux in shell due to shell current}\end{array}\right\}\begin{array}{l}\text{cutting}\\ \text{the core.}\end{array}\\ \left.\begin{array}{l}\text{(d) the flux in shell due to shell current}\\ \text{(e) the flux in shell due to core current}\end{array}\right\}\begin{array}{l}\text{cutting}\\ \text{the shell.}\end{array}\end{array}\right.$$

The elements in (a), (b), and (c) belong to the total inductive effect upon the core and similarly (d) and (e) belong to the shell. We may designate them as self and mutual inductances in the following manner:

(a) + (b) $= L'_1 + L''_1 = L_1 =$ self-inductance of the core.
(c) $= M_{2,1} =$ mutual inductance of the shell on the core.
(d) $= L_2 =$ self-inductance of the shell.
(e) $= M_{1,2} =$ mutual inductance, core on shell.

In computing the values of these elements, when the flux lies within the cross section of that part of the conductor upon which its inductive effect is being considered, it must be weighted in the integration in the proportion of the area of the conductor cut by the flux to the total area. Expressions for several, if not all, of these quantities have been given in other places, but they are derived here for the sake of completeness.

In the following r_1 is the radius of the core which has a permeability μ, r_2 is the outside radius of the shell of permeability unity, and x is the distance from the axis of a cylindrical shell of thickness dx. H_1 and H_2 are the magnetic intensities due, respectively, to the currents I_1 in the core and I_2 in the shell. Their values are determined from the fact that the lines of force are circles concentric with the axis of the conductor and lie in a plane perpendicular to the axis, and the law that the integral of the magnetic intensity around such a line of force is equal to 4π times the current flowing through the surface bounded by the line. The expressions are then derived for unit length as follows:

(a) $H_1 = \frac{2I_1x}{r_1^2}$. Weighting the flux in proportion to the area of the core cut $I_1 L'_1 = \mu \int^{r_1} \left(\frac{2I_1 x}{r_1^2}\right) \frac{x^2}{r_1^2} dx = I_1 \frac{\mu}{2}$; $L'_1 = \frac{\mu}{2}$ the familiar expression for the internal self-inductance per cm of length.

(b)
$$H_1 = \frac{2I_1}{x}$$

$$I_1 L_1'' = \int_{r_1}^{r_2} \frac{2I_1\,dx}{x} = 2I_1 \log\frac{r_2}{r_1}$$

$$L_1'' = 2\log\frac{r_2}{r^1};\ L_1 = L_1' + L_1'' = \frac{\mu}{2} + 2\log\frac{r_2}{r^1}$$

(c)
$$H_2 = \frac{2I_2}{r_2^2 - r_1^2}\cdot\frac{x^2 - r_1^2}{x}$$

$$\begin{aligned}
I_2 M_{2,1} &= \int_{r_1}^{r_2} \frac{2I_2}{r_2^2 - r_1^2}\,\frac{x^2 - r_1^2}{x}\,dx \\
&= \frac{2I_2}{r_2^2 - r_1^2}\left(\frac{r_2^2 - r_1^2}{2} - r_1^2 \log\frac{r_2}{r_1}\right) \\
&= I_2\left(1 - \frac{2r_1^2}{r_2^2 - r_1^2}\log\frac{r_2}{r_1}\right)
\end{aligned}$$

$$M_{2,1} = 1 - \frac{2r_1^2}{r_2^2 - r_1^2}\log\frac{r_2}{r_1}$$

(d) $H_2 = \frac{2I_2}{r_2^2 - r_1^2}\cdot\frac{x^2 - r_1^2}{x}$ Weighting the flux in proportion to the area of the shell cut

$$\begin{aligned}
I_2 L_2 &= \int_{r_1}^{r_2}\left(\frac{2I_2}{r_2^2 - r_1^2}\cdot\frac{x^2 - r_1^2}{x}\right)\cdot\frac{x^2 - r_1^2}{r_2^2 - r_1^2}\,dx \\
&= \frac{2I_2}{(r_2^2 - r_1^2)^2}\int_{r_1}^{r_2}\left(x^3 - 2xr_1^2 + \frac{r_1^4}{x}\right)dx \\
&= I_2\left[\frac{1}{2} - \frac{r_1^2}{r_2^2 - r_1^2}\left(1 - \frac{2r_1^2}{r_2^2 - r_1^2}\log\frac{r_2}{r_1}\right)\right]
\end{aligned}$$

$$L_2 = \frac{1}{2} - \frac{r_1^2}{r_2^2 - r_1^2}\left(1 - \frac{2r_1^2}{r_2^2 - r_1^2}\log\frac{r_2}{r_1}\right)$$

(e) $H_1 = \frac{2I_1}{x}$ Weighting according to area of shell cut.

$$I_1 M_{1,2} = \int_{r_1}^{r_2}\left(\frac{2I_1}{x}\right)\frac{x^2 - r_1^2}{r_2^2 - r_1^2}\,dx = I_1\left(1 - \frac{2r_1^2}{r_2^2 - r_1^2}\log\frac{r_2}{r_1}\right)$$

$$M_{1,2} = M_{2,1} = 1 - \frac{2r_1^2}{r_2^2 - r_1^2}\log\frac{r_2}{r_1} = M$$

The above expressions may also be obtained from the expression given by Fowle [16] for the internal inductance of a bimetallic wire for direct current. To do this, we will treat the core and shell as two circuits in parallel, each of which contains self-inductance and which act upon each other mutually. The equivalent inductance of two such circuits in parallel is given for zero frequency by

$$L_{p=0}=\frac{(L_1I_1+L_2I_2)(I_1+I_2)-I_1I_2(L_1+L_2-2M)}{(I_1+I_2)^2}$$

The expression for the self-inductance when the shell is of unit permeability and the core of permeability μ becomes

$$L_{p=0}=\frac{\frac{\mu}{2}I_1^2+2I_1I_2+\frac{1}{2}I_2^2\left(1-\frac{2r_1^2}{r_2^2-r_1^2}\right)+2\left(I_1-I_2\frac{r_1^2}{r_2^2-r_1^2}\right)^2\log\frac{r_2}{r_1}}{(I_1+I_2)^2}$$

where I_1 is the core current and I_2 the shell current.

If we equate the two expressions for $L_{p=0}$ and also equate the coefficients of I_1^2, I_1I_2, and I_2^2, we get

$$L_1=\frac{\mu}{2}+2\log\frac{r_2}{r_1} \tag{5}$$

$$M=1-2\frac{r_1^2}{r_2^2-r_1^2}\log\frac{r_2}{r_1} \tag{6}$$

$$L_2=\frac{1}{2}-\left(\frac{r_1^2}{r_2^2-r_1^2}\right)\left[1-\frac{2r_1^2}{r_2^2-r_1^2}\log\frac{r_2}{r_1}\right] \tag{7}$$

which are identical with the expressions obtained above.

We can now extend the treatment of the bimetallic wire as two circuits in parallel to the case of the frequency greater than zero and compute the effective resistance and inductance of the conductor for a given frequency. If we consider the distribution of the current in the shell to remain uniform, it is then necessary to take into account only the effects of change of distribution in the core. Since a change of distribution in the core will for the same core current affect only the field inside the core itself, only $L'_1=\frac{\mu}{2}$ of the above inductances will be altered. We can, however, compute the value of L'_1 for a given frequency from the formula already given for a straight cylindrical wire of constant permeability.

---

[16] Elect. World, 56, p. 1523; 1910.

The resistance of the core will also be altered by the change of distribution, but the effective resistance for a given frequency can also be computed by the formula previously given.

We obtain then from the expressions for the equivalent resistance and inductance of the divided circuit at a frequency $f=\frac{p}{2\pi}$ the following formulas for the effective resistance and inductance of a bimetallic wire:

(8)

$$R'=\frac{[R_1R_2-p^2(L_1L_2-M^2)](R_1+R_2)+p^2(R_1L_2+R_2L_1)(L_1+L_2-2M)}{(R_1+R_2)^2+p^2(L_1+L_2-2M)^2}$$

and

(9)

$$L'=\frac{(R_1L_2+R_2L_1)(R_1+R_2)-[R_1R_2-p^2(L_1L_2-M^2)](L_1+L_2-2M)}{(R_1+R_2)^2+p^2(L_1+L_2-2M)^2}$$

Where R_1 is the computed effective resistance of the core and R_2 is the (assumed) constant or direct current resistance of the shell and $L_1=L'_1$ (computed) $+L''_1$.

The formulas were checked and estimates of the errors due to the assumption of uniform distribution in the shell were obtained by computing the effective resistance and inductance of a *solid* hard-drawn copper conductor. The core and shell were given the same dimensions as those used later in the computation of a No. 4 bimetallic wire. In effect the computation gave the effective resistance and inductance of a larger wire from the effective resistance and inductance of a wire of smaller radius. The values could then be directly calculated for the large wire by the exact formulas and the two sets compared. The following values were used in the computation: $r_1=0.2607$ cm, $r_2=0.2231$ cm, $\sigma=1770.6$ abs. units.

$R_2=3.0983$ ohms per kilometer.
$L''_1=31.14$ microhenrys per kilometer.
$L_2=9.56$ microhenrys per kilometer.
$M=14.78$ microhenrys per kilometer.

Effective Resistance and Inductances of Core

Frequency (cycles per second)	R_1 (ohms per kilometer)	L'_1 (microhenrys per kilometer)
0	1.1323	50.00
1000	1.1608	49.37
3000	1.3538	45.19
5000	1.6205	39.64

The following values were obtained for the effective resistance and inductance of the larger wire by the formulas (5) and (6) and the exact formulas (3) and (4), respectively:

Cycles per second	0	1000	3000	5000
	Effective resistance (ohms per kilometer)			
Approximate formula	0.8293	0.8667	1.0912	1.3432
Exact formula	0.8293	0.8676	1.0972	1.3582
Per cent difference	0.0	0.1	0.6	1.1
	Effective inductance (microhenrys per kilometer)			
Approximate formula	50.00	48.86	42.20	35.30
Exact formula	50.00	48.85	42.14	35.14
Per cent difference	0.0	0.0	0.1	0.5

The error even at 5000 cycles per second is only 1.1 per cent for the resistance and 0.5 per cent for the inductance and in the direction to be explained by neglecting the changes in distribution of the current in the shell. The values given by the approximate formulas approach, as limiting values for increasing frequency, the resistance and inductance of the shell, while actually the resistance should become infinite and the (internal) inductance zero. For a given frequency the values for the bimetallic wire with a steel core would be more strongly determined by those for the shell than in the above case, and hence somewhat higher errors may be expected. A number of comparisons will be made later, between the values for actual wires as determined by experiment and those given by the formulas (8) and (9).

VI. EXPERIMENTAL. METHODS

1. THE BRIDGE

The Anderson bridge, used in the measurement of inductance with direct current, was also used, with a few modifications, for the determinations of effective resistance and inductance. *P*, *R*, *S*, *r*, and *C* (Fig. 12) were the same, but the *Q* arm contained in addition to the wire under measurement (or the substitute link) the manganin or copper links and the mercury slider, a low resistance variable inductance of about 50 microhenrys range. The indicating instrument was a vibration galvanometer of the Rubens type for frequencies of 50 or 100 cycles per second or a low resistance telephone for the higher frequencies. The current for the bridge was supplied by two generators, one for 50 and 100 cycles and the other for frequencies from 500 to 3000 cycles. These machines were run by storage batteries. A 12 to 1 or 24 to 1 step-down transformer of 600 watts capacity was used to reduce the voltage on the bridge and to increase the available current. The current in the bridge was read by Hartmann and Braun hot-wire ammeters of 1, 5, and 10 amperes range. A variable resistance ρ was introduced into the bridge in series with the condenser *C* for the following purpose: The balance of an ideal Anderson bridge—that is, a bridge made up of completely noninductive resistances, perfect condenser, etc.—is independent of the frequency and the equations for balance when $P = R$, are

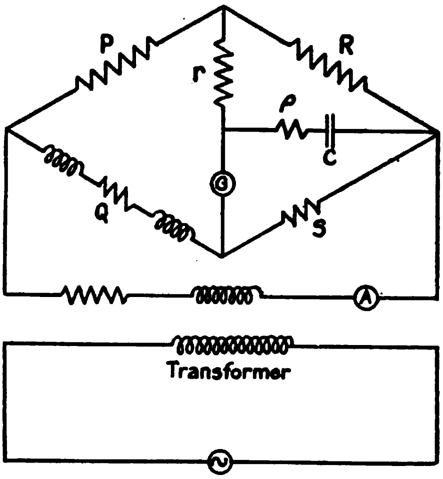


FIG. 12.—*Anderson bridge*

$$L = C\,S\,(R + 2r)$$
$$S - Q = 0$$

In an actual bridge, however, the resistance coils have residual inductance and the condenser is absorbing. On this account, the bridge balance at high frequencies and especially the resistance balance, will change considerably with the frequency. In a bridge such as that used in this work where the range of balance is somewhat limited, even though no error may be introduced, considerable changes in balance with the frequency are inconvenient.

If we assume the residual inductance l_3 in P and in R, l_4 in S and l_5 in r and a resistance ρ in series with a perfect condenser C, the equation[17] which gives the condition for a resistance balance may be brought into the form

$$S-Q=p^2C[(R+2r)l_4+Sl_3+2Sl_5-L\rho]$$

neglecting smaller terms.

In a low-resistance bridge the residual inductances are positive and the factor within the brackets can be reduced to zero by a proper choice of ρ. The resistance balance of the bridge then becomes independent of the frequency. However, the condenser may not be perfect and though it then is equivalent at a given frequency to a perfect condenser with a resistance in series, this resistance depends upon the frequency and an exact compensation for all frequencies is not possible. However, with a good mica condenser, as was used in this work, the changes in balance can be made very small.

2. RANGE OF THE MEASUREMENTS

The effective resistance and inductance of conductors which contain iron, depend in general both upon the frequency and the current strength. Also as in the case of the direct-current measurements, it was found that for solid iron wires consistent results could only be obtained when the iron was demagnetized. The procedure was to demagnetize the wire, using a low-frequency current by reducing the current slowly to one or two tenths of an ampere, from a considerably higher value than the maximum used in the experimental determinations. Then, at a given frequency, the measurements were made with successively increasing currents and the wire was again demagnetized before taking up another frequency. The range of current strengths was from 0.5

---

[17] The derivation is similar to that given by Rosa and Grover, this Bulletin, 1, p. 304; 1904–5 (Scientific Paper No. 14).

to 10 amperes and frequencies of 50, 100, 500, 1000, and 3000 cycles per second were used in addition to the direct current measurements.

A high accuracy in the measurements was considered unnecessary on account of the considerable variations that could be expected between different samples of the same grade of material. For this reason no attempt was made to control the temperature of the wire under investigation; the measurements were made at room temperatures which for most of the work varied from about 20° to 23° C.

3. METHOD USED FOR 50 AND 100 CYCLES

The measurements at 50 and 100 cycles were made in a manner similar to that used in the direct-current measurements, excepting that here the resistance as well as the inductance was considered. The resistance changes were measured by reading the settings of the calibrated mercury slider when the range of this was sufficient, otherwise the balance was brought within range of the mercury slider by substituting manganin links for similar copper links, the resistance changes due to the substitution having been determined by separate experiments. The inductance changes were determined from the settings of the resistance r. The frequency of the current was adjusted by means of the two vibration galvanometers used at these frequencies to determine the balance of the bridge. One was tuned to 50 and the other to 100 cycles and the frequency of the current was always adjusted to obtain the maximum sensibility. This determined the frequency to well within a per cent.

The method of measurement was to balance the bridge with a substitute link in place of the wire under measurement and with, say, 3 amperes current at 50 cycles in the bridge. The iron wire was then introduced and another balance obtained. Finally the bridge was balanced on direct current of the same strength. The difference in the setting of the resistance r between the first two balances gives the inductance at 50 cycles and 3 amperes when corrected for the inductance of the substitute link. The difference in the resistances of the mercury slider and manganin or copper links between the last two balances gives the change in resistance of the wire from direct current to 50 cycles and 3 amperes.

4. METHOD USED FOR 500, 1000, AND 3000 CYCLES

At the higher frequencies the above method would be liable to considerable error and the following procedure was used:

(1) The bridge containing the wire under measurement was balanced at 100 cycles and desired current. Readings of r and mercury slider were recorded.

(2) The bridge was balanced at the higher frequency by means of r and the mercury slider and if necessary by substituting a copper for a manganin link and these were recorded.

(3) The substitute link was introduced into the bridge and another balance at the higher frequency obtained. This time the inductance balance was obtained by means of the variable self-inductance, r remaining unchanged. The mercury slider was read.

(4) A final balance at 100 cycles with the substitute link was obtained by means of r and slider.

By this procedure a resistance and inductance which do not change appreciably with the frequency are substituted for the resistance and inductance of the iron wire at the higher frequency, the other parts of the bridge remaining unchanged. Then the determinations at 100 cycles serve to compare the values for high and low frequency or to measure the changes from 100 cycles to the higher frequency.

The resistance change is given by the difference in the readings of the mercury slider in going from (1) to (2) and then again from (3) to (4). Any changes in resistance links in the first interval must also be considered. In the latter interval the mercury slider alone will suffice on account of the resistance ρ in series with the condenser.

The inductance change is given by the difference in the settings of r in (1) and (4). No calibration of the variable inductance is necessary.

The method has the advantage of measuring the *changes* due to frequency in the wire under investigation and eliminates changes due to frequency in all parts of the bridge excepting the substitutes for the wire. The latter consisted of a variation of the mutual inductance between the two coils of a so-called variable self-inductance, and a variation of resistance by means of the mercury slider, the resistance links of manganin wire and the substitute link. The self-inductance was wound with stranded wire, each strand insulated, and would probably have introduced no error even if wound with solid wire, for we are here interested in the mutual inductance between the two coils which, in general, is not strongly influenced by frequency. The mercury slider and manganin links would not show appreciable skin effect at 3000 cycles due to high resistivity and small cross section. In the measurements upon solid iron wire where straight pieces of about 1.4 meters were included in the bridge, the substitute link was a copper wire which was made fairly heavy for convenience in handling. This had an appreciable skin effect at 1000 and 3000 cycles and the small corrections for this were calculated and applied. In the case of the solid copper and copper-steel bimetallic wires of which considerable lengths were used and strung in a rectangular shape, the ends were close together and a short substitute link of manganin was used for which no correction was necessary.

It was found that errors could be introduced into the measurement by a drift in the resistance balance of the bridge between (1) and (2), and (3) and (4). This error was reduced to a negligible amount by putting the measuring current on the bridge for a sufficient time before taking a measurement and by the procedure in which the wire under measurement was first included in the bridge and then the low temperature coefficient substitutes.

Errors could be introduced also by mutual inductance effects between different parts of the bridge and upon the indicating instrument, which changed when the substitution was made. These were guarded against in the arrangement of the bridge and investigated experimentally.

5. A TEST MEASUREMENT

Probably the most conclusive evidence of the freedom of the method from errors is shown by the test of the accuracy in measuring the small changes in a No. 10 solid copper wire. These could also be computed and compared with the measured values.

A length of about 32.5 m of the above wire was strung in a rectangular shape with about 30 cm spacing between the wires. The total resistance was 0.108 ohm and the inductance as strung 37.0 microhenrys. The changes in resistance and inductance as measured and computed for 100 to 2000 and 100 to 3000 cycles were as follows:

Cycles	Data	Change in resistance (ohms)	Change in inductance (centimeters)
100 to 2000	Experimental	+0.00116	−13
	Computed	+0.00132	−11
100 to 3000	Experimental	+0.00304	−26
	Computed	+0.00300	−25

The measurements were made with 3 amperes bridge current; the values for 2000 cycles are the mean of seven determinations, those for 3000 cycles the mean of nine. The agreement is in general better than would be expected from the sensitivity of the bridge under the conditions of measurement. Small corrections (about 2 cm) were made for changes in inductance with the setting of the mercury slider; in later work this correction was of no consequence.

VII. THE EFFECTIVE RESISTANCE AND INDUCTANCE OF IRON WIRES

1. EXPERIMENTAL RESULTS

Measurements of the effective resistance and inductance of iron wires were carried out upon the samples of telegraph and telephone wires previously mentioned. Three wires each of the grades E. B. B. (Extra Best Best) and Steel were used and Table 1 gives data with respect to them. The resistance measurements were made at about 20° C

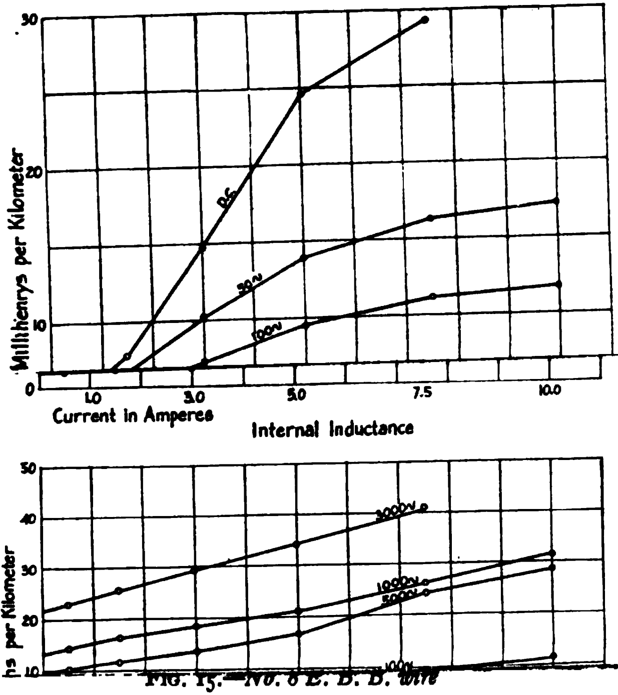


FIG. 15.—No. 8 E. B. B. wire

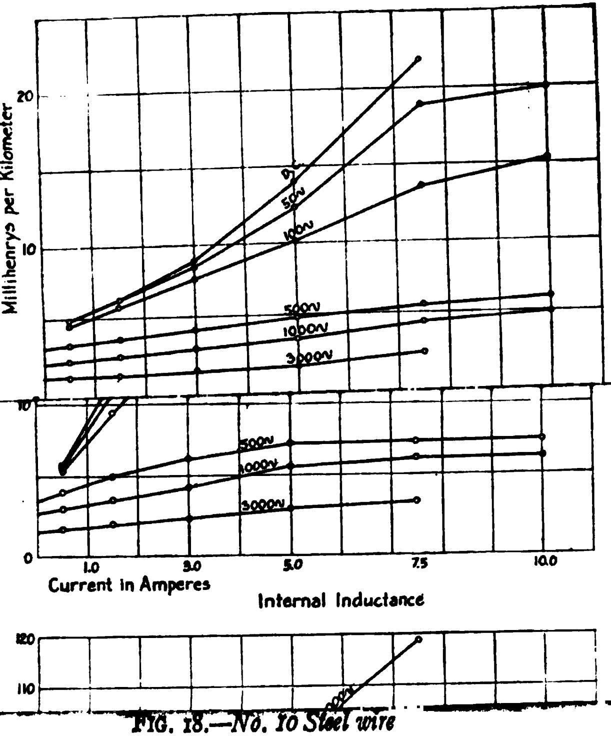


FIG. 18.—No. 10 Steel wire

FIGS. 13 TO 18 DIAGRAMS SHOWING EFFECTIVE RESISTANCE AND INTERNAL INDUCTANCE OF IRON TELEPHONE WIRES

10397°—15. (To face page 243.)

TABLE 1

Wire	Gage	Nominal diameter (millimeters)	Measured diameter (millimeters)	D. C. resistance (ohms per kilometer)	Resistivity (microhm-centimeters)	Per cent conductivity of H. D. copper
4 E. B. B.	B. W. G.	6.0	6.04	3.74	11.7	15.1
8 E. B. B.	B. W. G.	4.2	4.18	7.70	10.6	16.8
10 E. B. B.	B. W. G.	3.4	3.39	11.9	10.8	16.5
4 Steel	St. W. G.	5.7	5.68	5.34	13.5	13.1
8 Steel	B. W. G.	4.2	4.16	9.69	13.2	13.4
10 Steel	B. W. G.	3.40	3.41	14.7	13.4	13.2

The results of the measurements of effective resistance and inductance are given in Tables 2 to 7 and plotted in Figs. 13 to 18. The resistances are given in ohms per kilometer and the internal inductances in millihenrys per kilometer. The values for the D. C. inductances correspond to currents which are equal to the maximum values of the R. M. S. currents in the headings.

From the measurements the following points of interest may be noted:

1. Due to the lower permeability of the material, the Steel wires rapidly become better conductors than the E. B. B. wires of the same size as the frequency and current strength increase. At 50 cycles and 5 amperes the No. 4 Steel has a lower resistance than the No. 4 E. B. B. and the same is true of the Nos. 8 and 10 at the same frequency and 3 amperes.

2. The internal inductance at the higher frequencies is but slightly dependent upon the current strength and is, roughly, the same in value for all of the conductors.

TABLE 2

No. 4 E. B. B.

R. M. S. current in amperes	0.5	1.5	3.0	5.0	7.5	10.0
	Resistance (ohms per kilometer)					
Frequency:						
D. C.	3.74	3.74	3.74	3.74	3.74	3.74
50	4.04	4.58	5.77	7.62	8.64	8.80
100	4.53	5.61	7.27	9.99	11.9	12.4
500	8.53	10.9	14.7	21.5	28.7	30.0
1000	11.9	15.7	19.9	26.1	33.3	35.9
3000	19.5	24.7	30.9	39.9	50.5	....
	Internal inductance (millihenrys per kilometer)					
D. C.[18]	5.1	7.8	14.7	24.6	29.3	....
50	4.5	6.7	10.1	13.9	16.3	17.2
100	4.0	5.5	7.3	9.5	11.1	11.7
500	2.2	2.6	3.0	3.6	3.9	4.2
1000	1.5	1.7	2.1	2.6	3.1	3.4
3000	0.9	1.0	1.1	1.3	1.6	....

TABLE 3

No. 4 Steel

R. M. S. current in amperes	0.5	1.5	3.0	5.0	7.5	10.0
	Resistance (ohms per kilometer)					
Frequency:						
D. C.	5.34	5.34	5.34	5.34	5.34	5.34
50	5.50	5.70	6.00	6.51	7.51	8.56
100	6.01	6.39	6.98	7.92	9.53	11.4
500	10.4	11.6	13.4	16.5	24.1	28.5
1000	14.1	16.1	18.3	21.0	26.1	31.4
3000	23.0	25.5	29.3	34.1	41.1	....
	Internal inductance (millihenrys per kilometer)					
D. C.[18]	4.9	5.9	7.6	10.2	15.0	....
50	4.6	5.7	7.0	8.9	11.8	14.5
100	4.4	5.1	6.1	7.3	9.1	10.8
500	2.6	2.8	3.1	3.5	3.9	4.3
1000	1.8	1.9	2.2	2.5	2.8	3.3
3000	1.0	1.1	1.2	1.4	1.6	....

---

[18] The values of inductance for D. C. correspond to the maximum values of the R. M. S. currents given in the headings.

TABLE 4

No. 8 E. B. B

R. M. S. current in amperes	0.5	1.5	3.0	5.0	7.5	10.0
	Resistance (ohms per kilometer)					
Frequency:						
D. C	7.70	7.70	7.70	7.70	7.70	7.70
50	8.10	9.27	11.8	14.5	15.1	14.7
100	8.86	11.0	14.4	19.0	20.5	20.1
500	15.0	19.8	27.6	40.1	47.9	46.8
1000	20.7	27.5	36.8	49.8	57.5	58.6
3000	32.0	41.9	54.5	74.5	91.5	....
	Internal inductance (millihenrys per kilometer)					
D. C.[19]	6.6	12.5	27.4	40.8	40.2	....
50	6.5	11.1	17.7	23.2	25.1	25.2
100	5.9	8.9	12.6	16.1	17.6	17.7
500	3.5	4.2	5.2	6.0	6.2	6.4
1000	2.5	2.8	3.6	4.6	5.2	5.3
3000	1.7	1.9	2.1	2.4	2.8	....

TABLE 5

No. 8 Steel

R. M. S. current in amperes	0.5	1.5	3.0	5.0	7.5	10.0
	Resistance (ohms per kilometer)					
Frequency:						
D. C	9.69	9.69	9.69	9.69	9.69	9.69
50	9.80	10.1	10.5	11.6	13.3	14.5
100	10.2	10.7	11.5	13.3	16.4	18.5
500	15.3	17.2	20.2	27.7	40.7	43.8
1000	20.5	23.6	27.5	34.3	45.7	51.4
3000	32.5	37.5	43.7	54.7	73.1	....
	Internal inductance (millihenrys per kilometer)					
D. C.[19]	5.2	6.6	9.0	14.0	21.8	....
50	5.15	6.5	8.6	12.3	17.2	19.9
100	4.9	6.0	7.7	10.1	13.6	15.3
500	3.6	3.9	4.4	5.1	5.8	6.3
1000	2.5	2.8	3.2	3.7	4.7	5.3
3000	1.5	1.6	1.8	2.0	2.7	....

---

[19] The values of inductance for D. C. correspond to the maximum values of the R. M. S. currents given in the headings.

TABLE 6

No. 10 E. B. B

R. M. S. current in amperes	0.5	1.5	3.0	5.0	7.5	10.0
	Resistance (ohms per kilometer)					
Frequency:						
D. C.	11.9	11.9	11.9	11.9	11.9	11.9
50	12.2	13.1	16.1	19.5	19.8	19.0
100	12.8	14.6	19.0	25.1	26.4	25.5
500	19.0	24.7	35.4	52.8	60.8	58.8
1000	25.9	34.3	46.9	65.1	73.9	73.0
3000	41.3	52.9	69.3	98.8	118.5	....
	Internal inductance (millihenrys per kilometer)					
D. C.[30]	6.4	11.5	26.4	42.4	43.7	....
50	6.3	10.7	19.5	27.2	29.2	28.9
100	6.0	9.4	14.7	19.5	21.1	21.0
500	4.3	5.3	6.4	7.4	7.5	7.6
1000	3.1	3.7	4.5	5.9	6.4	6.5
3000	1.8	2.1	2.4	3.1	3.6	....

TABLE 7

No. 10 Steel

R. M. S. current in amperes	0.5	1.5	3.0	5.0	7.5	10.0
	Resistance (ohms per kilometer)					
Frequency:						
D. C.	14.7	14.7	14.7	14.7	14.7	14.7
50	14.8	15.1	15.7	17.4	19.7	20.2
100	15.1	15.7	16.8	19.9	23.6	25.2
500	19.9	22.7	27.5	39.7	52.8	54.6
1000	26.3	30.7	36.7	47.8	61.0	64.9
3000	40.7	47.2	56.4	75.1	98.1	....
	Internal inductance (millihenrys per kilometer)					
D. C.[30]	5.2	6.8	10.0	17.4	27.8	....
50	5.2	7.0	9.8	15.6	21.0	22.9
100	5.0	6.5	8.8	12.9	16.5	17.7
500	4.0	4.6	5.4	6.6	7.0	7.3
1000	3.0	3.4	4.0	5.0	5.9	6.3
3000	1.8	1.9	2.3	2.8	3.4	....

---

[30] The values of inductance for D. C. correspond to the maximum values of the R. M. S. current given in the headings.

2. COMPARISON OF OBSERVED AND COMPUTED VALUES FOR ZERO CURRENT

From the curves of Figs. 7, 8, and 9 we can obtain the initial values of the permeability for circular magnetization and by means of these values compute the effective resistances and inductances of the wires for zero current. In Table 8 these *computed* values are compared with the corresponding *observed* values obtained by extrapolating the results of experiment to zero current. The agreement between the two sets of values is fair; the differences may be due partly to the error in extrapolating and partly to a lack of homogeneity of the iron in the wires, which would require a somewhat different value of the minimum permeability to be used at different frequencies.

TABLE 8

Comparison of Observed and Computed Values of Effective Resistance and Inductance of Iron Wires for Zero Current

Wire	Permeability	Frequency	Resistance		Internal inductance	
			Observed	Computed	Observed	Computed
4 E. B. B	78	500	7.4	7.5	2.1	2.2
		1000	10.0	10.2	1.4	1.5
		3000	17.0	16.8	0.8	0.9
4 Steel	94	500	9.9	10.3	2.4	2.8
		1000	13.2	14.0	1.7	2.0
		3000	21.9	23.1	1.0	1.1
8 E. B. B	98	500	12.8	12.9	3.2	3.4
		1000	17.2	17.5	2.4	2.4
		3000	27.0	28.7	1.7	1.4
8 Steel	90	500	14.3	14.2	3.4	3.5
		1000	19.0	19.2	2.4	2.6
		3000	30.0	31.2	1.5	1.5
10 E. B. B	93	500	16.2	16.2	3.7	3.8
		1000	21.5	21.8	2.8	2.9
		3000	35.8	35.4	1.7	1.7
10 Steel	90	500	18.4	18.3	3.7	3.9
		1000	24.1	24.2	2.7	3.1
		3000	38.0	39.2	1.7	1.8

3. EQUIVALENT PERMEABILITY—EFFECTS OF HYSTERESIS

The values of equivalent permeability were computed in the manner outlined on page 230. The values shown in the curves of Figs. 19 and 20 were calculated from the measurements upon the

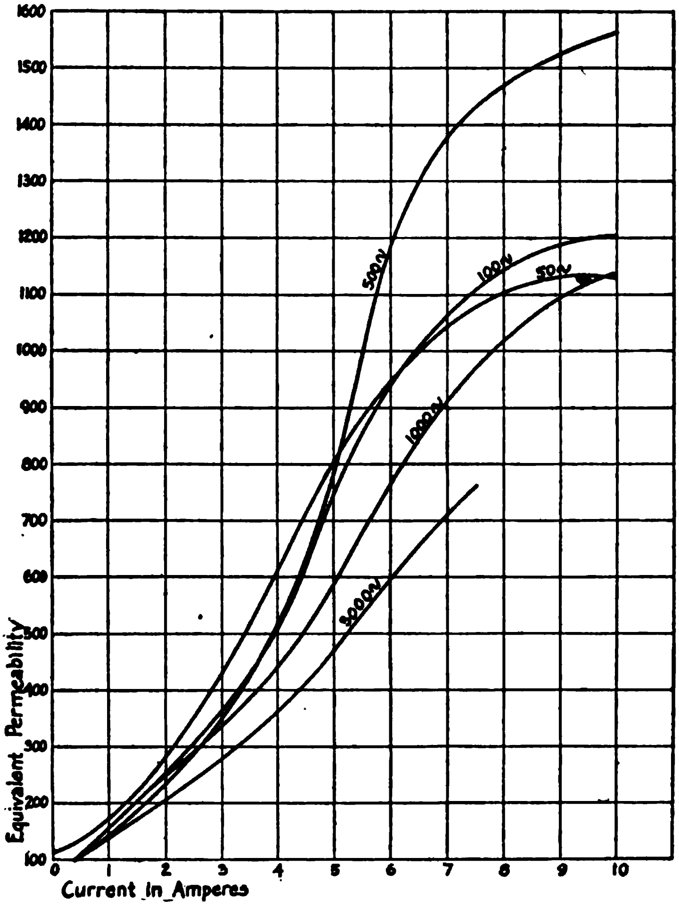


Fig. 19.—*Equivalent permeability from resistance data*

No. 4 E. B. B. wire. They are characteristic of the curves obtained for the other wires. It will be noted that the permeabilities calculated from resistance data reach extremely high values for 500 cycles, while at the same frequency the values obtained from the inductance data are low. From this we can obtain some interesting deductions with respect to the rôle played by hysteresis in determining the effective resistance and inductance of the wire.

The energy loss due to hysteresis increases the effective resistance of the conductor. Let us first assume that the distribution of current in the wire is unaltered as the frequency increases. Then the loss of energy per cycle would be a constant; the *loss per second* would increase proportional to the frequency. In such a case the resistance would increase proportional to the frequency.

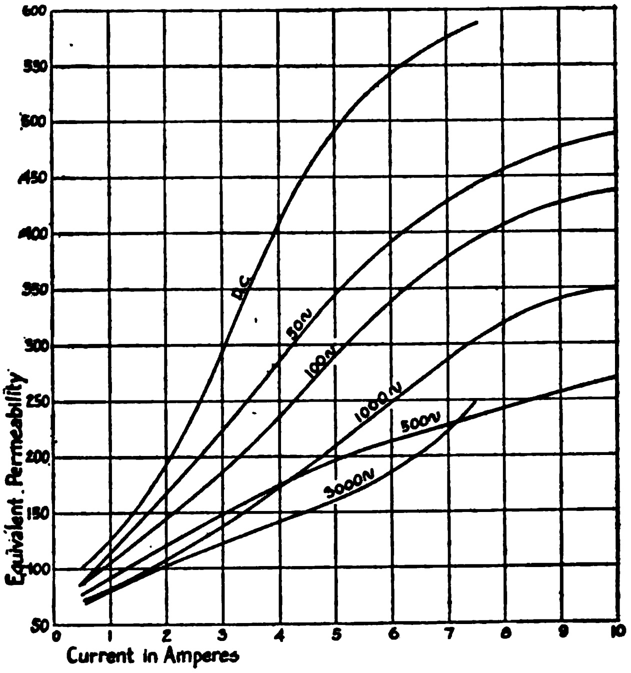


FIG. 20.—*Equivalent permeability from inductance data*

Actually, however, as the frequency increases, the current tends to concentrate more and more in the outer layers and the amplitude of the magnetizing force, and hence the hysteresis loss per cycle is diminished in the iron within the conductor. Thus, as the frequency increases these two effects oppose each other. At 500 cycles the hysteresis loss reaches a maximum, and on this account the equivalent permeability, as computed from the resistance data, attains high values.

Now let us consider the effect of hysteresis upon the inductance of the wire. The inductance is determined solely by the distribution of the current. We can consider that the hysteresis loss is due to a counter emf. This counter emf would be a maximum at the outside of the conductor since there the amplitude of the magnetizing force, and hence the hysteresis loss is a maximum. On this account the outer layers would have a higher effective resistance than those within, and this would tend to equalize the current distribution over the cross section, i. e., to diminish the skin effect. On this account the equivalent permeability values for 500 cycles, as calculated from the inductance data, would be low.

VIII. THE EFFECTIVE RESISTANCE AND INDUCTANCE OF COPPER-STEEL BIMETALLIC WIRES

1. EXPERIMENTAL RESULTS

Three sizes of copper-steel bimetallic wires, the so-called copper clad wires, were investigated. The sizes were No. 4, No. 8, and No. 10 A. W. G. Lengths of about 33 m of the wire were strung in a parallel wire arrangement with separations of about 30 or 40 cm. No effect of separation upon the changes in resistance and inductance with the frequency was expected or looked for with separations as large as these. The following data may be of interest:

TABLE 9

Copper Clad Wires and Steel Core of No. 4

Wire	Mean diameter (mm)	Calculated shell thickness (mm)	Resistance per kilometer-ohms	Resistivity (microhm-cm)	Per cent conductivity (H. D. copper)
4	5.22	0.38	2.24	4.79	37.0
8	3.27	0.36	4.42	3.71	47.7
10	2.57	0 23	8.15	4.23	41.9
Core of 4	2.46	....	26.5	12.63	14.0

The experimental results are given in Tables 10, 11, and 12, the resistance in ohms per kilometer and the internal inductance in microhenrys per kilometer. The external inductance in centimeters for parallel wires of radius ρ spacing d and length l all in centimeters is readily calculable from the formula

$$L = 4\, l \left[\log_e \frac{d}{\rho} - \frac{d}{l} \right]$$

where l is the length of each wire.

TABLE 10

No. 4 Copper Clad

R. M. S. current in amperes	0.5	1.5	3	5	10
	Resistance (ohms per kilometer)				
Frequency:					
D. C.	2.24	2.24	2.24	2.24	2.24
50	2.25	2.26	2.26	2.27	2.30
100	2.29	2.30	2.31	2.33	2.38
500	2.60	2.61	2.63	2.64	....
1000	2.73	2.75	2.75	2.76	....
3000	2.94	2.92	2.93	2.94	....
	Internal inductance (microhenrys per kilometer)				
D. C.[a]	293	318	355	392	....
50	288	306	335	369	446
100	274	284	298	316	361
500	123	122	118	112	....
1000	71	70	68	69	....
3000	43	41	39	39	....

[a] The values given for the D. C. inductance correspond to the maximum values of the R. M. S. currents given in the headings.

TABLE 11

No. 8 Copper Clad

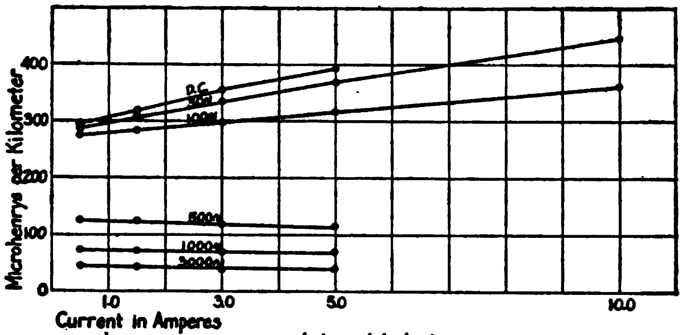
R. M. S. current in amperes	0.5	1.5	3	5	10
	Resistance (ohms per kilometer)				
Frequency:					
D. C.	4.42	4.42	4.42	4.42	4.42
50					
100	4.43	4.44	4.44	4.44	
500	4.58	4.59	4.60	4.62	
1000	4.77	4.78	4.78	4.80	
3000	5.04	5.04	5.04	5.02	
	Internal inductance (microhenrys per kilometer)				
50	141	145	151	158	174
100	140	144	149	155	
500	112	112	115	113	
1000	75	76	78	75	
3000	38	39	38	38	

TABLE 12

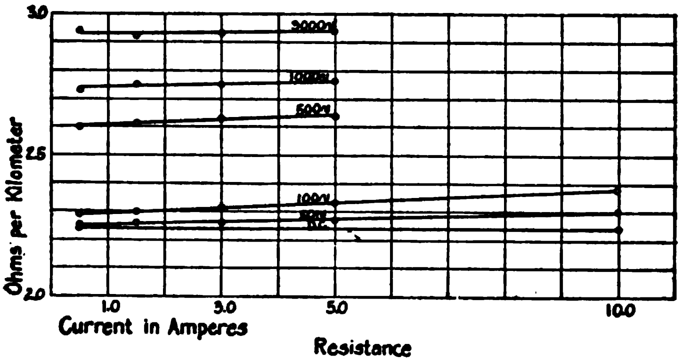
No. 10 Copper Clad

R. M. S. current in amperes	0.5	1.5	3	5
	Resistance (ohms per kilometer)			
Frequency:				
D. C.	8.15	8.15	8.15	8.15
100	8.16	8.16	8.17	8.18
500	8.36	8.39	8.43	8.50
1000	8.72	8.75	8.79	8.84
3000	9.38	9.38	9.39	9.39
	Internal inductance (microhenrys per kilometer)			
50	198	209	224	244
100	198	208	222	240
500	171	175	179	181
1000	122	123	122	121
3000	45	41	43	40

The results are plotted in Figs. 21, 22, and 23. For low frequencies there is an increase in resistance and inductance with increasing current. This is due to an increase in the flux of induction in the steel core which increases the inductance as a direct effect and increases the resistance indirectly on account of the skin effect. The latter tends to reduce the inductance, but is outweighed by the direct effect. As the frequency increases the resistance and inductance become independent of the current strength. This is due to the fact that the impedance of the core has become so high relative to the impedance of the shell that very large changes in the former would be required to produce appreciable changes in the combined impedance. At very high frequencies, of course, the copper clad wire would behave exactly the same as a solid copper conductor. Similarly, the higher the



Internal Inductance



Resistance

FIG. 21.—*No. 4 copper clad wire*

conductivity of the copper-steel wire the less the dependance of the resistance and inductance upon the current strength, in this case also approaching as a limit the solid copper conductor.

2. A COMPARISON OF OBSERVED AND COMPUTED VALUES

The experimental determinations of the effective resistance and inductance with a current of 0.5 ampere may very well be taken to represent the case where the permeability of the core is a minimum and constant. The *maximum* values of the magnetizing force in the core for direct current of 0.5 x $\sqrt{2}$ ampere are, roughly, 0.18, 0.20, and 0.32 C. G. S. units for the No. 4, No. 8, and No. 10 wires, respectively. From Fig. 11 it may be seen that the permeabilities for these magnetizing forces are practically the minimum.

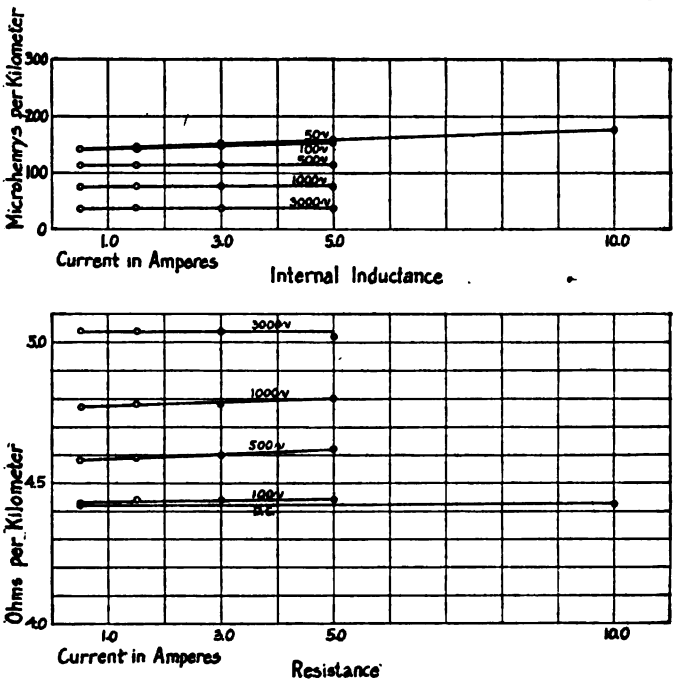
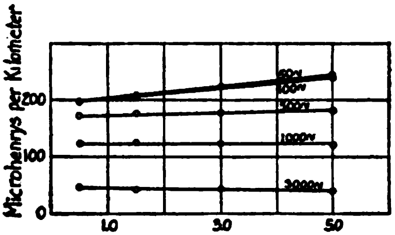


FIG. 22.—*No. 8 copper clad wire*

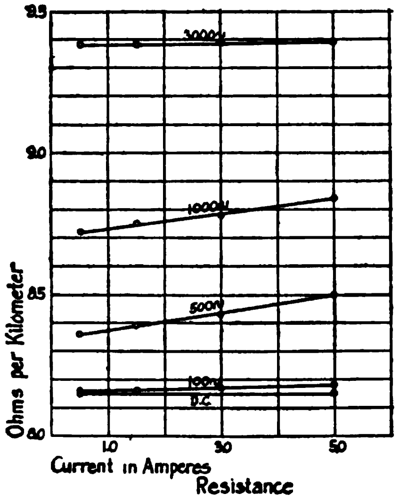
In order to compute the values of the effective resistance and inductance for a given frequency, it is necessary to know the radius, resistivity, and permeability of the core. Measurements upon the steel wire which was obtained by turning down the No. 4 Copper Clad wire gave the resistivity of the steel as 12.63 microhm-centimeter. The resistivity of hard-drawn copper was taken as 2.7 per cent higher than the international annealed copper standard;[23] i. e., as 1.771 microhm-centimeter. The resistivities of both the steel and the copper will vary irregularly from one wire to another, due to composition, heat treatment, and drawing. The above values were, however, assumed for all three wires in the calculations. From these values—the measured resistances on direct current and the radii of the wires—we obtain the data in Table 13.

The first calculations were made before the permeability curves of Fig. 11 were obtained for the No. 8 and No. 10 sizes, and the value $\mu = 90$ determined from the core of the No. 4 wire was used for all three wires.

Using the above data the effective resistance and internal inductance of the core was calculated for the frequencies 100, 500, 1000, and 3000 by the formulas (3) and (4) and the inductances L_1, L_2, and M by the formulas (5), (6), and (7). In formula (5) in place of $\frac{\mu'}{2}$ the computed value of the internal inductance of the core was substituted. Finally the effective resistances and internal inductances of the copper-clad wires were computed by formulas (8) and (9). The experimental values for 0.5 ampere



Internal Inductance



Resistance

FIG. 23.—*No. 10 copper clad wire*

---

[23] Copper Wire Tables, Circular No. 31, Bureau of Standards.

and the computed values are compared in Table 14, the resistances being given in ohms per kilometer and the internal inductances in microhenrys per kilometer.

TABLE 13

Wire	Radius (centimeter)	Radius of core (centimeter)	Thickness of shell (centimeter)	Direct-current resistance (ohms per kilometer)	
				Core	Shell
No. 4	0.261	0.223	0.038	8.06	3.10
No. 8	0.164	0.128	0.036	24.65	5.39
No. 10	0.129	0.106	0.023	35.95	10.53

It will be noted that two sets of computed values are given for the No. 10 wire. The first set (1) was computed from the assumed value $\mu=90$, as explained above. The determination of the permeability curves given in Fig. 11 was then made and resulted in values of 85 and 70 for the initial values of the permeabilities of the No. 8 and No. 10 wires, respectively. The calculation for the No. 10 wire was repeated, using the value 70, which resulted in the second set (2), which show considerably closer agreement between the observed and computed values, in particular for the self-inductance.

TABLE 14

Comparison of Observed and Computed Values of Effective Resistance and Internal Inductance of Copper-Clad Wires

Wire	Data		0~	100~	500~	1000~	3000~
No. 4	Resistance	Observed	2.24	2.29	2.60	2.73	2.94
		Computed	2.24	2.31	2.68	2.80	2.93
	Internal inductance	Observed	293	274	123	71	43
		Computed	360	316	99	46	18
No. 8	Resistance	Observed	4.42	4.43	4.58	4.77	5.04
		Computed	4.42	4.44	4.63	4.85	5.04
	Internal inductance	Observed	141	140	112	75	38
		Computed	163	160	118	71	30
No. 10	Resistance	Observed	8.15	8.16	8.36	8.72	9.38
		Computed (1)	8.15	8.16	8.41	8.85	9.58
		Computed (2)	8.15	8.16	8.32	8.67	9.46
	Internal inductance	Observed	198	198	171	122	45
		Computed (1)	246	245	208	146	50
		Computed (2)	195	194	176	137	53

3. EFFECT OF NONCIRCULARITY OF THE CORE

It is of interest to inquire into the possible effects of departures of the core of actual wires from the circular cross section assumed in the calculation. At low frequencies the core plays its most important part in determining the resistance and inductance of the wire, and this is especially true when the shell is thin. The inductance is the more strongly influenced, as may be seen from the above two values of the inductance for the No. 10 wire at zero frequency for $\mu=90$ and $\mu=70$. Here the change in μ is 22 per cent and the change in the inductance about 21 per cent. As the frequency increases, the resistance and inductance of the wire approach the values for the shell. It would seem that departure of the core from circularity would have the effect of decreasing the effective radius of the core and increasing the effective thickness of the shell; for when the core is circular the lines of force would run either entirely through steel or entirely through copper, the steel would have its maximum effect, making the total induction internal to the wire a maximum. When, however, the core is noncircular, some of the lines of force would run partly through steel and partly through copper, the reluctance of the path would be determined practically entirely by the copper, and the effect of this steel would hardly be any more than if it were a nonmagnetic material. It could then be considered a part of the shell. The effects due to noncircularity would be the following:

1. A decided decrease in the self-inductance for direct current and low frequencies, due to the decrease of the core inductance.
2. A reduction of the magnitude of the skin effect for intermediate frequencies, due to a decrease in the effective radius of the core.
3. An increase in the limiting value of the inductance for higher frequencies, due to the increase in the effective thickness of the shell.

The limiting value of the resistance for high frequency would, however, not be changed very much, due to the increase in the effective thickness of the shell, for this increase is brought about by the addition of a material of high resistivity; but the limiting value ought to be attained somewhat slower.

The differences between the computed and experimental values for the No. 4 wire, in which the core departs badly from circularity, are exactly such as to be explained from this cause. The agreement in the resistance at 3000 cycles is probably bettered somewhat by skin effect in the shell which brings the observed value up to the computed, in which this skin effect is neglected.

The noncircularity of the core is an advantage at low frequencies from the standpoint of practice, since it reduces both the inductance and resistance of the wire. It is probable that the increase in resistance and inductance due to increasing current strength is likewise diminished by the noncircularity of the core. This suggests the possible desirability of giving the core a cross section which would have the effect of a considerable departure from circularity, as might be accomplished by having a slotted cross section like that in Fig. 24. This would approximate somewhat a conductor having a core of high resistivity, nonmagnetic metal. In such a conductor the internal inductance would be lower than in a solid conductor, and both the resistance and inductance would be independent of the current strength. Further, as the frequency increased the per cent conductivity relative to a solid copper conductor would increase throughout the whole range, whereas the opposite is the case in the lower range of frequencies for the wire as now made.

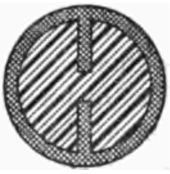


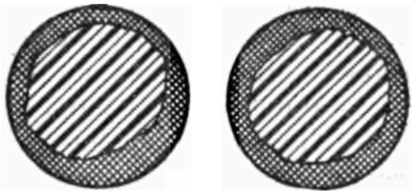
FIG. 24.—*Slotted cross section*

In Fig. 25 are shown the cross sections of the Nos. 4, 8, and 10 wires used in the measurements. These drawings were obtained from photomicrographs.[23] One sample was taken from each end of the 30-m length of wire. The photographs show a considerable deviation of the core from circularity (especially for the No. 4), together with a fair constancy of the cross section of the steel in a particular wire both in shape and area.

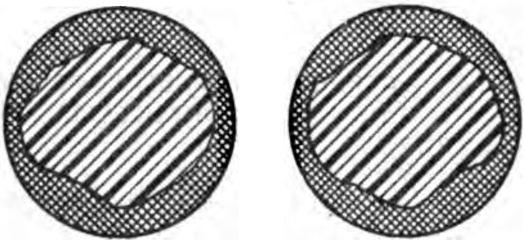
---

[23] The samples were prepared by the metallurgical division of the Bureau of Standards. The photographs were taken by this division and the Bureau photographer.

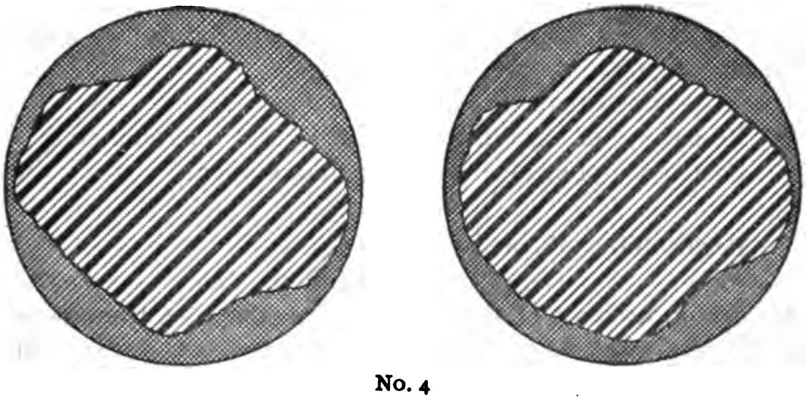
In making the calculations for effective resistance and inductance of the copper-clad wires it seemed reasonable to assume that the resistivity of the copper would be very nearly constant from wire to wire; the resistivity of the steel, however, was considered to be susceptible of considerable variations. In order to obtain some information in this respect, the photomicrographs were measured with a planimeter and the ratio of r_2 to r_1 determined for all three wires. This measured ratio agreed very closely with



No. 10



No. 8



No. 4

FIG. 25.—*Cross section of copper clad wires from photomicrographs*

that previously computed for the No. 8 wire, but for the Nos. 4 and 10 was several per cent higher. Since a direct measurement of conductivity had been made upon the steel of the No. 4 wire, this meant that the copper was considerably lower in conductivity than the assumed value—i. e., 97.5 per cent of standard annealed copper. A short strip of copper was then sawed from the No. 4 wire, and the conductivity was found to be only about 84.8 per cent of annealed copper. It is therefore probable that the copper in the No. 10 wire was also low in conductivity, but that the No. 8 was normal. The effect upon the computed effective resistances is small; the computed inductances, however, would be affected more strongly, and in such a way as to reduce the differences for the No. 4 wire.

IX. WIRE TABLES. EFFECTIVE RESISTANCE AND INDUCTANCE OF COPPER-STEEL BIMETALLIC WIRES FOR SMALL CURRENTS

By means of the formulas (8) and (9) the effective resistance and inductance of copper-steel bimetallic wires Nos. 0, 2, 4, 6, 8, 10, and 12 A. W. G. were calculated for 30, 40, and 50 per cent conductivities (of hard-drawn copper) and frequencies ranging from zero to 3000.

It was assumed that

Resistivity of hard-drawn copper = 1.771 microhm-cm.
Resistivity of steel core = 12.63 microhm-cm.

$$\text{Permeability of core}\left\{\begin{array}{l}=90 \text{ for Nos. } 0, 2, 4\\ =85 \text{ for Nos. } 6, 8\\ =70 \text{ for Nos. } 10, 12.\end{array}\right.$$

Mainly on account of the departure of the core from noncircularity the values of inductance for actual wires may differ considerably from the computed values. Deviations of the permeability of the core and conductivity of the core and shell from the assumed values may also cause differences. The resistance values are not as strongly affected by these variations as the inductance values and the computed values of resistance should fit actual wires to a few per cent.

Table 15 gives the resistance values in ohms per kilometer, Table 16 the per cent increase in resistance from the direct-current values, and Table 17 the internal inductance values to two significant figures in microhenrys per kilometer. In Fig. 26 the per cent increase in resistance is plotted with respect to frequency, and in Fig. 27 the internal inductance values are also plotted as a function of the frequency. Fig. 28 shows the variation of the per cent increase in resistance with the conductivity, and from this figure, by interpolation, values may be obtained for intermediate conductivities. It will be noted that ordinary deviations of the wire size from the nominal size for the gage number will not appreciably affect the per cent increase in resistance.

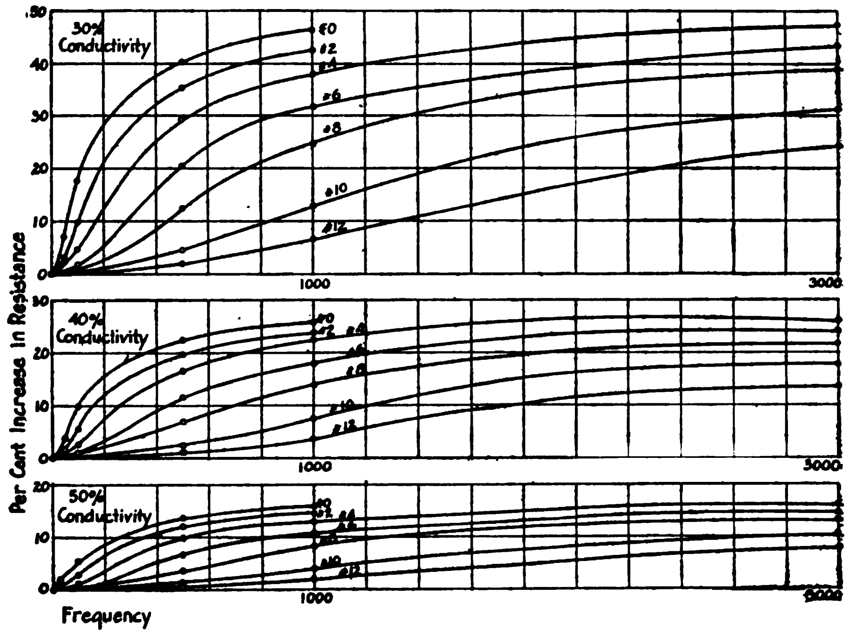


FIG. 26.—*Effective resistance of copper-steel wires. Computed*

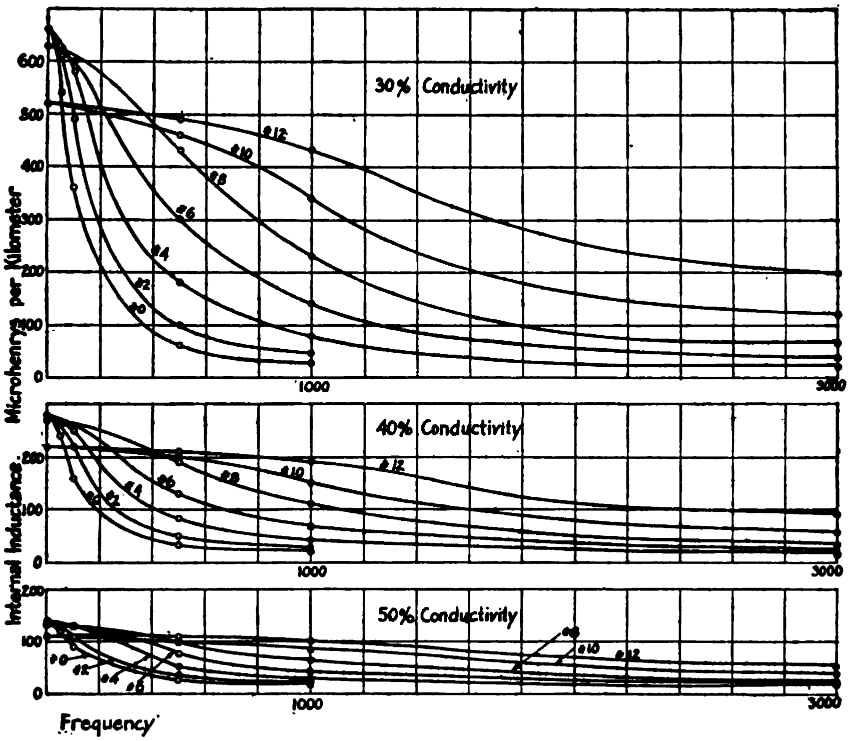


FIG. 27.—*Computed internal inductance. Copper-steel wires*

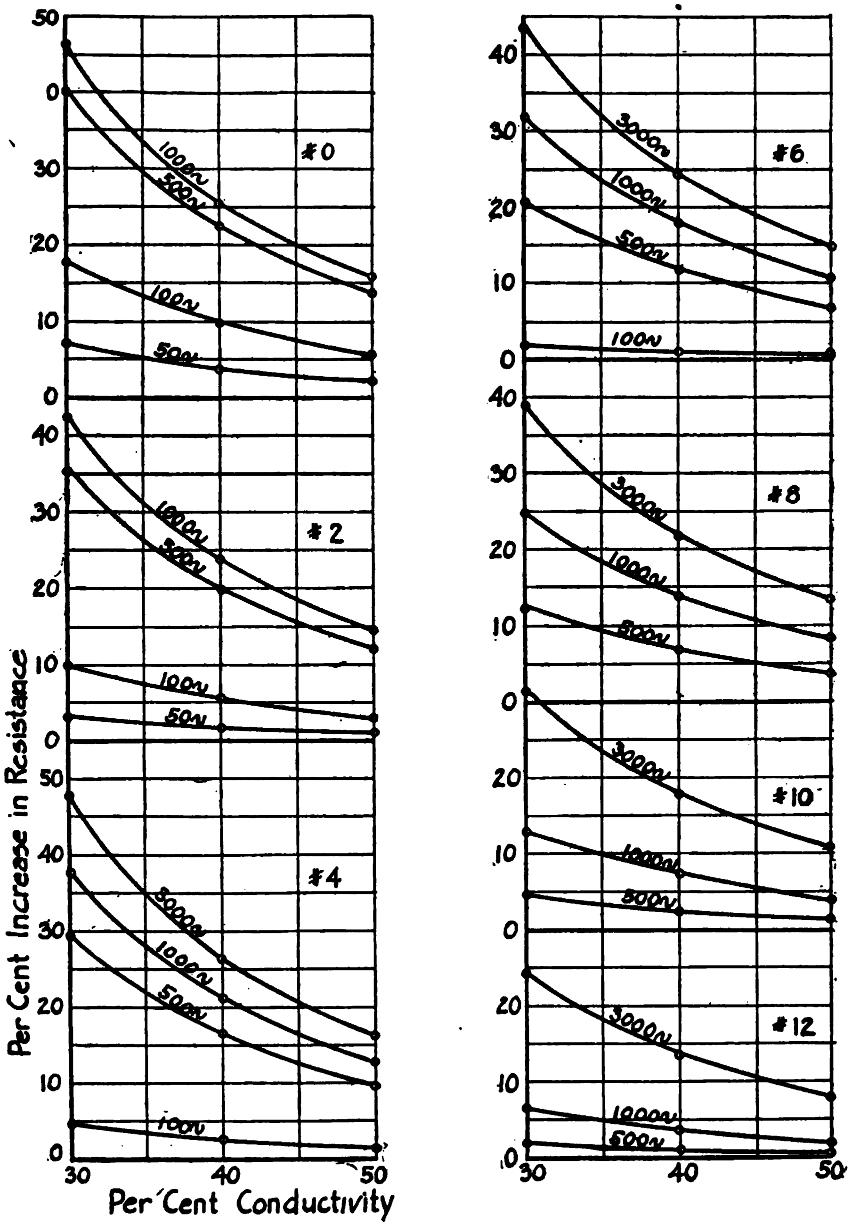


FIG. 28.—*Effective resistance of copper-steel wires as a function of the conductivity*

TABLE 15

Effective Resistance of Copper-Steel Bimetallic Wires for Small Currents

[Computed by the formulas 8 and 9, p. 235]

Wire (A. W. G.)	Conductivity	Frequency (cycles per second)					
		0~	50~	100~	500~	1000~	3000~
	Per cent	Ω/km	Ω/km	Ω/km	Ω/km	Ω/km	Ω/km
No. 0	30	1.10	1.18	1.30	1.55	1.61	....
	40	0.83	0.86	0.91	1.01	1.04	....
	50	0.66	0.68	0.70	0.75	0.77	....
No. 2	30	1.75	1.81	1.93	2.37	2.50	....
	40	1.32	1.34	1.39	1.58	1.63	....
	50	1.05	1.06	1.08	1.18	1.21	....
No. 4	30	2.79	....	2.92	3.61	3.85	4.12
	40	2.09	....	2.15	2.44	2.54	2.64
	50	1.67	....	1.70	1.84	1.89	1.94
No. 6	30	4.44	....	4.52	5.3	5.8	6.4
	40	3.33	....	3.36	3.71	3.92	4.13
	50	2.66	....	2.68	2.84	2.95	3.06
No. 8	30	7.1	....	....	7.9	8.8	9.8
	40	5.3	....	....	5.7	6.0	6.4
	50	4.23	....	....	4.39	4.57	4.80
No. 10	30	11.2	....	....	11.7	12.7	14.7
	40	8.4	....	....	8.6	9.0	9.9
	50	6.7	....	....	6.8	7.0	7.5
No. 12	30	17.8	....	....	18.2	19.0	22.1
	40	13.4	....	....	13.5	13.9	15.2
	50	10.7	....	....	10.8	10.9	11.6

TABLE 16

Increase in Effective Resistances of Copper-Steel Bimetallic Wires From Direct Current Values, for Small Currents

[Computed by the formulas 8 and 9, p. 235]

Wire (A. W. G.)	Conductivity	Frequency (cycles per second)					
		50~	100~	500~	1000~	3000~	∞[34]
	Per cent	Per cent	Per cent	Per cent	Per cent	Per cent	Per cent
No. 0	30	7	18	40	46	....	61
	40	4	10	22	26	....	32
	50	2	6	14	16	....	19
No. 2	30	3	10	35	42	....	61
	40	2	5	20	24	....	32
	50	1	3	12	14	....	19
No. 4	30	....	5	29	38	48	61
	40	....	3	17	21	26	32
	50	....	1	10	13	16	19
No. 6	30	....	2	21	32	43	61
	40	....	1	12	18	24	32
	50	....	0.5	7	11	15	19
No. 8	30	....	....	12	25	39	61
	40	....	....	7	14	22	32
	50	....	....	4	8	13	19
No. 10	30	....	....	4	13	31	61
	40	....	....	2	7	18	32
	50	....	....	1	4	11	19
No. 12	30	....	....	2	7	24	61
	40	....	....	1	4	14	32
	50	....	....	0.5	2	8	19

---

[34] The limiting value by this method of calculation. It is determined by the ratio of the resistance of the shell to the total resistance of the wire for direct current.

TABLE 17

Effective Internal Inductance of Copper-Steel Bimetallic Wires for Small Currents [25]

[Computed by the formulas 8 and 9, p. 235]

Wire A. W. G.	Conductivity	Frequency (cycles per second)						
		0~	50~	100~	500~	1000~	3000~	∞ [26]
	Per cent	μh/km	μh/km	μh/km	μh/km	μh/km	μh/km	μh/km
No. 0	30	660	540	360	61	28	....	7
	40	280	240	160	32	20	....	11
	50	140	120	90	27	20	....	16
No. 2	30	660	610	490	100	47	....	7
	40	280	260	220	50	27	....	11
	50	140	130	110	36	24	....	16
No. 4	30	660	....	580	180	79	24	7
	40	280	....	250	83	40	18	11
	50	140	....	130	53	31	19	16
No. 6	30	630	....	600	300	140	39	7
	40	270	....	260	130	66	24	11
	50	130	....	130	77	44	23	16
No. 8	30	630	....	....	430	230	66	7
	40	270	....	....	190	110	35	11
	50	130	....	....	100	65	28	16
No. 10	30	520	....	....	460	340	120	7
	40	220	....	....	200	150	58	11
	50	110	....	....	100	85	40	16
No. 12	30	520	....	....	490	430	200	7
	40	220	....	....	210	190	93	11
	50	110	....	....	110	100	57	16

---

[25] The values were calculated on the assumption of circular cross section of core and definite values of permeability of core material and of resistivity of core and shell. The deviations of the values for actual wires from those in the table may be considerable.

[26] The limiting value by this method of calculation. It is the internal inductance of the shell.

X. SUMMARY

1. A formula is derived which permits the exact permeability curve of a homogeneous iron wire for circular magnetization to be obtained from measurements of the self-inductance of the wire with direct current. Determinations and comparisons of axial and circular permeability curves are made for a number of wires. In some cases the two curves show fair agreement and in others considerable differences. The latter are probably due to the nonhomogeneity or nonistropy of the material.

2. The effective resistances and inductances of a number of samples of iron telegraph and telephone wires are determined by a bridge method in the range of frequencies from 50 to 3000 cycles per second and with current strengths varying from 0.5 to 10 amperes. The experimental results are interpolated to zero current and compared with values calculated from the minimum permeabilities of the materials.

3. Formulas are derived for the effective resistance and inductance of bimetallic wires for the case of constant permeability of the materials. A number of copper-steel bimetallic wires are investigated experimentally, and the results for low currents are compared with the values obtained by computation. A table of computed values of effective resistance and inductance for various frequencies is given for copper-steel wires of 30, 40, and 50 per cent conductivity and for sizes Nos. 0, 2, 4, 6, 8, 10, and 12 A. W. G.

In conclusion I desire to express my thanks to Dr. E. B. Rosa and Dr. H. L. Curtis for their many valuable suggestions.

WASHINGTON, April 14, 1915.

A DIRECT-READING DEVICE FOR USE IN COMPUTING CHARACTERISTICS OF VACUUM TUNGSTEN LAMPS (WITH COMPUTING CHART)

By J. F. Skogland

CONTENTS

		Page.
I.	Introduction	269
II.	Scales of the device	270
III.	How the scales are used	271
IV.	Examples illustrating scale settings and their precision	272
V.	Advantages and disadvantages	275
VI.	Construction of the scales	276
	1. Range selected	276
	2. Coordinates to which all points are referred	277
	3. Watts per candle scales	277
	4. Normal per cent candlepower scale	278
	5. Preliminary per cent voltage scale	278
	6. Preliminary volt scale	278
	7. Use of the scales constructed above	278
	8. Per cent candlepower curves	279
	9. Final volt scale	281
VII.	The computer in a modified form	281
VIII.	Tables of values used in constructing the scales	282
	1. Volt scale	282
	2. Watts per candle scale	283
	3. Per cent candlepower chart	284
IX.	Summary	286
	Instructions for chart (computing chart attached) facing	288

I. INTRODUCTION

Previous work in the derivation of characteristic equations of vacuum tungsten lamps[1] has led to the consideration of the possibility of designing some form of computing device, the construction of which would depend directly upon these equations, and which would yield values of acceptable precision, when compared with those obtained by use of the tables[2] which have been computed from the equations.

---

[1] Middlekauff and Skogland, "Characteristic equations of tungsten lamps and their application in heterochromatic photometry," this bulletin, 11, pp. 483–534; 1914. (Scientific Paper No. 238.)

[2] Pp. 527–534.

The characteristic equations here mentioned are employed in the determination from observed values of voltage, candlepower, and watts per candle the values of these variables at other points, thus obtaining values whose precision is as high as that of the observations used as the basis of the computations. Observations may thus be made at any convenient point; for example, at a voltage corresponding to a match in color with standards whose values are well known, and direct measurements at other points are eliminated. The advantage of measurements at color match may thus always be realized, and the disadvantages of color difference measurements (notably, large deviations of individual observations from their mean) may be eliminated. By this method the following steps are taken in obtaining a solution:

1. Factors read from a reduction table are applied to the observed values to obtain the so-called "normal" values; that is, values at normal wpc (1.20).
2. Voltage ratio is computed, using the normal value as a base.
3. The proper table is referred to at the point corresponding to the voltage ratio, and actual wpc and per cent cp are read.
4. Cp ratio is applied to normal cp to secure the value desired.

In case the voltage corresponding to a certain cp is desired, the second step is the computation of cp ratio, using the normal value as a base; this cp ratio is then the point of reference in the tables. This process, though simple, requires considerable time before a solution is obtained.

The device has passed through the experimental stage, improvements being made as suggested by considerations of convenience to the user. The result finally accomplished is the production of a chart of scales which evaluate any one of the variables—candlepower, watts per candle, or voltage—with a single setting to observed values at any wpc within the range of the device.

II. SCALES OF THE DEVICE

The scales, shown on the plate at the end of this paper, include horizontal scales of volts and wpc. These scales are found at the lower part of the plate. The horizontal lines which are ruled across the cp chart correspond to observed or rated wpc, and they fix the point at which the volt scale is set in computing per cent cp.

The volt scale should be cut from the chart and preferably mounted on a strip of dry hardwood or on metal, care being exercised to avoid shrinkage.[8] Shellacking of any of the scales is not recommended.

The remainder of the chart may be placed on a drawing board or other plane surface.

III. HOW THE SCALES ARE USED

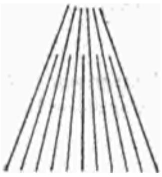


FIG. 1.—*Interpolating scale*

Observed voltage is set to the 100 per cent cp line at its intersection with the horizontal corresponding to observed wpc. Extreme care need not be exercised in interpolating wpc between scale divisions in this setting, but the volt scale should be exactly horizontal. The 100 per cent cp line (hereinafter called the initial cp line) has been drawn as a broken line, so that it is always easily found. From the setting made, per cent cp at any voltage may now be read at the point of intersection with the volt scale corresponding to the voltage at which a value is desired, or the converse problem may be solved by the same setting. For accurate interpolation on the cp chart a scale like that of Fig. 1 may be used. Its use is too obvious to require comment. In practice, this scale is copied on transparent material, such as celluloid or tracing cloth. A scale of this kind was used in obtaining values given in the examples of the following section.

If the volt scale has been set exactly to observed wpc on the initial cp line, wpc at any point desired may be read by dropping a perpendicular to the horizontal wpc scale; but since, as stated above, extreme accuracy is not required in setting to observed wpc on the cp chart, a better method is to apply the volt scale directly to the horizontal wpc scale, placing observed voltage opposite observed wpc. On this wpc scale interpolation may be easily and accurately made.

---

[8] If only the volt and wpc scales are to be generally used, both may be detached from the chart and mounted in any convenient slide-rule form, the cp chart still being available for occasional computations

A few examples illustrating different settings of the scales will now be given.

IV. EXAMPLES ILLUSTRATING SCALE SETTINGS AND THEIR PRECISION

In the following examples values read from the scales are compared with those obtained by use of Tables 20 and 22.[4]

EXAMPLE 1.—Per cent cp and wpc read with 100 volts set to the initial cp line and to 1.20 wpc. From this setting per cent cp and wpc at other volt scale divisions are read. This is evidently a percentage setting of the volt scale. The results are given in Table 1.

TABLE 1

Comparison of Values of Per Cent cp and of wpc at Various Percentages of the Voltage Corresponding to 1.20 wpc, as Read from the Scales and from the Tables

Per cent voltage	Per cent candlepower			Watts per candle		
	Scale	Table 20	Difference	Scale	Table 22	Difference
			Per cent			Per cent
94.0	80.0	79.94	0.08	1.360	1.361	0.07
97.5	91.4	91.29	.12	1.262	1.263	.08
103.0	111.3	111.2	.09	1.131	1.131	.00
107.5	129.4	129.4	.00	1.040	1.040	.00
112.5	151.9	151.8	.07	.953	.9525	.05
117.0	174.0	174.0	.00	.884	.8840	.00
120.0	190.0	189.9	.05	.843	.8431	.01
125.5	221.4	221.4	.00	.776	.7762	.03
128.0	237.0	236.7	.13	.749	.7487	.04
132.0	262.8	262.6	.08	.708	.7084	.06
Mean			.06			.03

EXAMPLE 2.—Readings of per cent cp and wpc made from setting observed, or rated, values at 1 wpc, and comparison with values computed by the tables.

---

[4] Middlekauff and Skogland, loc. cit., pp. 532, 534.

TABLE 2

Volts		Per cent candlepower			Watts per candle		
From observed or rated	To scale	Scale	Table 20	Difference	Scale	Table 22	Difference
				Per cent			Per cent
105	111	121.4	121.4	0.00	0.899	0.899	0.00
113	121	127.0	127.0	.00	.878	.878	.00
115	124	130.0	130.1	.08	.866	.866	.00
117	107	72.9	72.8	.14	1.192	1.193	.08
117	125	126.2	126.0	.16	.882	.881	.11
118	111	80.7	80.6	.12	1.127	1.127	.00
120	110	73.5	73.4	.14	1.187	1.187	.00
Mean				0.09			0.03

EXAMPLE 3.—Observations: 110 volts, 25 candles, 1.35 wpc. Required: (*a*) Cp and wpc at 125 volts; (*b*) volts and wpc for 20 candles.

Solution.—(*a*) Set 110 volts to the initial cp line at its intersection with the horizontal through 1.35 wpc. Read at 125 volts, 158 per cent cp. $1.580 \times 25 = 39.50$ candles, the required value. A similar setting to the horizontal wpc scale gives 1.045 wpc.

(*b*) $\frac{20}{25} = 0.80 = 80$ per cent cp. Set the scales as in (*a*) above and read at 80 per cent cp, 103.5 volts, the required value for 20 candles. Set 110 volts to 1.35 on the wpc scale, and opposite 103.5 volts read 1.532 wpc.

The solution by the much longer method employing Tables 19, 20, and 22 is (*a*) 39.48 candles, 1.045 wpc; (*b*) 103.5 volts, 1.532 wpc.

Summary of results.—(1) Scale values: (*a*) 39.50 candles, 1.045 wpc; (*b*) 103.5 volts, 1.532 wpc.

(2) Table values: (*a*) 39.48 candles, 1.045 wpc; (*b*) 103.5 volts, 1.532 wpc.

EXAMPLE 4.—Observations: 110 volts, 88 candles, 1.05 wpc. Required: Volts and candles at 0.7 wpc.

Solution.—Set 110 volts to 1.05 wpc. Read at 0.7 wpc, 136.6 volts, the required value. Set 110 volts to the initial cp line on the horizontal through 1.05 wpc. Read at 136.6 volts, 211.2 per cent cp. 88×2.112=185.9 candles, the value required. This solution is identical with that obtained by employing Table 19.[6]

EXAMPLE 5.—The following example, which has been given in part in Table 8, page 506, of the paper above referred to (note 1), illustrates the application of the scales in the adjustment of several observations. Three solutions by the same method are given, so that the scales (the cp scales in particular) may be tested on several wpc lines.

This method is somewhat different from that given in the discussion of characteristic equations, but, although not quite so precise, it results in securing very nearly the same adjustment of values. It is based on the validity of the arithmetical mean and therefore appears to be a sound method. One observed value is selected as a base for scale settings. After reading from the scales the values at each voltage, these values are adjusted to give the same mean as the observed values. This is accomplished by increasing or decreasing each observed value by the same percentage as that required to correct the mean of the observed values to the mean of the computed values, a procedure familiar in other applications to all who have had laboratory experience in any line.

Results and comparisons are given in Table 3.

TABLE 3

Adjustment of observed cp Values of a Group of Seven 40-watt Drawn Wire Lamps

Observations				Scale values applied as factors to—			Adjusted values, from mean based on—		
Number	Volts	Candles	(wpc)	Obs. 1	Obs. 4	Obs. 7	Obs. 1	Obs. 4	Obs. 7
1	96	18.20	(1.633)	18.20	18.33	18.18	18.27	18.30	18.24
2	104	24.48	....	24.43	24.58	24.46	24.52	24.54	24.54
3	112	32.15	....	31.94	32.04	31.98	32.06	31.98	32.09
4	120	41.07	(1.032)	40.82	41.07	40.89	40.97	41.00	41.03
5	128	51.46	....	51.25	51.55	51.32	51.44	51.46	51.50
6	136	63.48	....	63.16	63.45	62.96	63.39	63.33	63.18
7	144	76.71	(0.786)	76.66	77.10	76.71	76.94	76.96	76.97
Mean	....	43.94	....	43.78	44.02	43.79	43.94	43.94	43.94

---

[6] Middlekauff and Skogland, loc. cit. This example is given on p. 524.

TABLE 3—Continued

Mean of adjusted values	Values obtained from Tables 19 and 20
18.27	18.26
24.53	24.51
32.04	32.05
41.00	40.97
51.47	51.37
63.30	63.33
76.96	76.91
43.94	43.91

Maximum difference, 0.19 per cent.
Average difference, 0.07 per cent.

The differences which appear in these examples between values read from the scales and those computed by employing Tables 19, 20, and 22 are certainly so small that no valid objection can be raised against the use of the device as here presented without raising a similar objection to the tables of the previous paper; for the observed values are checked very nearly as closely by one method as by the other. Such differences as do occur may be attributed to slight errors in construction, width of line of the original drawing necessary to secure a plain reproduction, and errors in interpolation.

V. ADVANTAGES AND DISADVANTAGES

When compared with the method involving use of Tables 19 to 22, the great advantage that is first apparent in favor of this device is a saving of time, a fact which will be evident to any who solve the examples of the preceding section by each method.

This results because of the following facts, namely, reduction to normal wpc is eliminated; direct and converse problems may be solved with equal ease; voltage ratios need not be computed; the volt and wpc scales constitute a complete slide rule from which, for example, life-test voltages may be read directly for any wpc, or the wpc at the nearest rack voltage may be read from the same setting.

An advantage over any method employing exponents is that use of the exponential rule or of logarithm tables is unnecessary. This method, in common with the exponential method, avoids reduction to normal wpc. In the exponential method, however, several charts of curves, constructed as previously described,[6] are necessary in solving both direct and converse problems. For example, curves giving exponents which, when applied to voltage ratio, evaluate cp ratio can not be employed in solving the converse problem. Further, voltage and wpc ratios must be known before the exponential method can be used.

There are but few disadvantages, of which the following may be mentioned: A device which would include the whole range in wpc covered by the tables would either be inconveniently large, or, if reduced to a convenient size, there would be some sacrifice of precision; the values obtained even from scales perfectly constructed are modified by personal errors in interpolation between scale divisions. These errors may, however, be greatly reduced by using the interpolating scale shown in Fig. 1.

VI. CONSTRUCTION OF THE SCALES

1. RANGE SELECTED

In order to minimize the disadvantages just mentioned, the scales have been designed to include a range in wpc of from 0.70 to 2.05, with a volt scale extending from 94 to 166 volts, these limits being determined by the size of section-ruled paper available, with due consideration of the voltages and efficiencies most generally employed in laboratory and factory practice. It is believed that the range of these scales is sufficient to include settings and solutions for 105 to 130 volt lamps in standardizing and life-test problems.

A full description of the design of these scales will now be given. The method outlined may be used in the construction of scales of any range and is general in its application to functions which may be expressed by related equations of the same form as those expressing the characteristics here considered.[7] By extension of the method, all similarly related quadratic equations involving either x or y as independent variable, if these variables are of exponential significance, may be similarly represented after the proper choice of coordinates and may be used in exactly the same way as are the scales here designed.

---

[6] Middlekauff and Skogland, loc. cit., p. 513. [7] For characteristic equations, see pp. 496, 498.

2. COORDINATES TO WHICH ALL POINTS ARE REFERRED

All of the scales are based upon scales of equal parts corresponding to logarithmic per cent voltage, which may be considered as a whole number (for convenience, it is here so considered) or as a ratio.

From reciprocals of values of "Volts" given in Table 19,[8] which contains factors for reduction to 1.20 wpc, it was found that a range in wpc of from 0.70 to 2.05 includes values of per cent voltage of from 76 to 133, approximately. The corresponding logarithms are included within the range 1.880 to 2.130. These numbers, 1.880 to 2.130, were indicated in pencil in a horizontal direction as the *X* scale of equal parts. For reasons which will appear later, the same scale was repeated in a vertical direction as a *Y* scale. Due to limitations imposed by the width of the plotting paper, the total length of the *Y* scale was made one-half that of the *X* scale. Any practical value may be assigned to the ratio of these scales.

3. WATTS PER CANDLE SCALES

From reciprocals of values of "Volts" given in Table 19[8] the horizontal wpc scale was constructed in steps of 0.01, each point being placed at the corresponding value of logarithmic per cent voltage. The same scale was plotted vertically in steps of 0.05 wpc from 2.05 to 1 and of 0.02 wpc for the remainder of the scale. The last-named scale divisions were extended horizontally across the chart, as shown on the plate. On drawings of smaller range the *Y* scale may be made equal in length and in number of scale divisions to the *X* scale, but this is a matter of no great importance since, as stated in Section III above, extreme accuracy in interpolation on this scale is not required.

These wpc scales are not logarithmic,[9] as they are based upon a second-degree logarithmic equation.[10]

---

[8] Middlekauff and Skogland, loc. cit., p. 527. See also p. 500, sec. 6, lines 19–23.

[9] "Logarithmic"; that is, two such scales would not form scales *A* and *B* of the ordinary slide rule.

[10] Middlekauff and Skogland, loc. cit., p. 496, eq. 12.

4. NORMAL PER CENT CANDLEPOWER SCALE

From Table 20[11] may be read per cent cp for any voltage ratio on the normal basis (1.20 wpc), so that tabulated values may be plotted in the same manner as was done in constructing the wpc scales; that is, steps of per cent cp may be plotted opposite the corresponding logarithmic per cent voltage, this construction giving a normal per cent cp scale. Assume this scale drawn.

5. PRELIMINARY PER CENT VOLTAGE SCALE

If, now, a per cent voltage scale be drawn in with points placed at the corresponding logarithms X, there is completed a set of scales from which values of per cent cp and of wpc may be read for any percentage of normal voltage; that is, the tabulated values have been transferred to the scales, and the device is, as yet, no improvement over the tables, because it could at best yield only the tabulated values, with little, if any, saving of time. It need hardly be stated that the present choice of coordinates would not have been necessary if this were the final result to be achieved. The purpose of the choice made becomes evident in what follows.

6. PRELIMINARY VOLT SCALE

If, now, the per cent voltage scale just constructed be free to move—that is, detached from the chart—it may be used as a volt scale of range from 76 to 133 volts, which is a considerable improvement, as voltage ratios need not be computed. Obviously, this transformation of the per cent voltage scale to a volt scale follows directly from the fact that the scale is logarithmic, and hence is, by construction, a computer of voltage ratio for any setting.

7. USE OF THE SCALES CONSTRUCTED ABOVE

These three scales (wpc, volts, and normal per cent cp) may be used in connection with reduction Table 19 (see note 2), normal cp only being computed by applying to the observed value the factor (F_0) read from the table, or the reciprocal (F_1) of this factor may be read from the normal per cent cp scale at the observed voltage, which is set to observed wpc. At the required voltage is read the cp factor (F_2) on the normal basis and the actual wpc. If P_0 is the observed value in candles, and P_1 is the required value, $P_1 = P_0F_0F_2 = \frac{P_0}{F_1}F_2$. It is to be noted that the reduction $P_0F_0 = \frac{P_0}{F_1}$ is a reduction of observed cp to normal cp, but that the corresponding normal voltage need not be read. There then remains only the reduction to normal cp to dispense with before being free from use of the tables or from taking two readings from a normal per cent cp scale. This is accomplished by the construction of the per cent cp curves, which, as will be proven, are straight lines.

---

[11] Middlekauff and Skogland, loc. cit., p. 532.

8. PER CENT CANDLEPOWER CURVES

As the second characteristic equation (see note 7) is employed in proving some of the relations which are derived below, it is here quoted.

This equation is:

$$y_2 = -\ .946x^2 + 3.592x \qquad (1)$$

in which $y_2 = \log$ cp ratio and $x = \log$ voltage ratio, the conditional relation between x and y_2 being that $x = y_2 = \log 1 = 0$ at actual wpc $= 1.20$ (the normal value).

From equation 1 is derived by differentiation,

$$\frac{dy_2}{dx} = -1.892x + 3.592, \qquad (2)$$

which shows that $\frac{dy_2}{dx}$ decreases as x increases above its normal value. Since wpc and consequently logarithm wpc also decrease, it follows that at wpc less than 1.20 the change of logarithm cp ratio with logarithm voltage ratio, and hence of cp ratio with voltage ratio, is less than at the normal value of 1.20 wpc; hence the distance in scale divisions from 100 per cent to any other per cent cp on a scale constructed in the manner given in paragraph 4, page 278, but based on a wpc less than 1.20, would be greater than on the normal scale. By similar reasoning, since x is negative for voltage ratios less than 1, this distance would be less at wpc greater than 1.20. For example, a cp scale constructed with its

100 per cent point at 0.8 wpc will contain fewer scale divisions for a given range in wpc than will a similar scale constructed at 1.6 wpc.

Suppose, now, that the normal cp scale constructed above be moved up to the 1.20 line of the Y wpc scale constructed in paragraph 3, page 277. It will evidently bear the right relation to all of the scales so far constructed. Also, the 100 per cent cp point for any wpc may be traced as a straight line drawn through corresponding points of the horizontal and vertical wpc scales. By construction this line also passes through corresponding values of logarithmic per cent voltage. This line is then the first of the cp curves and is designated as the initial cp line. If other cp scales be constructed at points above and below the normal scale just placed, values being obtained from the tables, a series of curves may be drawn through corresponding points of the different scales. These curves are straight lines, which greatly simplifies their construction and makes it evident why the vertical scale was based on a logarithmic scale of equal parts. That these curves are straight lines may be proven as follows: Equation (1) above may be written

$$Y_2 = X^{-.946x+3.592} = X^{u_2} \qquad (3)$$

in which Y_2 = cp ratio and X = voltage ratio, u_2 being a variable function of x expressed by the equation

$$u_2 = -.946x + 3.592, \qquad (4)$$

from which is obtained by differentiation

$$du_2 = -.946dx, \qquad (5)$$

or, expressed in words, u_2 changes uniformly with x.

It therefore follows that the curve for any per cent cp must be displaced from the initial cp line by a distance which is constantly changing with x, since an increase in exponent corresponds to an increase in distance on a logarithmic scale. To illustrate: Log $8 = \log 2^3 = 3 \log 2$. If log 2 be plotted as one scale division, log 8 would be found at three scale divisions; the exponent is increased from 1 to 3 and the distance is increased in like degree. These curves are therefore straight lines which, as shown above, diverge from the initial cp line in passing to the lower wpc.

So that in construction two points only need be accurately placed for each line, preferably one above and one below the normal scale, throughout its range, and either two or three points for the lines which do not cross the normal scale. A third point in any case is a check on the accuracy of placing the other two, and is not absolutely necessary.

9. FINAL VOLT SCALE

The volt scale designed in sections 5 and 6, page 278, might be used, but a scale of different range constructed, however, in exactly the same manner is given on the plate. Logarithms corresponding to the voltages chosen (94 to 166) were substituted for the original per cent voltage logarithms of the *X* scale, the same distance between successive points being of necessity adhered to. The volt and half-volt divisions were drawn in at points corresponding to their logarithms. Any volt scale desired may be constructed in a similar manner.

The actual values through which the lines of the chart were drawn are given in Section VIII, so that the entire chart or any section of it may be drawn to any scale desired.

VII. THE COMPUTER IN A MODIFIED FORM

From these data it is possible to construct a circular or disk computer, although the construction presents points of considerable difficulty. Volt and wpc scales may be easily so arranged, and their construction is not difficult.

Since, however, the cp curves of a circular computer would be drawn through points determined by radial and circumferential voltage logarithms, they would no longer be straight lines, and more than three points would have to be placed for each curve. As a result their construction would be rather difficult.

VIII. TABLES OF VALUES USED IN CONSTRUCTING THE SCALES

Scale divisions are here considered as whole numbers; e. g., log 94 is given instead of log 0.94 as the first point of the volt scale. Rectangular cartesian coordinates are used and *X* and *Y* have their usual graphical significance.

1. VOLT SCALE

The values of *X* for each scale division on a reference scale of equal parts are given in Table 4. This is the only logarithmic scale of the device.

TABLE 4

Values of X Corresponding to Values of Voltage Indicated by Scale Divisions

Scale	X	Scale	X	Scale	X	Scale	X
94.0	1.97812	110.5	2.04336	127.0	2.10380	143.5	2.15685
94.5	543	111.0	532	127.5	551	144.0	836
95.0	772	111.5	727	128.0	721	144.5	986
95.5	1.98000	112.0	921	128.5	890	145.0	2.16136
96.0	227	[12] 112.5	2.05115	129.0	2.11058	145.5	286
96.5	452	113.0	307	129.5	226	146.0	435
97.0	677	113.5	499	130.0	394	146.5	583
97.5	900	114.0	690	130.5	561	147.0	731
98.0	1.99122	114.5	880	131.0	727	147.5	879
98.5	343	115.0	2.06069	131.5	892	148.0	2.17026
99.0	563	115.5	258	132.0	2.12057	148.5	173
99.5	782	116.0	445	132.5	221	149.0	318
100.0	2.00000	116.5	632	133.0	385	149.5	464
100.5	216	117.0	818	133.5	548	150.0	609
101.0	432	117.5	2.07003	134.0	710	151	897
101.5	646	118.0	188	134.5	872	152	2.18184
102.0	860	118.5	371	135.0	2.13033	153	469
102.5	2.01072	119.0	554	135.5	193	154	752
103.0	283	119.5	736	136.0	353	155	2.19033
103.5	494	120.0	918	136.5	513	156	312
104.0	703	120.5	2.08098	137.0	672	157	589
104.5	911	121.0	278	137.5	830	158	865
105.0	2.02118	121.5	457	138.0	987	159	2.20139
105.5	325	122.0	635	138.5	2.14144	160	412
106.0	530	122.5	813	139.0	301	161	682
106.5	734	123.0	990	139.5	457	162	951
107.0	938	123.5	2.09166	140.0	612	163	2.21218
107.5	2.03140	124.0	342	140.5	767	164	484
108.0	342	124.5	516	141.0	921	165	748
108.5	542	125.0	691	141.5	2.15075	166	2.22010
109.0	742	125.5	864	142.0	228		
109.5	941	126.0	2.10037	142.5	381		
110.0	2.04139	126.5	209	143.0	533		

---

[12] Example: Log 112.5=2.05115, the point on the *X* scale of equal parts at which the 112.5 volt division is placed.

2. WATTS PER CANDLE SCALE

In Table 5 are given values of X corresponding to wpc scale divisions. These values of X are in part repeated as ordinates Y, through which horizontals are drawn across the per cent cp chart. All of these values may be repeated for placing horizontals, if it is so desired. The values given are based on a reference scale of equal parts. The length of the Y scale may bear any convenient ratio to that of the X scale.

TABLE 5

Values of X Corresponding to Values of wpç Indicated by Scale Divisions

Scale	X	Scale	X	Scale	X	Scale	X
0.70	2.12346	1.04	2.03138	[18] 1.38	1.97021	1.72	1.92478
.71	003	.05	2.02926	.39	1.96869	.73	361
.72	2.11663	.06	715	.40	718	.74	245
.73	332	.07	507	.41	569	.75	129
.74	002	.08	301	.42	421	.76	013
.75	2.10682	.09	098	.43	273	.77	1.91900
.76	363	.10	2.01897	.44	126	.78	786
.77	053	.11	701	.45	1.95983	.79	673
.78	2.09745	.12	501	.46	841	.80	560
.79	444	.13	308	.47	699	.81	450
.80	146	.14	114	.48	558	.82	339
.81	2.08854	.15	2.00924	.49	419	.83	229
.82	565	.16	735	.50	280	.84	119
.83	283	.17	549	.51	142	.85	013
.84	002	.18	364	.52	005	.86	1.90904
.85	2.07727	.19	181	.53	1.94869	.87	796
.86	455	.20	000	.54	734	.88	689
.87	188	.21	1.99822	.55	600	.89	584
.88	2.06923	.22	645	.56	466	.90	480
.89	663	.23	465	.57	336	.91	375
.90	405	.24	294	.58	207	.92	271
.91	153	.25	121	.59	078	.93	169
.92	2.05902	.26	1.98949	.60	1.93949	.94	066
.93	656	.27	782	.61	823	.95	1.89965
.94	412	.28	615	.62	697	.96	863
.95	172	.29	449	.63	571	.97	763
.96	2.04934	.30	284	.64	446	.98	664
.97	701	.31	122	.65	323	.99	565
.98	469	.32	1.97960	.66	200	2.00	466
.99	241	.33	800	.67	078	.01	367
1.00	014	.34	642	.68	1.92956	.02	272
.01	2.03792	.35	484	.69	834	.03	177
.02	571	.36	326	.70	713	.04	079
.03	354	.37	173	.71	595	.05	1.88981

---

[18] Example: In Table 19 at 1.38 wpc find 107.1 "Volts." $\text{Log}\ \frac{100}{1.071} = 1.97021$, the point on the X scale of equal parts at which the 1.38 wpc division is placed.

In computing tables similar to Table 5 above, values of X for every scale division need not be determined as given in the example. Values of sufficient accuracy may be obtained by computing X for wpc points in steps of 0.12. These values are then differenced to the second order, interpolated twice to halves and once to thirds, thus completing the table except the points near the beginning and end, which may be interpolated by differencing to the fifth order and employing Newton's or Stirling's interpolating coefficients. Rules for these interpolations may be found in any text on the subject.

3. PER CENT CANDLEPOWER CHART

The curves of this chart are straight lines. Although two points accurately placed determine a straight line, three points are given for the longer lines, so that errors due to slight displacement of points and to possible inaccuracies of a straight edge used in ruling in the lines, are minimized.

All of these lines are referred to the horizontal and vertical scales of equal parts, the points chosen falling on even divisions of the vertical scale, so that possible errors due to interpolation will occur only in the X direction. These errors are minimized by choosing a long X scale. The construction must be very carefully carried out to place these lines in proper relation to the other scales. Values for each line of this chart are given in Table 6.

TABLE 6

Values of X Corresponding to Values of Y

[A straight line drawn through the points given in the body of the table places the per cent cp line corresponding to the scale division indicated in the first column of the table]

Scale	Values of X			Scale	Values of X		
		$Y=2.110$	$Y=2.120$			$Y=2.100$	$Y=2.120$
15.0		1.88117	1.89001	17.0		1.88653	1.90437
15.5		1.88489	1.89376	17.5		1.88982	1.90771
16.0		1.88851	1.89742	18.0		1.89305	1.91096
16.5		1.89204	1.90093	18.5		1.89618	1.91414
				19.0		1.89924	1.91721
				19.5		1.90223	1.92025

TABLE 6—Continued

Values of X Corresponding to Values of Y—Continued

Scale	Values of X		
		$Y=2.080$	$Y=2.100$
20		1.88708	1.90514
21		1.89266	1.91076
22		1.89800	1.91614
23		1.90311	1.92129
24		1.90800	1.92624
25		1.91271	1.93100
26		1.91724	1.93559
27		1.92162	1.94001
		$Y=2.040$	$Y=2.100$
28		1.88891	1.94429
29		1.89293	1.94840
30		1.89682	1.95239
31		1.90057	1.95627
32		1.90423	1.96002
33		1.90778	1.96367
34		1.91122	1.96721
35		1.91458	1.97066
36		1.91783	1.97402
37		1.92101	1.97729
38		1.92412	1.98048
39		1.92712	1.98359
	$Y=1.$[illegible]	$Y=2.000$	$Y=$[illegible]$.00$
40		1.8[illegible]27	1.98664
42		1.89786	1.99249
44	1.88416	1.90321	1.99810
46	1.88923	1.90832	2.00346
48	1.89410	1.91324	2.00864
50	1.89879	1.91796	2.01360
52	1.90331	1.92251	2.01837
	$Y=1.960$	$Y=2.000$	$Y=2.100$
54[14]	1.88835	1.92689	2.02301
56	1.89252	1.93113	2.02755
58	1.89653	1.93524	2.03177
60	1.90042	1.93921	2.03595
62	1.90420	1.94304	2.03997

Scale	Values of X		
	$Y=1.940$	$Y=2.000$	$Y=2.100$
64	1.88836	1.94677	2.04391
66	1.89189	1.95040	2.04774
68	1.89530	1.95393	2.05142
70	1.89863	1.95734	2.05503
72	1.90187	1.96068	2.05854
74	1.90501	1.96394	2.06198
76	1.90810	1.96709	2.06532
78	1.91108	1.97018	2.06856
	$Y=1.920$	$Y=2.000$	$Y=2.100$
80	1.89427	1.97321	2.07173
85	1.90128	1.98044	2.07936
90	1.90781	1.98730	2.08661
95	1.91406	1.99380	2.09346
100	1.92000	2.00000	2.10000
	$Y=1.890$	$Y=2.000$	$Y=2.100$
105	1.89559	2.00590	2.10623
110	1.90092	2.01155	2.11220
115	1.90604	2.01697	2.11792
120	1.91095	2.02216	2.12342
	$Y=1.890$	$Y=2.000$	$Y=2.050$
125	1.91567	2.02716	2.07791
130	1.92022	2.03199	2.08287
135	1.92462	2.03664	2.08764
140	1.92885	2.04112	2.09225
145	1.93293	2.04547	2.09673
150	1.93689	2.04967	2.10105
155	1.94074	2.05373	2.10524
160	1.94447	2.05770	2.10931
165	1.94808	2.06153	2.11327
170	1.95159	2.06528	2.11711
175	1.95502	2.06889	2.12084
180	1.95835	2.07243	2.12447
185	1.96159	2.07588	
190	1.96476	2.07925	
195	1.96785	2.08253	

---

[14] Example: Number corresponding to 1.960 is 91.20 per cent voltage. In Table 20 find at 91.20 "volts" 71.59 "cp." 54 per cent of 71.59 = 38.66. In Table 20 find 77.33 "volts" corresponding to 38.66 "cp." Log 77.33 = 1.88835, a point on the X scale of equal parts and in the 54 per cent cp line.

TABLE 6—Continued

Values of X Corresponding to Values of Y

Scale	Values of X		
	$Y=1.890$	$Y=2.000$	$Y=2.050$
200	1.97085	2.08572	....
210	1.97668	2.09191	....
220	1.98225	2.09785	....
230	1.98757	2.10353	....
240	1.99269	2.10897	....
250	1.99761	2.11421	....
260	2.00234	2.11926	....
270	2.00694	2.12415	....
	$Y=1.890$	$Y=1.980$	
280	2.01136	2.10741	....
290	2.01561	2.11193	....
300	2.01974	2.11628	....
310	2.02375	2.12051	....
320	2.02764	2.12463	....
	$Y=1.890$	$Y=1.960$	
330	2.03145	2.10694	....
340	2.03511	2.11076	....
350	2.03870	2.11451	....
360	2.04218	2.11813	....
370	2.04556	2.12166	....
380	2.04887	2.12509	....
	$Y=1.890$	$Y=1.940$	
390	2.05212	2.10660	....
400	2.05526	2.10985	....
420	2.06134	2.11611	....
440	2.06715	2.12212	....
	$Y=1.890$	$Y=1.920$	
460	2.07273	2.10575	....
480	2.07809	2.11123	....
500	2.08325	2.11647	....
520	2.08821	2.12153	....
540	2.09300	2.12642	....
	$Y=1.890$	$Y=1.910$	
560	2.09764	2.11995	....
580	2.10209	2.12450	....
	$Y=1.870$	$Y=1.890$	
600	2.08404	2.10643	....
620	2.08817	2.11066	....
640	2.09223	2.11474	....
660	2.09614	2.11869	....
680	2.09996	2.12254	....

IX. SUMMARY

As described above, the characteristic equations of vacuum tungsten lamps, derived and discussed in a previous paper (see note 1), have been employed in constructing a computing device consisting of volt, wpc, and per cent candlepower scales, from which may be read, by a single setting of the movable volt scale to observed values within the range of from 0.70 to 2.05 wpc, values of any one of the variables at other points. The volt scale has a range of from 94 to 166 volts. The limits in wpc and voltage were chosen to include values most generally used in laboratory and factory practice.

A test of the device, illustrated in part by examples given in this paper, indicates that, on an average, values of per cent cp and of wpc read therefrom deviate from those obtained by use of Tables 20 and 22 (see note 2, p. 269) by amounts not exceeding 0.10 per cent and 0.05 per cent, respectively; also that values read from the scales do, on an average, check observed values very nearly as closely as those computed by use of the tables. Such differences as occur when compared with values computed by the tables may be traced to slight errors in construction, width of lines, and modification of values by personal error in interpolation. Differences due to this last cause may be minimized by using an interpolating scale. In scales of smaller range the interpolating scale may be dispensed with, as a greater number of scale divisions may be drawn for a given range; for example, the candlepower chart may be constructed in steps of 1 per cent.

The chief advantage which this device possesses over other methods of characteristic evaluation, including the use of the tables referred to above, is a great saving of time, this resulting because of the directness of solutions without reference to normal wpc, voltage ratios, exponents, etc.

All of the points (referred to horizontal and vertical scales of equal parts) through which lines of the scales of this device were drawn have been tabulated, so that similar scales of the same or different range may be constructed directly from these values.

Sufficient description of the derivation of these values and of the relative position of the scales has been given to permit the construction of scales not included within the wpc range here employed.

Mention has been made of the possibility of constructing this device in circular or disk form, the volt and wpc scales being well adapted to this construction.

The device as given on the plate is ready for use as soon as the volt scale is cut off from the drawing.

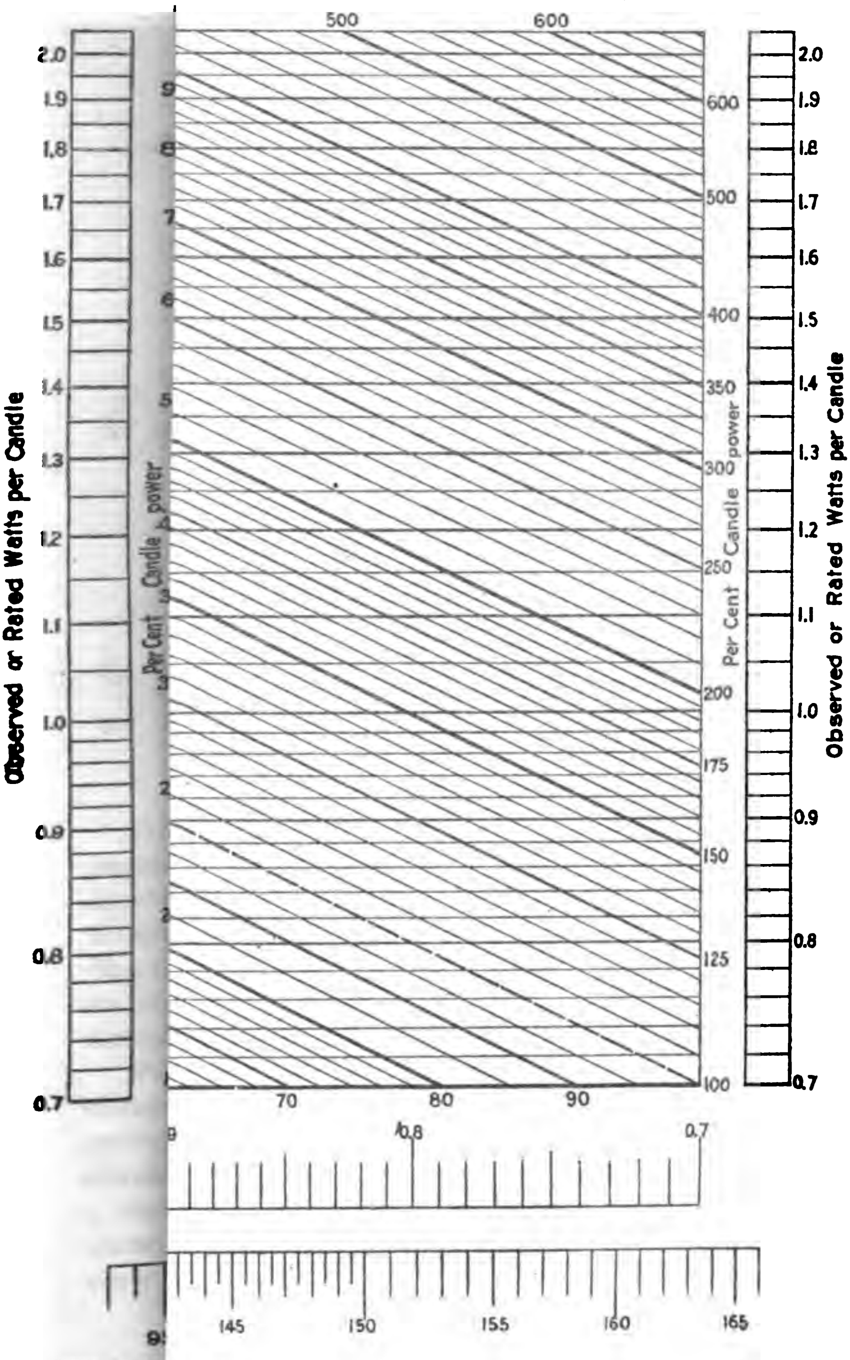
It is hoped that this device will lessen the labor of any who have to compute characteristics of vacuum tungsten lamps and that the general efficiency of testing bureaus where it may be used will be raised by the saving of time, which is its chief merit. The device has been made as simple as possible, and even the novice may use it after a little preliminary practice.

WASHINGTON, April 9, 1915.

INSTRUCTIONS FOR CHART

Remove the scale marked "Volts" from the plate. Place on the dotted diagonal of the candlepower chart (upper part of the plate) at its intersection with the horizontal wpc line nearest that corresponding to the observed value (preferably a value obtained from an observation made at color match with the standards used) the point on the volt scale corresponding to the observed value of voltage. Read on the cp chart the value of per cent cp at any desired value of voltage; or, conversely, the voltage for any per cent cp.

Apply observed voltage to observed wpc (the horizontal scale just below the cp chart) and read the value of wpc at the desired voltage; or, conversely, read the voltage for any desired wpc.


500
600
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
Observed or Rated Watts per Candle
Per Cent Candle power
600
500
400
350
300
250
200
175
150
125
100
70
80
90
0.8
0.7
145
150
155
160
165

10397°—15.

A STUDY OF THE QUALITY OF PLATINUM WARE

By George K. Burgess and P. D. Sale

CONTENTS

Page
I. Introduction 289
II. Thermoelectric determination of platinum purity 290
III. Quality of platinum crucibles 298
  1. Method of determining losses on heating 299
    (a) Acid treatment 300
  2. Experimental results 301
    (a) Microscopic observations 301
    (b) Presence of iron 307
    (c) Other conclusions from tables 308
  3. Prediction of loss in weight due to heating 309
  4. Suggestions as to specifications 310
IV. Nature of disintegration 311
V. Summary and conclusions 313
VI. Selected bibliography 314

I. INTRODUCTION

At the request of Dr. W. F. Hillebrand, Chairman of the Committee on Quality of Platinum Utensils of the American Chemical Society, experiments on the loss in weight associated with continued and repeated heating of platinum crucibles of varying degrees of purity have been undertaken, in continuation of similar experiments carried out under the immediate supervision of members of the above-mentioned committee.

From some of these earlier experiments, and from the work of other observers on the evaporation of metals of the platinum group, it appears to have been hoped that it would be possible to classify platinum ware as to purity in terms of its evaporation at a definite temperature, say 1200° C. This seemed plausible, in view of the fact that the usually predominant impurity, iridium, is very much more volatile than platinum. Even if this method, however, does give an indication of the platinum purity, it is at best a somewhat tedious and delicate operation to carry out.

In view of the results obtained from this investigation, in which is shown the relatively great effect of extremely minute quantities of admixtures of iridium and iron on the properties of platinum, especially vapor pressure, it may be questioned if some of the results obtained by other experimenters, who have studied quantitatively the volatilization of the platinum metals, are not subject, in certain cases, to some slight modification, as in general there has been no check on the purity of the materials used by them.

Systematic observations on the disintegration of the platinum metals under the action of high temperatures have been made by a number of experimenters, as shown in the appended bibliography, but a quantitative study of the effects of impurities on the disintegration of platinum, we believe, had not been made before the experiments herein described.

II. THERMOELECTRIC DETERMINATION OF PLATINUM PURITY

The preliminary experiments showed the desirability of having an accurate and rapid method for determining platinum purity, and one that could be applied to crucibles without defacing them and which would serve also as a ready means of classification of the several grades of platinum.

The most exact method for such determination appears to be by measurement of the temperature coefficient of electrical resistance, which quantity has a mean value of about 0.00391 per degree centigrade for the interval 0° to 100° C for the purest obtainable platinum, and decreases with the addition of anything to the platinum. This measurement can be made conveniently and exactly only with wires, and is therefore of little interest for the determination of the purity of platinum ware such as crucibles.

The thermoelectromotive force of platinum against many of its alloys has also been determined with considerable exactness.[1]

---

[1] See, in particular, W. Geibel, Zs. Anorg. Chem., 69 (1910), p. 38; 70 (1911), p. 240; Burgess and Le Chatelier, "The measurement of high temperatures," 3d ed., 1912, p. 171.

This property may evidently be made use of, therefore, in devising a method for the determination of platinum purity and one that possesses, furthermore, the advantages of accuracy, speed, convenience, and preservation intact of the objects tested. It is only necessary to have at hand a chart or table of the emf's of pure platinum against its alloys with rhodium and iridium and a convenient experimental arrangement for measuring these emf's.

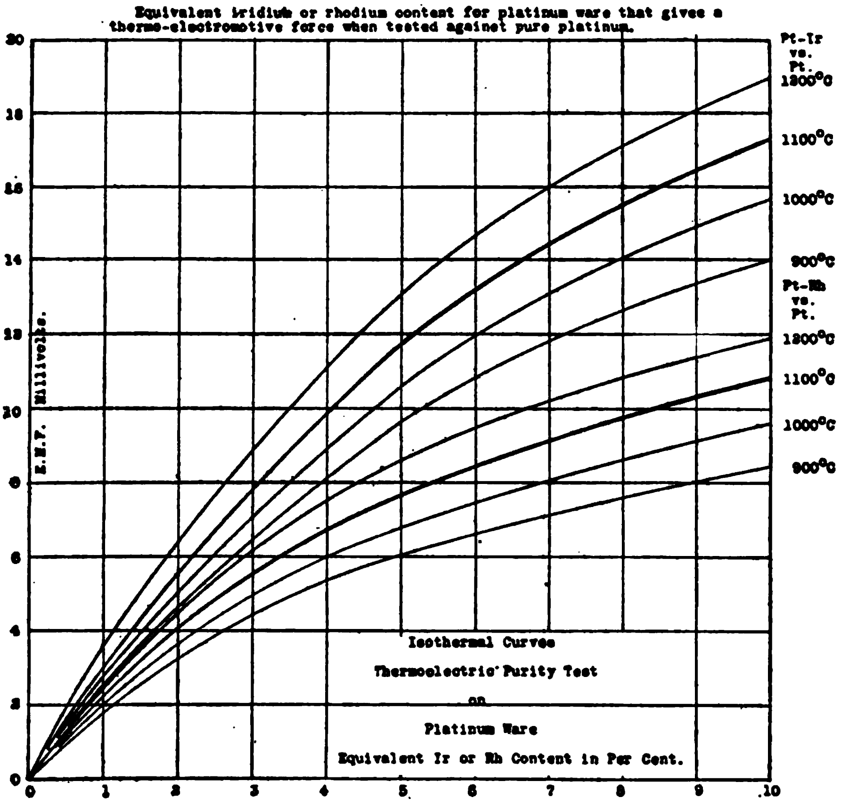


FIG. 1.—*Isothermal Curves for Ir or Rh Content of Pt*

Such a chart is shown in Fig. 1, in which the emf in millivolts of the alloy against pure platinum is plotted in terms of the equivalent iridium or rhodium content. There are thus shown the isothermal curves for each alloy up to 10 per cent of the alloying element and for each 100° C from 900° to 1200°.

The experimental realization of the emf method as developed for use with crucibles is shown in Fig. 2. To the rim of the crucible *C* are arc-soldered, at *e* and *f*, two pure platinum wires of small diameter (0.1 or 0.2 mm); these wires are connected to an ordinary pyrometer-galvanometer or millivoltmeter, *G*; the junction *e* is heated by a small oxy-gas or other blast flame from *d*, and the junction *f* is kept cool by an air blast, *c*; a sheet of asbestos, *A*, cut as shown, serves to prevent radiation from the heated portion of the crucible reaching the cold junction *f*.

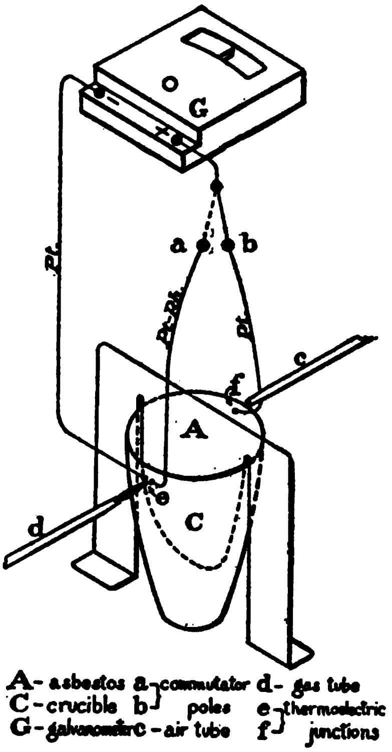

G
a
b
Pt
Pt-Rh
Pt
c
f
A
e
C
d
A - asbestos a ⎤ commutator d - gas tube
C - crucible b ⎦ poles e ⎤ thermoelectric
G - galvanometer c - air tube f ⎦ junctions

FIG. 2.—*Thermoelectric purity test*

Temperatures are measured by means of a 90Pt-10Rh, Pt thermocouple using the cold junction as above, and a Pt-Rh wire arc-soldered to the crucible near *c*, most conveniently adjacent to (0.5 to 1 mm distant) but not touching the Pt wire at *e*.

The wires are attached at *e* and *f* by the well-known arc-soldering method, which consists in making the crucible one terminal of an electric circuit of about 40 volts and a sharpened graphite pencil the other, with a rheostat in series. The operation of soldering consists in drawing a minute arc between pencil and crucible and at the same instant touching the wire to the crucible at this point. The end of the wire is thus fused to the crucible. With a little practice this operation may be made so that there is hardly any noticeable effect on the appearance of the crucible after the wire is removed.

Quite satisfactory results may also be obtained by simply touching the Pt wires to the crucible at *e* and *f*, or by clamping with platinum clips, without any soldering. This last has been the method usually employed in these tests. With the apparatus once set up, a test may then be made in a few seconds, and the crucible remains, of course, absolutely intact. The homogeneity of the crucible may also be determined by this method.

By means of a commutator at *a b* measurements may be made alternately of the temperature at *e* and of the emf developed across the crucible when *e* is at this temperature. The commutator may also be designed so as to reverse *e* and *f* if it is desired to make *f* the hot junction. Suitable variations from the above arrangement may evidently be made for different types of ware, such as covers, gauze, needles, etc.

It is, of course, essential that the two platinum testing wires be made of strictly pure platinum. For this purpose use is made of Heraeus normal thermoelement wire drawn down. This platinum has shown itself to be a standard, uniform product, the purity of which is easily controlled by the electric resistance method above mentioned.

As an example of the uniformity which may be expected by the thermoelectric examination by the most convenient method, in which the testing wires are held by platinum clips against the crucible or other object, may be cited the observations of Table 1. Observer No. 1 was experienced in the operation, observer No. 3 had had little, and No. 2 no experience in this manipulation. A portion of the irregularities noted are due to inhomogeneity of the material. This method has been found useful by ourselves and others for the ready classification of platinum utensils of various kinds and especially for the determination, in the case of purchases of platinum ware, whether or not such articles conform to specifications as to platinum purity. This thermoelectric test, which in no way mars the crucible or other article examined, is now a routine test made by the Bureau of Standards, and many samples of platinum have been so tested the past year.

TABLE 1

Electromotive Force Measurements (Millivolts at 1100° C.) on Platinum Ware by Three Observers

Description of ware	Observer	Maximum	Minimum	Probable
Crucible APW-C	1	7.00	6.95	7.00
	2	6.23	5.90	
	3	7.00	6.50	
Crucible 9	1	2.80	2.60	2.90
	2	2.18	1.92	
	3	3.00	2.40	
Crucible 11	1	1.88	1.38	1.94
	2	1.40	1.23	
	3	2.00	1.60	
Crucible cover 11	1	1.88	1.85	1.87
	2	1.86	1.52	
	3	2.20	1.50	
Crucible cover 9[a]	1	2.05	1.06	1.03 to 2.05
	2	1.21	1.08	
	3	2.00	1.70	

[a] Found to be inhomogeneous.

In Table 2 is shown the quality of platinum utensils of various sorts and from many sources expressed in terms of equivalent iridium content as measured thermoelectrically at this Bureau. Of the 164 pieces examined, which are typical of such material as found on the market, it is seen that nearly 75 per cent have impurities equivalent to over 0.5 per cent of iridium or in quantity to cause considerable losses on heating, which would be troublesome in exact analytical work; and of the crucibles 65 per cent have over 0.5 per cent equivalent iridium content. Dishes and miscellaneous ware, in which stiffness is usually required and which are often not subjected to heating and weighing, are heavily charged with impurities, usually iridium, the miscellaneous ware showing, for example, nearly half of the samples with over 4 per cent iridium. The 22 pieces of miscellaneous ware consisted of wire, boat, gauze, spatula, bottle, stopper, tubes, specially shaped ware, and scrap.

TABLE 2

Classification of Platinum Ware submitted to Bureau of Standards for Thermoelectric Purity Test

Class of ware investigated	Number pieces of each class	Per cent in each class in terms of equivalent iridium content in per cent				
		0.0 to 0.5	0.5 to 1.0	1.0 to 2.0	2.0 to 4.0	4.0 to 25
Crucibles	84	36	17	34	13	0
Crucible cover	47	26	23	38	13	0
Dishes	11	9	36	19	36	0
Miscellaneous	22	0	18	9	27	45
Total number	164	26	20	31	17	6

In Table 3 is shown the results of a thermoelectric test of the first 23 crucibles examined by this method.

It will be noted that all impurities are, for convenience, expressed in terms of iridium content. Of particular interest is the comparison of columns 2 and 6, the former giving the stated iridium content (and often accompanied by the statement that there are no other impurities) and the latter giving the iridium content as determined thermoelectrically by experiment and use of Fig. 1. In several instances, notably for the crucible of normal thermoelement platinum (*a*) of Heraeus and for Baker's crucible (*i*) of 2.37 per cent iridium, the stated and found iridium contents agree exactly. On the other hand, there is a wide divergence from the supposed and actual iridium content for several of the crucibles; for example, *j* of the American Platinum Works, supposed to contain not over 0.50 to 1.5 per cent iridium, actually contains an iridium equivalent of 2.50 per cent; and even in the cases of refined, specially refined, and best crucible ware the equivalent iridium content is not inconsiderable.

In comparing the stated contents (column 2 of the table), it should be borne in mind that some of these crucibles, excepting, perhaps, those marked refined or specially refined, may contain iridium which is put or left in the material purposely, usually for the purpose of stiffening, or, in the case of commercial ware, because such Pt always carried Ir. The method here described, however, provides a delicate means of determining whether or not

TABLE 3

Summary of Thermoelectric Tests of 23 Platinum Crucibles

Source or maker	Stated Ir content	Previously heated	Temp. of test	EMF Pt vs crucible	Equivalent Ir content	EMF from Pt-Ir curve (see fig. 1)	EMF excess due to Ir or impurities	Reference letter	Remarks	Date of purchase
	Per cent	Hours	°C	Mv	Per ct	Mv	Mv			
Heraeus	0.0	17	1100	0.10	0.05	0.00	0.10	a	Normal thermoelement Pt	1911
	.7	Heated	1050	2.00	.70	2.00	.00	e	Normal Tiegel platin	1911
	.7	0	1050	2.60	.90	2.00	.60	p	.....do	1911
American Platinum Works	<.2	24	1050	.63	.19	.10	.53	d	"Tiegel platin" Heraeus	1911
	<.2	16	1050	1.00	.35	.10	.90	q	.....do	1911
	.5-1.5	11	1075	6.60	2.55	1.50-4.10	2.50-5.10	j	Commercial	1911
	.5-1.5	4	1085	6.40	2.50	1.50	4.90	r	.....do	1911
	?	.5	1100	3.50	1.19	....	3.50	s	Loaned by purchaser	1913
Baker & Co.	?	10	1050	6.75	2.72	....	6.75	k	Commercial	1908
	?	20	1050	6.95	2.78	....	6.95	c	.....do	1908
	2.37	0	1030	6.00	2.37	6.00	.00	i	Special analyzed make	1912
	.702	12	1050	1.90	.66	2.05	—.15	f	.....do	1912
	Refined	0	1040	.33	.10	....	.33	t	Specially refined	1911
	?	[a]3	1100	3.00	1.01	....	3.00	h	Used three years in laboratory	1909
	?	0	1100	1.71	.55	....	1.71	....	Average of seven commercial crucibles.	1914
Johnson Matthey & Co.	?	67	1050	.48	.15	....	.48	b	Best crucible ware	1911
	?	0	1070	.68	.23	....	.68	u	....do	1911
	?	20	1050	.63	.22	....	.63	c	....do	1913
J. Bishop	?	[b]9	1100	2.10	.68	....	2.10	v	Loaned by purchaser. Used continuously 9 months.	1913
	?	.5	1100	2.20	.72	....	2.20	w	....do	1913
	Refined	31	1050	.60	.21	....	.60	x	Specially refined	1911

Quenessen, De Belmont, Legendre et Cie.	?	10	1050	2.79	.98	....	2.79	x	....	1911
	?	0	1060	2.80	.98	....	2.80	y	....	1911
	?	4	1030	1.60	.56	....	1.60	z	....	1911

[a] Years.

[b] Months.

the desired limitation of impurity, expressed in terms of iridium content and as measured thermoelectrically, has been met by the manufacturer. By the experimental arrangement above described, the amount of impurity may be determined to 0.01 per cent, and this accuracy might be improved upon slightly if necessary.

The method does not distinguish the various possible impurities from each other, but is nevertheless a certain check on the platinum purity. Thus, all the metals found associated with platinum, such as palladium, iridium, rhodium, etc., when alloyed with platinum (up to 90 per cent only of palladium) give, at high temperatures, an emf of the same sign against pure platinum. Therefore, there is no ambiguity in balancing one impurity against another. Although the above-described thermoelectric method will not alone distinguish between small amounts of rhodium and iridium, it may nevertheless be combined oftentimes for this purpose with other methods of examination which do not injure the crucible.

One of the most undesirable impurities often found in commercial platinum ware is iron; this is readily detected by ignition and subsequent washing with hot HCl and applying the usual tests for Fe. Iron, if present in relatively considerable quantity, will also discolor the crucible on ignition. The presence of iron is discussed at length in the following paragraphs.

III. QUALITY OF PLATINUM CRUCIBLES

In addition to data on the purity of platinum ware now in use, more exact information was desired by the committee concerning the losses on heating. The experiments here outlined, it is believed, furnish information concerning losses sustained by platinum crucibles of several grades of purity when subjected to continued heating. By suitable acid treatment after heating it has been possible to give an estimate of the relative amount of iron present, and the crucibles have also been examined microscopically and, through the kindness of Dr. Burrows, magnetically.

As will be shown, a combination of these several methods, or some of them, usually gives sufficient data for the ready classification of various grades of platinum ware.

1. METHOD OF DETERMINING LOSSES ON HEATING

For an exact determination of the loss in weight of a crucible on heating, ignition over the blast lamp is not satisfactory. A preliminary series of experiments, using electric resistance furnaces with platinum and other metals as heating coils, carried out by members of the chemical staff of this Bureau, showed that one could not expect to obtain consistently reliable results in a furnace containing metal. A considerable number of measurements of losses on heating at several temperatures were so taken, but no very definite conclusions can be drawn from these preliminary observations which were obtained before the thermoelectric method of classification was put into practice.

The method finally adopted for determining heat losses in crucibles is based on the use of a metal-free, electrically heated furnace, somewhat similar to one which had been in use for other purposes at the Geophysical Laboratory, accompanied by exact weighings of the crucible before and after each heating of two hours' duration at 1200° C, together with a weighing to determine the iron loss after washing in acid.

The essentials of the furnace are shown in Fig. 3. The heating spiral *H* is of graphite set into brass, water-cooled terminals. The crucible *C* is supported on a Marquardt porcelain stand *S*, as shown, and inclosed within a tube *T* of the same material. Temperatures were measured by a platinum, platinum-rhodium thermocouple inclosed in Marquardt. At 1200° C the furnace, which was operated on alternating current, took about 3.5 kw.

For a region of about three crucible heights at the center of the furnace there was maintained a region of practically uniform temperature, 1200° C, and this inner portion of the furnace never became blackened. A slight current of air was maintained about the crucible, as shown, but great care was taken, by means of a sand and paraffin seal at *P*, to prevent vapors passing from the heating spiral into the inner chamber. The crucible was not placed in the furnace until this had reached 1000° C; a temperature of 1200° C was then attained in about 10 minutes and held constant for exactly two hours and cut off, the crucible being removed 10 minutes later, or at about 900° C.

Crucible weighings were made on an inclosed precision balance, at first to 0.001 mg and later to 0.01 mg, as this latter was found

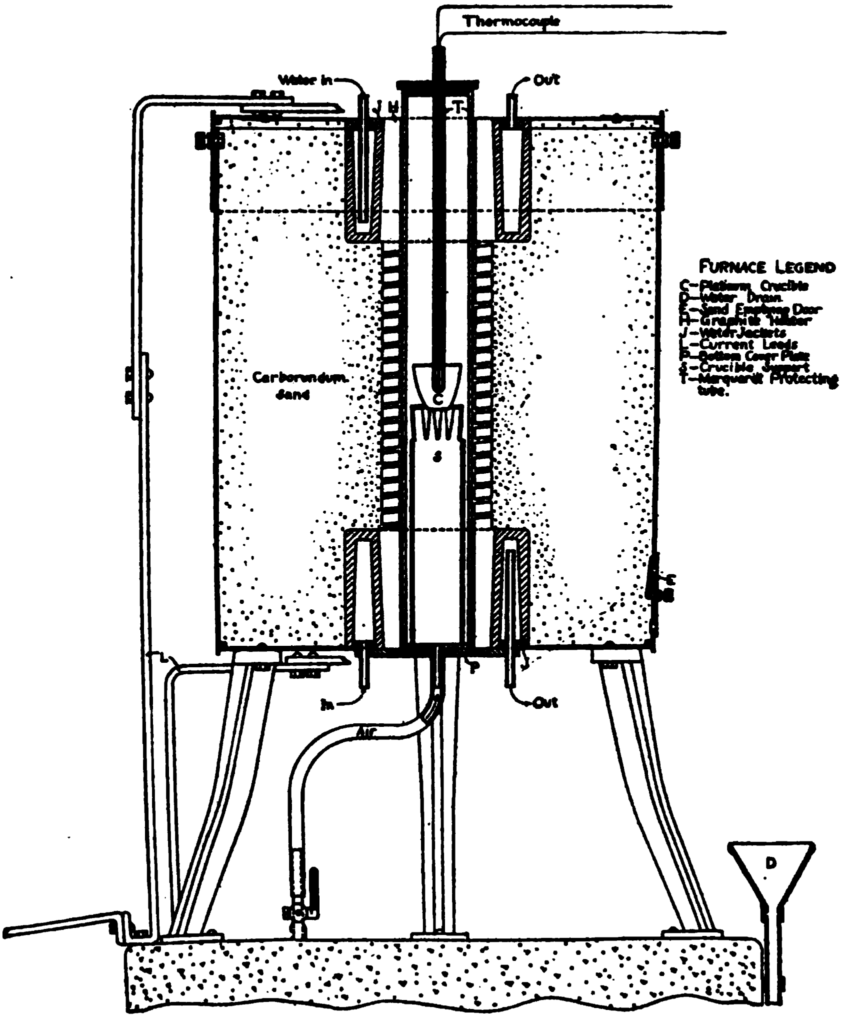


FIG. 3.—*Furnace for determining losses in weight of Pt*

sufficient. All weights are reduced to a common basis of loss per 100 cm^2 of total crucible surface.

(*a*) ACID TREATMENT.—Iron and other soluble materials were removed and determined by weighing after each heating. For several of the crucibles examined the characteristic reddish coating due to iron oxide appeared after each heating. The acid treatment consisted in boiling the crucible for five minutes in 25 per cent hydrochloric acid, contained in a large covered porcelain crucible. The iron was determined by precipitation with ammonia upon ashless filter paper, ignited in platinum for one minute at about 1200° C, and weighed as ferric oxide.

2. EXPERIMENTAL RESULTS

The 14 crucibles described in Table 4 were carried through a series of identical operations, consisting of a determination of their equivalent iridium or rhodium content by the thermoelectric method (see lines I and I′ of Table 5); the determination of loss of weight after each of three heatings of two hours each at 1200° C (see lines II, III, and IV), each followed by determination of loss by acid treatment (lines II′, III′, and IV′), and of iron content (lines IX, X, and XI). As seen from Table 6, these 14 crucibles range in equivalent iridium content from 0.05 (a) to 2.65 (l) per cent; crucibles *m* and *n* were supposed to be 90 platinum 10 rhodium, but as measured thermoelectrically contain 7.30 and 7.95 per cent rhodium, respectively, the former having been used considerably and the latter an unused crucible; some of the others are of unknown composition and the nonplatinum content of each is expressed both in terms of iridium and rhodium (see lines 2 and 3 of Table 6). In Table 7 the heating and acid losses are given for four crucibles which underwent additional heatings.

(*a*) MICROSCOPIC OBSERVATIONS.—The microscopic observations were helpful in finally classifying the crucibles in terms of their main impurity, iridium or rhodium (see last line of Table 6), as it was found that the crystalline structures characteristic of these alloying metals, in general, imparted their appearance to the heated but otherwise unetched crucible even when present in small percentages.

The several photomicrographs shown in Figs. (a) to (n) are all of heat-etched crucibles, with a magnification of 100 diameters. They are typical of the four classes of platinum ware here dealt with, namely, pure platinum, platinum containing rhodium, iridium, and considerable iron. The letters of the figures correspond to the lettering of the crucibles in Table 4.

Fig. (a) presents a typical appearance of the purest platinum ware obtainable. Figs. (b), (m), and (n) give the appearance of Pt-Rh ware with Rh equivalents of 0.19, 7.30, 7.95, respectively. This snarled, irregular appearance was also noted in the case of crucible *h*. It will be noticed in Fig. 4 that these crucibles gave very low volatilization on heat treatment; in fact, less than the pure platinum. It is seen (Table 4) that crucibles *m* and *n* were originally special Rh ware and that *m* had been used pretty severely in the chemical laboratory while *n* was practically new when these microphotographs were taken. Figs. (f) and (i) are typical of Pt-Ir with Ir equivalent of 0.55 and 2.30 per cent, respectively. This regular, mottled appearance of finer grain (f) contrasts with the irregular snarled appearance of the Rh ware, as noted above. Though this characteristic mottled effect was not obtained on the surface of the crucible *i* on heat and ordinary acid treatments, the small, rounded, crystalline structure with heavy boundaries is itself characteristic of Pt-Ir (see Holborn und Henning in bibliography). Figs. (e) and (l) with Ir equivalents of 0.21 and 2.65 per cent and an appreciable iron content (see l, Table 6) constitute a fourth quality of platinum ware, viz, Pt-Ir-Fe. The size of the crystal as governed by the Ir content and also the heavy boundaries are brought out nicely here as in other photomicrographs of these last two classes. The spots and hazy appearance here are thought to be due to the presence of iron, which is in the form of iron oxide and shows itself generally after heat treatment by a dark or reddish tinge which it gives to the surface of ware so treated. All crucibles having the appearance just noted gave appreciable iron on acid treatment between heatings.

The photomicrographic analysis is not always sufficient in itself for classifying platinum ware as to quality, although it will, in general, corroborate the results of the emf, heating, and acid tests, and used with the emf test alone will usually be of sufficient certainty to decide between the predominance of iridium or rhodium impurity.

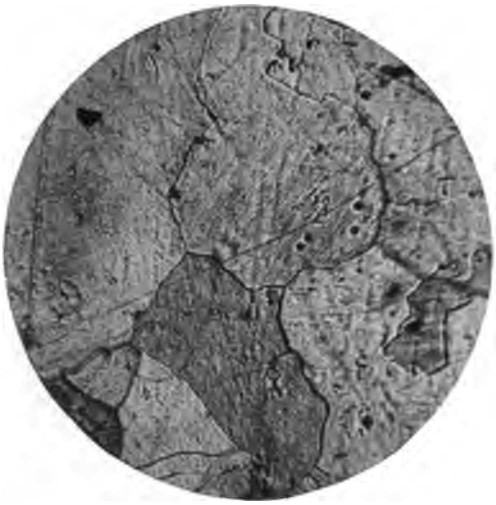


FIG. (a).—*Equivalent Ir.* = *0.1%. Highest purity Pt. ware obtainable*

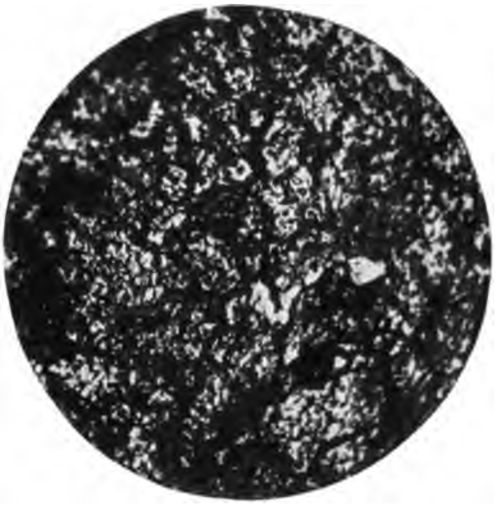


FIG. (b).—*Equivalent Rh.* = *0.2%. Practically new ware*

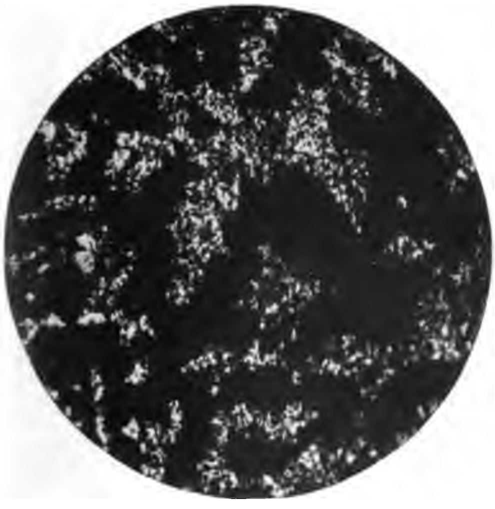


FIG. (m).—*Equivalent Rh.* = *7.30%. After 50 severe ignitions*

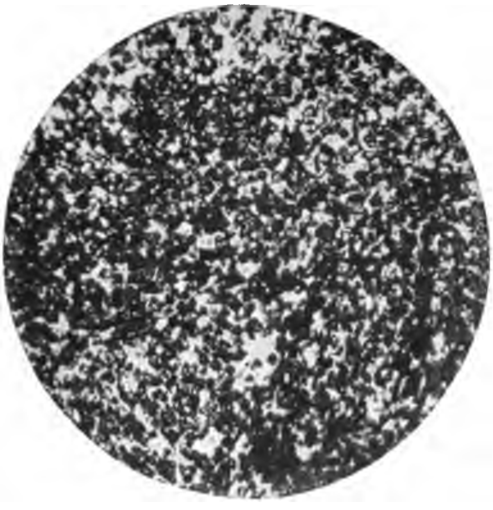


FIG. (n).—*Equivalent Rh.* = *7.95%. Practically new ware*

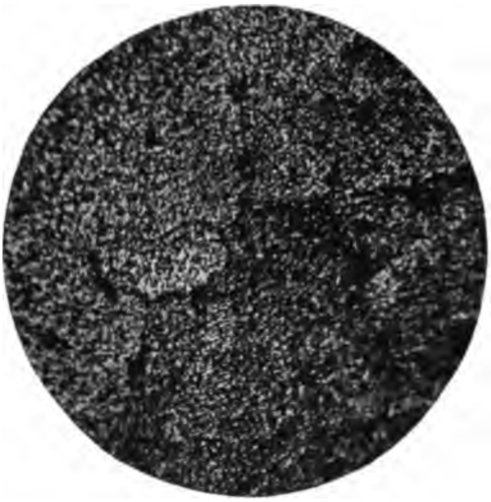


FIG. (f).—*Equivalent Ir. = 0.6%*

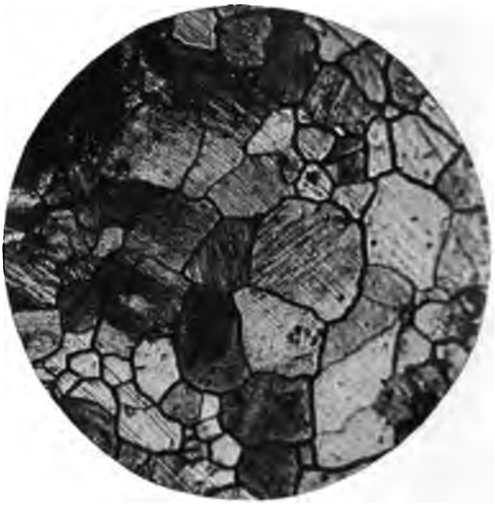


FIG. (i).—*Equivalent Ir. = 2.3%*

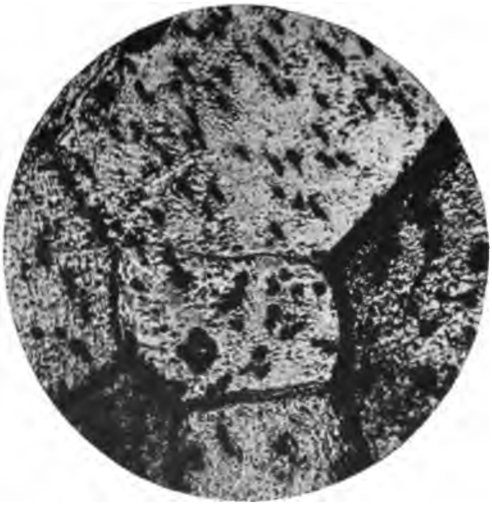


FIG. (e).—*Equivalent Ir. = 0.2%. Highly contaminated with iron*

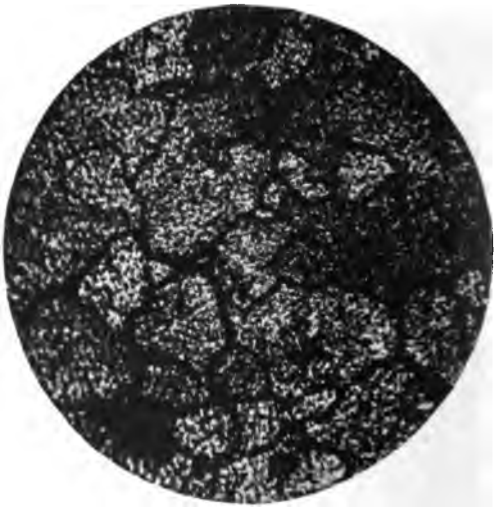


FIG. (l).—*Equivalent Ir. = 2.7%. Highly contaminated with iron*

TABLE 4

Descriptions of Platinum Crucibles (a to n)

Source or maker	Crucible reference letter	Date of purchase	Weight	Volume	Average thickness	Mean emf	Magnetic suscep- tibility	Previously heated	Factor for reduction to 100 cm² surface
			grams	cm³	cm	MV		hours	
Heraeus, normal thermoelement Pt.	a	1911	17.2	26.6	0.0208	0.09	1	17	1.30
American Platinum Works, Heraeus Tiegel platin	d	1911	16.2	21.3	.0168	.70	14	24	1.12
American Platinum Works, commercial ware	j	1911	16.9	21.7	.0285	6.40	125	11	1.81
Baker & Co., special analyzed make	f	1912	19.9	32.5	.0187	1.88	....	12	1.01
Baker & Co.	h	1909	10.8	16.4	.0183	2.98	11	([a])	1.83
Baker & Co., special analyzed make	i	1912	20.1	30.6	.0194	6.18	....	0	1.04
Baker & Co., commercial ware	k	1908	23.9	33.7	.0305	6.50	....	10	1.27
Do	l	1908	39.6	55.5	.0342	6.90	6	20	.93
Baker & Co., special Rh ware	m	1913	19.9	24.0	.0235	9.30	....	([b])	1.27
Do	n	1913	19.9	24.0	.0235	9.70	....	0	1.27
Johnson Matthey Co., best crucible ware	b	1911	26.2	33.3	.0229	.50	26	67	.94
Do	c	1913	19.3	33.0	.0167	.64	....	20	.93
J. Bishop, specially refined ware	e	1911	15.9	25.5	.0190	.70	8	31	1.28
Quennessen, De Belmont, Legendre et Cie	g	1911	16.3	23.7	.0193	2.80	....	10	1.28

[a] 3 years laboratory use.

[b] 50 ignitions (bisulphate and cleanings).

TABLE 5

Losses in Weight due to Heating and to Action of Acid on Platinum Crucibles (Milligrams per 100 cm^2)

[Emf measurements: Millivolts at 1100° C, Crucible v. Pure Pt (I, I′). Heat treatment loss: Milligrams per 100 cm^2 for 2 hours, heating at 1200° C (II, III, IV). Acid treatment loss: Milligrams per 100 cm^2 for 5 minutes, boiling in 25 per cent HCl (II′, III′, IV′). Recovered Fe_2O_3: Milligrams per 100 cm^2 per treatment with NH_4OH and ignition on ashless paper (IX, X, XI, XII)]

	Crucible reference letter	a	b	c	d	e	f	g	h	i	j	k	l	m	n
I	Emf at beginning	0.10	0.50	0.65	0.70	0.70	1.90	2.80	3.00	6.40	6.70	7.00	7.30	9.30	9.70
I′	Emf at end	.07	.50	.62	.70	.70	1.85	2.80	2.95	5.95	6.10	6.00	6.50	9.30	9.70
	Emf mean I and I′	.09	.50	.64	.70	.70	1.88	2.80	2.98	6.18	6.40	6.50	6.90	9.30	9.70
II	First heat, 2 hours at 1200° C	2.18	1.98	2.16	1.27	2.46	2.00	1.08	1.38	7.47	1.94	6.24	2.16	1.74	.61
II′	First acid, 25 per cent HCl	.05	.24	.06	.19	.83	.16	1.76	3.82	.81	9.50	1.14	5.33	.46	.02
	II+II′	2.23	2.22	2.22	1.46	3.29	2.16	2.84	5.20	8.28	11.44	7.38	7.49	2.20	.63
III	Second heat, 2 hours at 1200° C	.84	.96	1.93	1.36	.26	2.20	2.18	.43	4.42	4.93	5.45	2.32	1.08	1.44
III′	Second acid, 25 per cent HCl	.30	.35	.18	.17	.56	.09	.99	1.24	.16	1.84	.39	2.47	.13	.13
	III+III′	1.14	1.31	2.11	1.53	.82	2.29	3.17	1.67	4.58	6.77	5.84	4.79	1.21	1.57
IV	Third heat, 2 hours at 1200° C	1.85	2.04	1.61	1.65	2.02	2.16	2.36	.75	4.38	3.23	2.94	5.08	1.00	1.21
IV′	Third acid, 25 per cent HCl	.06	.15	.11	.09	.26	.07	.40	.34	.08	.66	1.01	2.79	.10	.04
	IV+IV′	1.91	2.19	1.72	1.74	2.28	2.23	2.76	1.09	4.46	3.89	3.95	7.87	1.10	1.25
V	ΣII, III, IV, heat loss	4.87	4.98	5.70	4.28	4.74	6.36	5.62	2.56	16.17	10.10	14.63	9.56	3.82	3.26
VI	ΣII′, III′, IV′, acid loss	.41	.74	.35	.45	1.65	.32	3.15	5.40	1.05	12.00	2.54	10.59	.69	.19
VII	ΣV, VI, total loss	5.28	5.72	6.05	4.73	6.39	6.68	8.77	7.96	17.22	22.10	17.17	20.15	4.51	3.45
VIII	Total change emf	.03	.00	.03	.00	.00	.05	.00	.05	.45	.60	1.00	.80	.00	.00
IX	Iron as Fe_2O_3 after II′	.147	.125	....	.099	.544	.005	1.080	.794	.052	2.520	....	....	....	.090
X	Iron as Fe_2O_3 after III′	.216	.202	....	−.022	.357	.113	.272	.714	.026	.603	....	....	....	.168

XI	Iron as Fe_2O_3 after IV′	.103	.102	....	.123	.628	.270	.072	.454	.057	.295	....	....	....	.004
XII	Total iron as Fe_2O_3	.466	.429	....	.200	1.529	.388	1.424	1.962	.135	3.418	....	....	....	.262
XIII	Total iron compared to T. L. in per cent	8.82	7.50	....	5.16	23.95	5.81	16.25	24.65	.78	15.46	....	....	....	7.59
XIV	Total iron compared to acid L. in per cent	100.00	58.00	....	53.30	92.60	100.00	45.40	36.35	12.87	28.45	....	....	....	100.00
XV	Total acid compared to T. L. in per cent	7.76	12.93	....	9.50	25.80	4.78	35.95	67.80	6.10	54.30	....	....	....	5.51

TABLE 6

Classification of Ware by EMF, Volatilization, Chemical, Magnetic, and Photographic Data

Crucible Reference letter	a	b	c	d	e	f	g	h	i	j	k	l	m	n
Mean emf millivolts	0.09	0.50	0.64	0.70	0.70	1.88	2.80	2.98	6.18	6.40	6.50	6.90	9.30	9.70
Equivalent Ir content, per cent	.05	.10	.20	.21	.21	.55	.90	1.00	2.30	2.40	2.42	2.65	3.75	3.95
Equivalent Rh content, per cent	.05	.19	.29	.30	.30	.85	1.29	1.40	3.53	3.70	3.75	4.15	7.30	7.95
Equivalent Fe (magnetic), per cent	.0001	.0022		.0012	.0007			.0010		.0110		.0050		
Weight of Fe remaining, magnetic test	.018	.576		.194	.112			.106		1.859		.198		
Makers' chemical statement, per cent				<.2 Ir		.70 Ir			2.37 Ir	.5-1.5 Ir			10.0 Rh	10.0 Rh
Relative Fe by VI, Table 5	1	1.8	.9	1.1	4.0	.8	7.7	8.5	2.6	29.0	6.3	26.0	1.7	.5
Relative Fe by susceptibility	1	26.0		14.0	8.0			11.0		125.0		6.0		
With reference to fig. 4	Pt	Rh or Pt	Pt-Ir	Pt-Ir or Rh	Pt-Ir	Pt-Ir	Pt-Ir-Fe	Pt-Rh	Pt-Ir	Pt-Ir-Fe	Pt-Ir-Fe	Pt-Ir-Fe	Pt-Rh	Pt-Rh
With reference to photomicrographs	Pt	Pt-Rh	Pt-Ir	Pt-Ir-Fe	Pt-Ir-Fe	Pt-Ir	Pt-Ir-Fe	?-Fe	Pt-Ir	Pt-Ir-Fe	Pt-Ir	Pt-Ir-Fe	Pt-Rh	Pt-Rh

TABLE 7

Heat and Acid Losses on Four Crucibles, in Milligrams per 100 cm², After Successive Two-Hour Heatings at 1200° C

Treatment	Crucible (n)			Crucible (l)			Crucible (k)			Crucible (c)		
	Heat	Acid	Total	Heat	Acid	Total	Heat	Acid	Total	Heat	Acid	Total
First	0.98	0.00	0.98	4.30	0.77	5.07	6.24	1.14	7.38	2.16	0.06	2.22
Second	2.30	.18	2.48	2.32	2.47	4.79	5.45	.39	5.84	1.93	.18	2.11
Third	.61	.02	.63	5.08	2.19	7.27	2.94	1.01	3.95	1.61	.11	1.72
Fourth	1.44	.13	1.57	7.40	2.60	10.00	3.78	.60	4.38	2.34	.09	2.43
Fifth	1.21	.04	1.25	4.04	2.37	6.41	......	......	......	......	......	......
Sixth	.81	.06	.87	......	......	......	......	......	......	......	......	......
Total	7.35	.43	7.78	23.14	10.40	33.54	18.41	3.14	21.55	8.04	.44	8.48
Average values	1.23	.07	1.30	4.63	2.08	6.71	4.60	.79	5.39	2.01	.11	2.12

As an additional method for detecting rhodium, it may be mentioned that on fresh platinum ware containing considerable rhodium a characteristic bluish color appears after heating to a dull red. This test may fail, however, with ware that has been used considerably.

(*b*) PRESENCE OF IRON.—The presence of iron in several crucibles was also determined after the last heating and washing by measurement of the magnetic susceptibility, by a solenoid and balance method in a field of 60 gausses, and an estimate made of the relative amounts of iron present, assuming it to be free iron (see Table 6). The crucibles would hardly be arranged even approximately in the same order by chemical analysis following heating and by the magnetic measurements (see Table 6). It is also of interest to note that iron-free platinum appears to have a susceptibility of zero, and that the susceptibility of "platinum" ranges from 1 to 125. A value of about 20, in the same units, has usually been found for the susceptibility of pure platinum, due undoubtedly to the presence of iron. The total amount of iron in a crucible can not be determined readily from the magnetic measurements, as is seen by comparison of the magnetic and chemical data of Table 6; and the iron, which is supposedly in solution in the platinum, on heating diffuses to the surface as fast as removed by oxidation, thus tending to establish an equilibrium between the iron oxide, oxygen, and the iron-platinum solution at the temperature of the experiment. The iron was found nonuniformly distributed in some of the crucibles. The iron washed out in three acid treatments is seen to be greater than the total iron content calculated as iron from the magnetic data, showing that the Fe-Pt alloy containing only a few hundredths per cent iron is paramagnetic and also, from the data, that the susceptibility is not proportional to the iron content. The discoloration sometimes noticed on first heating a new crucible, but not subsequently, may be due to iron from the finishing tools and polishing material.

(*c*) OTHER CONCLUSIONS FROM TABLES.—Considering further the inferences to be drawn from the results recorded in Tables 5, 6, and 7, it is seen that, taking the emf as a criterion or purity, there is a purification of all crucibles containing over 0.5 per cent iridium after heating. The platinum-rhodium crucibles (*m* and *n*) do not change composition on heating (see line VIII, Table 5); that is, the iridium passes out of the crucible and the rhodium does not appreciably.

The losses on heating and after acid treatment are seen to be fairly uniform for each crucible. For some of the less pure crucibles the loss in washing is greater than the heating loss, notably for crucibles *h*, *j*, and *l*. The total heating losses per 100 cm^2 range from 4.28 to 16.17 mg for six hours at 1200°, and the total acid losses (due mainly to iron) from 0.32 to 12 mg, for the crucibles containing iridium. For the platinum-rhodium crucibles, *m* and *n*, containing 7 to 8 per cent rhodium, the total losses on heating and washing together are one-fourth less than for the heating loss for purest platinum, crucible *a*. In general, there is no relation between the acid loss and loss on heating of crucibles containing both iridium and iron, showing the independence of the determination of loss of soluble and insoluble materials by these methods, although relatively large amounts of iron appear to lower somewhat the losses on heating that would otherwise be observed (see crucibles *h*, *j*, *l*). The acid losses are seen to vary from all iron to only 13 per cent iron. The remaining loss on acid treatment could be due to soluble alkaline materials, as observed first by

Dr. Hillebrand and Dr. Weber (see bibliography). However, in a series of nine crucibles tested by us, no alkalinity of the surface of the crucibles was apparent after heating. Drs. Hillebrand and Weber state: "It has been noted in varying degrees of intensity and is not always apparent."

3. PREDICTION OF LOSS IN WEIGHT DUE TO HEATING

The question arises, Can the probable loss on heating of a given crucible of unknown composition be predicted from these data? Although a definite answer can not be given, nevertheless the observations point to a reasonable probability of affirmative reply.

Consider Fig. 4, in which the heating losses are plotted against emf at 1100° C against pure platinum for each of the 14 crucibles. The purest crucibles, *a*, *b*, *c*, *d*, and *e*, all lie close together at 5 mg loss and less than 1 millivolt. The platinum-rhodium crucibles *m* and *n* lie on a line, inclined to the left of 5 mg, and which would reach to the heat loss for pure rhodium as a limit. The platinum-iridium crucibles *f* and *i*, containing negligible iron, lie on a line inclined to the right of 5 mg. The platinum-iridium crucibles, containing considerable iron, *l*, *j*, and *k*, lie between the iridium and rhodium lines, and the crucible *h*, containing considerable iron, lies to the left of the rhodium line; *h* may contain rhodium, as is also indicated by its photomicrograph.

It would follow, therefore, that one would expect a crucible, nearly free from iron (which fact is readily tested by heating over the blast lamp), and with a measured emf of, say, 8 millivolts against pure platinum, to lose 4 mg per 100 cm^2 of surface in six hours at 1200° C if made of a rhodium alloy of platinum, and would expect it to lose about 20 mg if of an iridium alloy. The distinction between these iridium and rhodium alloys may usually be made microscopically, as already shown. When iron is present, these heat losses will be reduced somewhat and may be halved, since the effective disintegration area is reduced by the layer of iron oxide. Fig. 4 may therefore be used, at least approximately, as a basis for estimating losses on heating of platinum crucibles for the usual products found on the market and Fig 4 is probably reliable for practically iron-free crucibles containing up to 40 per cent rhodium or to perhaps 10 per cent iridium.

In view of the fact that a satisfactory determination of the loss on heating for any type of crucible can be made only by elaborate experimentation, it is evidently of interest to be able to substitute for this the simple thermoelectric and microscopic tests.

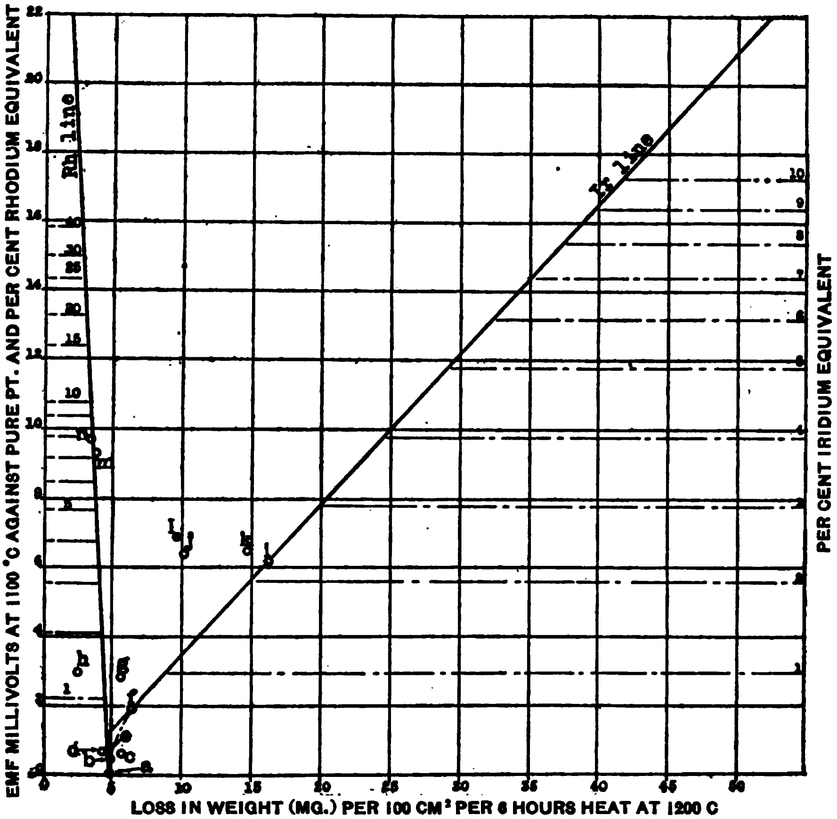


FIG. 4.—*Prediction of losses in weight on heating Pt crucibles containing Ir or Rh*

4. SUGGESTIONS AS TO SPECIFICATIONS

What is desired in a crucible for exact analytical work is a material which will strictly maintain its weight on heating and on treating with strong acid after heating; which will not exude any oxidizable or soluble matter on heating; which will not crack or develop other mechanical defects; and which is stiff enough to handle conveniently with tongs. Some of these requirements appear to be simultaneously difficult of realization in the present state of our science and of the art of working platinum and its alloys.

Iridium which adds stiffness to platinum renders the crucible subject to proportionably greater losses of weight on heating. The presence of iron appears to lower materially the heat losses, but is very objectionable on account of the formation of a soluble oxide coating. The purest platinum is usually not stiff enough, although two or three of the apparently purest crucibles here examined were unaccountably stiff, perhaps because of the presence of a small amount of osmium or silica. Rhodium both stiffens the crucible and lowers the heat and acid losses and is much to be preferred to iridium as an alloying element, and crucibles containing small amounts of rhodium, say from 3 to 5 per cent, are to be preferred to crucibles of pure platinum. As the rhodium content is increased the crucible may develop cracks in service.

A requirement which might reasonably be met, at not too great expense for highest grade crucibles, would be platinum containing 3 to 5 per cent rhodium, practically free from iron and iridium, and containing no other detectable impurities. For most purposes these requirements could be checked by assuring one's self that the emf at 1100° C against pure platinum was less than 8 and greater than 5 millivolts, that the characteristic crystal structure was that of rhodium and not iridium, and that no iron hydroxide precipitate was obtained after ignition for two hours over strong blast and applying the acid treatment as above described. If pure platinum were preferred, the emf at 1100° C should be less than 1 millivolt. If other stiffening ingredients than rhodium be allowed, the 1-millivolt emf requirement should be maintained.

IV. NATURE OF DISINTEGRATION

It has been assumed or generally held until recently by investigators of the disintegration of the platinum metals on heating that the loss of weight observed is a true volatilization, but J. H. T. Roberts offers evidence to show that there is an oxidation of platinum, iridium, etc., at high temperatures, followed by a subsequent reduction, so that the deposit carried to the cooler regions is metallic not because of volatilization but on account of an endothermic reaction between the platinum metal and oxygen forming at high temperatures an oxide which is unstable at lower temperatures.

In favor of this view appear to be the facts that these metals are less volatile at low pressures than at high, and that the presence of oxygen is essential. Against this is to be set the fact that metals, in general, have a vapor pressure which increases with temperature, and the region of stability of these oxides does not appear to have been determined.

The experiments here recorded do not give crucial evidence in favor of either of these views. The disintegration in air is shown to have a constant value for any given type of material—platinum with small additions of iridium or rhodium—and this disintegration is found to be approximately proportional to the amount of the alloying element present in platinum. Iron, when also present, as we have seen, has a special rôle of its own in retarding the disintegration.

It is possible, as Berthelot pointed out, that there might be a quantitative as well as a qualitative difference in these disintegration phenomena at high temperatures, dependent upon whether the metal is heated from outside or whether it is heated electrically from within. The experiments of Langmuir on electrically heated metal, however, indicate in high vacua an evaporation only one-twentieth of that here observed for pure platinum thermally heated at 1200° C.

The phenomena at high vacua are not, however, comparable with those at atmospheric pressure. It appears probable, therefore that there is a slight evaporation characteristic of the metal accompanied, in the presence of oxygen or air, by reaction between oxygen and platinum. Langmuir considers that "the oxygen in addition to combining with the vapor from the platinum, also attacks the platinum at a rate that increases with the pressure."

That the iridium alloys of platinum disintegrate more rapidly than platinum or its alloys with rhodium, may be due either to the greater vapor pressure of the former or to a more ready reaction with oxygen, or a combination of these two causes.

V. SUMMARY AND CONCLUSIONS

There has been devised a simple, thermoelectric method suitable for the determination of the purity of platinum ware. This method does not mar the article tested and gives data for the classification of platinum in terms of its equivalent iridium (or rhodium) content.

There were examined by the thermoelectric method 164 pieces of platinum ware, of which 26 per cent contained less than 0.5 per cent iridium and 67 per cent less than 2 per cent of iridium. Of 84 crucibles, 36 per cent contained less than 0.5 per cent iridium and 87 per cent less than 2 per cent iridium.

A method has been developed for determination of the exact loss on heating of platinum crucibles by means of a suitable electric furnace containing no heated metal parts.

Fourteen crucibles of various makes and grades were examined for loss in weight on heating and after acid treatment following each heating. Their magnetic susceptibilities were also determined. The susceptibility of pure platinum is zero and the range of susceptibility for seven crucibles is 1 to 125.

The loss of weight due to heating per 100 cm^2 of practically iron-free crucible surface at 1200° C, ranged from 0.71 mg to 2.69 mg per hour, the lesser losses being for crucibles containing rhodium and the greater losses being associated with iridium.

Iron appears to lessen somewhat the loss of weight on heating, but its presence is objectionable on account of the soluble oxide formed on the crucible surface. The chemical analysis and magnetic measurements place the crucibles in only approximately the same order as to iron content; the magnetic susceptibility is not, however, proportional to the iron content.

It appears to be possible, from a thermoelectric and microscopic examination of a crucible, to predict its probable loss of weight on heating within limits close enough for analytical purposes.

Suggestions are offered concerning the specifications of highest grade platinum crucibles, including the substitution of rhodium to 5 per cent for iridium, and the practical elimination of iron.

Whether crucibles have been long in use or not, after the first two or three heatings and acid washings, appears to make little or no difference in their behavior as to losses on heating and washing.

The nature of the process of disintegration of platinum and its alloys is briefly discussed.

We wish to express our obligation to Dr. W. F. Hillebrand, who has furnished many valuable suggestions which have aided the progress of this investigation.

VI. SELECTED BIBLIOGRAPHY

"On the volatilization and recrystallization of the platinum metals," by Prof. L. Holborn and Dr. F. Henning, Sitzber, Berlin Akad. Wiss., p. 936; 1902.

Experiments were conducted on Ir, Rh, Pt, Pd, and Au at temperatures near their melting points; also on Pt-Rh, Pt-Ir, and Ir-Ru.

A volatilization figure was obtained for each, but data are not reduced to one basis for comparison; however, the Ir and its alloys were found more volatile than Rh and its alloys. The Pt and also Rh crystals are larger than those of Ir or Pd. Concerning the alloys, the greater the percent of Ir the smaller the crystal, and prolonged heating increases the size of the boundaries between these. The Rh alloys with Pt give a smaller and more irregular crystallization than Ir alloys. Heat etching brings out the crystallization, though etching with aqua regia is more effective.

"The volatilization of metals of the platinum group," by Sir William Crookes, Pro. Roy. Soc. Lon., 86-A, p. 461; 1911-12.

Data presented in this paper show relation between volatilization of metals of platinum group. Crucibles and strips were heated in an Heraeus furnace for two-hour increments at 1300° C, 1000° to 1400° C, and 900° C in an atmosphere of still air.

The volatilization order was found to be as follows: Ru, Ir, Pd, Pt, and Rh, the last being the least volatile.

"The disintegration of metals at high temperatures" (condensation nuclei from hot wires), by J. H. T. Roberts, Phil. Mag., 25, p. 270; 1913.

Condensation nuclei could be more readily produced in O than in air, and could not be produced in pure N. Some conclusions are as follows: (1) The rate of loss of Pt is zero in N, H, or a vacuum; (2) at low pressures of O the nuclei are very small; (3) the nuclei begin to be formed, i. e., disintegration begins, at a fairly definite temperature (500° C); (4) the nuclei for a given temperature and pressure are persistent and do not alter in size; they are unaffected by light or by an electric field.

The rate of loss of weight of the platinum wire being roughly proportional to the oxygen pressure points to the formation of an endothermic oxide. "This theory of the formation of an endothermic, volatile, and dissociable oxide explains all the observed facts in a perfectly simple manner and without any other assumptions."

"Disintegration of the platinum metals in different gases," by L. Holborn and L. W. Austin, Phil. Mag., 7, p. 388; 1904.

Observations on each metal were made in the following order: (1) In air at atmospheric pressure; (2) in vacuum; (3) in slow stream of commercial O or N; (4) in H at different pressures.

Conclusions were as follows: Pt and Rh disintegration about five times as great in O as in air; in air at 25 mm about one-half that at 760 mm; and in N disintegration

is very small. Ir disintegration in air about ten times as great as Pt. At 25 mm Ir loss in weight equals one-eighth that at 760 mm, while in O it is about eleven times as great as in air.

"On the condensation nuclei produced in air and hydrogen by heating a platinum wire," by Gwilym Owen, Phil. Mag., 6, p. 306; 1903.

Condensation nuclei from heated Pt wire were produced in an atmosphere of air at 300° C. The size and number of nuclei are increased with increased temperature. In the case of hydrogen the results are obtained at temperatures some 600° or 700° higher than in air; in fact, not until the wire is luminous.

Platinum laboratory utensils, by Percy H. Walker and F. W. Smithers, Bu. of Chem., Bull. No. 137, p. 180, November, 1910.

Apparently the pioneer paper demonstrating the desirability of determining the quality of platinum ware purchased for exact analytical purposes.

Preliminary report of the committee on "Quality of platinum laboratory utensils,' by W. F. Hillebrand, Percy H. Walker, and E. T. Allen, J. Indus. & Eng. Chem., 8, No. 9, September, 1911.

Report of progress by committee on "Quality of platinum laboratory utensils," April 9, 1914, J. Indus. & Eng. Chem., June, 1914.

"The vapor pressure of the metals platinum and molybdenum," by Irving Langmuir and G. M. J. Mackay, Phys. Rev., IV, No. 4, p. 377, October, 1914.

The method consists in determining the loss of weight undergone by wires of the two metals maintained electrically at various temperatures for definite periods of time in glass vessels exhausted to an exceptionally good vacuum.

The calculation of the vapor pressure from the rate of loss of weight at a definite temperature is based upon the kinetic theory of gases and the Clausius-Clapeyron formula giving the relation between the vapor pressure of any substance and the temperature.

The rate of evaporation of heated platinum and molybdenum wires in high vacuum were determined over wide ranges of temperature. From these data the vapor pressures of these metals were calculated. The evaporation per square centimeter per second was determined and also the latent heat of vaporization in calories per gram atom.

"Chemical reactions at low pressures," by Dr. Irving Langmuir, J. Indus. & Eng. Chem., 7, p. 349, 1915; J. Am. Chem. Soc., 37, p. 1139, 1915.

Dr. Langmuir advances the "theory of molecular films" to explain heterogeneous reactions occurring between heated filaments and the surrounding gases. The adsorption film is taken to be in a state of kinetic equilibrium with the gases around it, and is in a state of constant change. The majority of gas molecules striking the bare surface of the film are held by cohesive forces until they evaporate. The rate of formation of the adsorption film is proportional to the gas pressure and to the area of the uncovered surface. The reactions observed may be divided into four classes:

1. The filament is attacked by the gas.
2. The gas reacts with vapor given off by the filament.
3. The filament acts catalytically on the gas, producing a chemical change in the gas without any permanent change in the filament.

4. The gas is chemically changed or reacts with the filament as a result of electrical discharge through the gas.

The application of this theory to particular cases was considered and included platinum reactions under classes 2 and 3.

"The sublimation of metals at low pressures," by G. W. C. Kaye, B. A., D. Sc., and Donald Ewen, M. Sc., Pro. Roy. Soc., Lon., 89–A, p. 58, 1913–14.

The "Rectilinear emission of particles," from the surface of electrically heated strips of Ir, Cu, Fe, W, is discussed. There appear to be two main classes of vapor given out when a metal volatilizes, one kind, which is associated with evaporation as usually understood by the term, the other, made up of particles of metal, which travel in straight lines from the surface of the metal. It is suggested that the difference between the two classes is that the latter are charged electrically, while the former are electrically neutral. A deposit taking the outline of the heated strip or the shape of an interposed screen could be obtained readily in the case of the above metals heated to about 1000° C in nitrogen at about 20 mm pressure.

MISCELLANEOUS REFERENCE LIST

Author	Subject or title	Reference
Moissan	Volatilization of metals	Ann. d. Chem. et d. Phys., 8, pp. 145–181; 1906.
Reboul and De Bollement.	Disintegration of metals by heat	Eng., 94, p. 577; 1912.
Stewart, Walter	Disintegration Pt wires at high temperatures, white and red heat.	Phil. Mag., 48, p. 481; 1899.
Wartenburg	Determination of vapor pressure Pb, Ag, Tl, Cu, and Pt.	Zs. f. Elektrochemie, 19, p. 482; 1913.
Rosenhain and Ewen	Intercrystalline cohesion of metals	J. Inst. of Metals, 8, p. 149; 1912.
F. Emich	Microanalysis	Sitzungsber, Akad. Wien, Abtl. II b. 124, p. 427; 1915.
Burgess and Sale	A thermoelectric method for the determination of the purity of platinum ware.	J. Indus. & Eng. Chem., 6, p. 452, 1914; and July, 1915. J. Wash. Acad., 4, p. 282; 1914.

WASHINGTON, March 28, 1915.

PUBLICATIONS ISSUED BY THE BUREAU OF STANDARDS

SCIENTIFIC PAPERS

1. Recomparison of the United States Prototype Meter . . . . *L. A. Fischer*
2. A Study of the Silver Voltameter . . . . *K. E. Guthe*
3. The So-called International Electrical Units . . . . . . *Frank A. Wolff*
4. The Spectra of Mixed Gases. . . . . *P. G. Nutting*
5. On Secondary Spectra and the Conditions under which They May Be Produced . . . . *P. G. Nutting*
6. Some New Rectifying Effects in Conducting Gases . . . . . *P. G. Nutting*
7. On Fibers Resembling Quartz in Their Elastic Properties . . . *K. E. Guthe*
8. On the Temperature of the Arc . . . . *C. W. Waidner and G. K. Burgess*
9. The Absolute Measurement of Inductance . . *E. B. Rosa and F. W. Grover*
10. The Absolute Measurement of Capacity . . . *E. B. Rosa and F. W. Grover*
11. Optical Pyrometry . . . . *C. W. Waidner and G. K. Burgess*
12. On the Theory of the Matthews and the Russell-Leonard Photometers for the Measurement of Mean Spherical and Mean Hemispherical Intensities . . . . *E. P. Hyde*
13. The Testing of Clinical Thermometers . . . *C. W. Waidner and L. A. Fischer*
14. Measurement of Inductance by Anderson's Method, Using Alternating Currents and a Vibration Galvanometer . *E. B. Rosa and F. W. Grover*
15. Use of Serpentine in Standards of Inductance . . *E. B. Rosa and F. W. Grover*
16. The Silver Coulometer . . . . *K. E. Guthe*
17. History of Standard Weights and Measures of the United States . . *L. A. Fischer*
18. Wattmeter Methods of Measuring Power Expended upon Condensers and Circuits of Low Power Factor . . . . *E. B. Rosa*
19. The Relative Intensities of Metal and Gas Spectra from Electrically Conducting Gases . . . . *P. G. Nutting*
20. The Use of White Walls in a Photometric Laboratory . . . . *E. P. Hyde*
21. Influence of Wave Form on the Rate of Integrating Induction Wattmeters . . . . *E. B. Rosa, M. G. Lloyd, and C. E. Reid*
22. Detector for Small Alternating Currents and Electrical Waves . . *L. W. Austin*
23. The Positive Charges Carried by the Canal Rays . . . . . *L. W. Austin*
24. Radiation from Platinum at High Temperatures . . . . . . *G. K. Burgess*
25. A Five-Thousand-Volt Generator Set . . . . *P. G. Nutting*
26. Talbot's Law as Applied to the Rotating Sectored Disk . . . *E. P. Hyde*
27. A New Determination of the Electromotive Force of Weston and Clark Standard Cells by an Absolute Electrodynamometer . . *K. E. Guthe*
28. The Gray Absolute Electrodynamometer . . . . *Edward B. Rosa*
29. Construction and Calculation of Absolute Standards of Inductance . *J. G. Coffin*
30. An Efficiency Meter for Electric Incandescent Lamps . . . . *E. P. Hyde and H. B. Brooks*
31. Calculation of the Self-Inductance of Single-Layer Coils . . . *Edward B. Rosa*
32. Heat Treatment of High-Temperature Mercurial Thermometers . . . . *Hobert C. Dickinson*
33. A New Potentiometer for the Measurement of Electromotive Force and Current . . . . *H. B. Brooks*
34. Spectrum Lines as Light Sources in Polariscopic Measurements . *Frederick Bates*
35. Polarimetric Sensibility and Accuracy . . . . *P. G. Nutting*
36. On the Platinum-Point Electrolytic Detector for Electrical Waves . *L. W. Austin*
37. The Influence of Frequency upon the Self-Inductance of Coils . . *J. G. Coffin*
38. Experiments on the Heusler Magnetic Alloys . . *K. E. Guthe and L. W. Austin*
39. A Pocket Spectrophotometer . . . . *P. G. Nutting*

40. Preliminary Measurements on Temperature and Selective Radiation of Incandescent Lamps . . . . . . *C. W. Waidner and G. K. Burgess*
41. Revision of the Formulæ of Weinstein and Stefan for the Mutual Inductance of Coaxial Coils . . . . *Edward B. Rosa*
42. The Mutual Inductance of Two Circular Coaxial Coils of Rectangular Section . . . . *Edward B. Rosa and L. Cohen*
43. On the Determination of the Mean Horizontal Intensity of Incandescent Lamps by the Rotating Lamp Method . . . *E. P. Hyde and F. E. Cady*
44. Purity and Intensity of Monochromatic Light Sources . . . . *P. G. Nutting*
45. Radiometric Investigations of Infra-Red Absorption and Reflection Spectra . . . . *W. W. Coblentz*
46. A Vacuum Radiomicrometer . . . . *W. W. Coblentz*
47. On the Geometrical Mean Distances of Rectangular Areas and the Calculation of Self-Inductance . . . . *E. B. Rosa*
48. The Compensated Two-Circuit Electrodynamometer . . . . *E. B. Rosa*
49. Complete Form of Fechner's Law . . . . *P. G. Nutting*
50. A Comparison of the Unit of Luminous Intensity of the United States with Those of Germany, England, and France . . . . . . *E. P. Hyde*
51. Geometrical Theory of Radiating Surfaces, with Discussion of Light Tubes . . . . *E. P. Hyde*
52. The Influence of Basic Lead Acetate on the Optical Rotation of Sucrose in Water Solution . . . . *F. J. Bates and J. C. Blake*
53. On the Calorimetric Determination of Iron, with Special Reference to Chemical Reagents . . . . *H. N. Stokes and J. R. Cain*
54. On Sulphocyanic Acid . . . . *H. N. Stokes and J. R. Cain*
55. Radiation from and Melting Points of Palladium and Platinum . . . . *C. W. Waidner and G. K. Burgess*
56. The Mutual Inductance of a Circle and a Coaxial Single-Layer Coil—The Lorenz Apparatus and the Ayrton-Jones Absolute Electrodynamometer . . . . *E. B. Rosa*
57. On the Establishment of the Thermodynamic Scale of Temperature by Means of the Constant-Pressure Thermometer . . . *Edgar Buckingham*
58. An Exact Formula for the Mutual Inductance of Coaxial Solenoids . . . . *Louis Cohen*
59. The Mutual Inductance of Coaxial Solenoids . . . *E. B. Rosa and L. Cohen*
60. The Production of High-Frequency Oscillations from the Electric Arc . . . . *L. W. Austin*
61. An Explanation of the Short Life of Frosted Lamps . . . . *E. P. Hyde*
62. Melting Points of the Iron Group Elements by a New Radiation Method . . . . *G. K. Burgess*
63. On the Determination of the Mean Horizontal Intensity of Incandescent Lamps . . . . *Edward P. Hyde and F. E. Cady*
64. Simultaneous Measurement of the Capacity and Power Factor of Condensers . . . . *F. W. Grover*
65. A New Determination of the Ratio of the Electromagnetic to the Electrostatic Unit of Electricity . . . . . *E. B. Rosa and N. E. Dorsey*
66. A Comparison of the Various Methods of Determining the Ratio of the Electromagnetic to the Electrostatic Unit of Electricity . . . . *E. B. Rosa and N. E. Dorsey*
67. Preliminary Specifications for Clark and Weston Standard Cells . . . . *F. A. Wolff and C. E. Waters*
68. Calorimetric Resistance Thermometers and the Transition Temperature of Sodium Sulphate . . . . . *H. C. Dickinson and E. F. Mueller*
69. On the Standard Scale of Temperature in the Interval 0° to 100° C. . . . . *C. W. Waidner and H. C. Dickinson*
70. Clark and Weston Standard Cells . . . . . *F. A. Wolff and C. E. Waters*
71. The Electrode Equilibrium of the Standard Cell . *F. A. Wolff and C. E. Waters*
72. A Comparative Study of Plain and Frosted Lamps . *E. P. Hyde and F. E. Cady*
73. The Variation of Resistances with Atmospheric Humidity . . . . *E. B. Rosa and H. D. Babcock*
74. On the Self-Inductance of a Toroidal Coil of Rectangular Section . . . . *Edward B. Rosa*

75. On the Self-Inductance of Circles . . . . *Edward B. Rosa and Louis Cohen*
76. The Influence of Frequency on the Resistance and Inductance of Solenoidal Coils . . . . *Louis Cohen*
77. The Atomic Weight of Hydrogen . . . . *W. A. Noyes*
78. The Best Method of Demagnetizing Iron in Magnetic Testing . *C. W. Burrows*
79. A Deflection Potentiometer for Voltmeter Testing . . . . . *H. B. Brooks*
80. The Self and Mutual Inductance of Linear Conductors . . . *Edward B. Rosa*
81. The Atomic Weight of Chlorine . . . . *W. A. Noyes and H. C. P. Weber*
82. The Preparation of Chloroplatinic Acid by Electrolysis of Platinum Black . . . . *H. C. P. Weber*
83. The Self-Inductance of a Coil of any Length and any Number of Layers of Wire . . . . *Edward B. Rosa*
84. Self-Inductance of a Solenoid of any Number of Layers . . . *Louis Cohen*
85. Instruments and Methods Used in Radiometry . . . . . *W. W. Coblentz*
86. A Quartz Compensating Polariscope with Adjustable Sensibility . . *F. J. Bates*
87. An Apparatus for Determining the Form of a Wave of Magnetic Flux . . . . *M. G. Lloyd and J. V. S. Fisher*
88. Effect of Wave Form upon the Iron Losses in Transformers . *Morton G. Lloyd*
89. The Luminous Properties of Electrically Conducting Helium Gas . . . . *P. G. Nutting*
90. Function of a Periodic Variable Given by the Steady Reading of an Instrument, with a Note on the Use of the Capillary Electrometer with Alternating Voltages . . . . *Morton G. Lloyd*
91. Selective Radiation from the Nernst Glower . . . . . . *W. W. Coblentz*
92. The Testing of Glass Volumetric Apparatus . . *N. S. Osborne and B. H. Veazey*
93. Formulæ and Tables for the Calculation of Mutual and Self-Inductance . . . . *Edward B. Rosa and Louis Cohen*
94. Some Contact Rectifiers of Electric Currents . . . . *L. W. Austin*
95. A Method for Producing Feebly Damped High-Frequency Electrical Oscillations for Laboratory Measurements . . . . *L. W. Austin*
96. On the Advantages of a High Spark Frequency in Radio-Telegraphy . . . . *L. W. Austin*
97. Selective Radiation from Various Solids . . . . *W. W. Coblentz*
98. Remarks on the Quartz Compensating Polariscope with Adjustable Sensibility . . . . *Frederick Bates*
99. Methods of Obtaining Cooling Curves . . . . *George K. Burgess*
100. Note on the Approximate Values of Bessel's Functions for Large Arguments . . . . *Louis Cohen*
101. The Influence of Terminal Apparatus on Telephonic Transmission . *Louis Cohen*
102. The Principles Involved in the Selection and Definition of the Fundamental Electrical Units to be Proposed for International Adoption . . . . *F. A. Wolff*
103. The Luminous Equivalent of Radiation . . . . *P. G. Nutting*
104. The Temperature Formula of the Weston Standard Cell . . . *F. A. Wolff*
105. Radiation, Constants of Metals . . . . *W. W. Coblentz*
106. Dependence of Magnetic Hysteresis upon Wave Form . . . *Morton G. Lloyd*
107. A New Form of Standard Resistance . . . . *Edward B. Rosa*
108. Errors in Magnetic Testing with Ring Specimens . . . . *Morton G. Lloyd*
109. The Testing of Transformer Steel . . . . *M. G. Lloyd and J. V. S. Fisher*
110. A New Method of Determining the Focal Length of a Converging Lens . . . . *Irwin G. Priest*
111. A New Method for the Absolute Measurement of Resistance . *Edward B. Rosa*
112. The Theory of Coupled Circuits . . . . *Louis Cohen*
113. A Volt Scale for a Watts-per-candle Meter . . . . . . *Herbert E. Ives*
114. The Coefficient of Reflection of Electrical Waves at a Transition Point . . . . *Louis Cohen*
115. A Tungsten Comparison Lamp in the Photometry of Carbon Lamps . . . . *Herbert E. Ives and L. R. Woodhull*
116. The Determination of the Ratio of Transformation and of the Phase Relations in Transformers . . . . . . *E. B. Rosa and M. G. Lloyd*
117. The Determination of the Magnetic Induction in Straight Bars . . . . *Charles W Burrows*

118. A Method for Constructing the Natural Scale of Pure Color . . *P. G. Nutting*
119. An Approximate Method for the Analysis of EMF. Waves . . *P. G. Agnew*
120. The Thermoelectric Properties of Tantalum and Tungsten . *W. W. Coblentz*
121. The Estimation of the Temperature of Copper by Means of Optical Pyrometers . . . . . . *George K. Burgess, assisted by J. F. Crowe*
122. The Resolving Power of Objectives . . . . *P. G. Nutting*
123. The Theory of the Hampson Liquefier . . . . . . *Edgar Buckingham*
124. Platinum Resistance Thermometry at High Temperatures . . . . *C. W. Waidner and G. K. Burgess*
125. The Daylight Efficiency of Artificial Illuminants . . . . . *H. E. Ives*
126. Coupled Currents in which the Secondary has Distributed Inductance and Capacity . . . . *Louis Cohen*
127. Effect of Phase of Harmonics upon Acoustic Quality . . . . *M. G. Lloyd and P. G. Agnew*
128. White Light from the Mercury Arc and its Complementary . *Herbert E. Ives*
129. The Regulation of Potential Transformers and the Magnetizing Current . . . . *M. G. Lloyd and P. G. Agnew*
130. The Determination of the Constants of Instrument Transformers . . . . *P. G. Agnew and T. T. Fitch*
131. Selective Radiation from Various Solids, II . . . . . . *W. W. Coblentz*
132. Luminous Efficiency of the Firefly . . *Herbert E. Ives and W. W. Coblentz*
133. Luminosity and Temperature . . . . *P. G. Nutting*
134. A Theoretical and Experimental Study of the Vibration Galvanometer . . . . *F. Wenner*
135. Specific Heat of Some Calcium Chloride Solutions between −35° C. and +20° C . . . . *H. C. Dickinson, E. F. Mueller, and E. B. George*
136. On the Definition of the Ideal Gas . . . . *Edgar Buckingham*
137. Mica Condensers as Standards of Capacity . . . . . . *Harvey L. Curtis*
138. The Mutual Induction of Two Parallel Coaxial Circles in Terms of Hypergeometrical Series . . . . *Frederick W. Grover*
139. A New Method for the Absolute Measurement of Electric Quantity . . . . *Burton McCollum*
140. The Comparative Sensitiveness of Some Common Detectors of Electrical Oscillations . . . . *Louis W. Austin*
141. Photometric Units and Nomenclature . . . . *E. B. Rosa*
142. A Modified Method for the Determination of Relative Wave Lengths, Especially Adapted to the Establishment of Secondary Standards . . . . *Irwin G. Priest*
143. Note on the Temperature Scale between 100 and 500° C. . . . . *C. W. Waidner and G. K. Burgess*
144. A New Form of Direct-Reading Candlepower Scale and Recording Device for Precision Photometers . . . . . . *George W. Middlekauff*
145. A Device for Measuring the Torque of Electrical Instruments . . . . *F. G. Agnew*
146. The Intensities of Some Hydrogen, Argon, and Helium Lines in Relation to Current and Pressure . . . *P. G. Nutting and Orin Tugman*
147. The Temperature Coefficient of Resistance of Copper . . . *J. H. Dellinger*
148. The Electrical Conductivity of Commercial Copper . . . . *F. A. Wolff and J. H. Dellinger*
149. On the Constancy of the Sulphur Boiling Point . . . . *C. W. Waidner and G. K. Burgess*
150. Note on Oscillatory Interference Bands and Some of their Practical Applications . . . . *G. O. Squier and A. C. Crehore*
151. The Effect of Preliminary Heat Treatment upon the Drying of Clays . . . . *A. V. Bleininger*
152. The Reflecting Power of Various Metals . . . . *W. W. Coblentz*
153. The Action of Sunlight and Air upon Some Lubricating Oils . *C. E. Waters*
154. The Visibility of Radiation. A Recalculation of Konig's Data . *P. G. Nutting*
155. A Photometric Attachment for Spectroscopes . . . . *P. G. Nutting*
156. Selective Radiation from Various Substances, III . . . . . *W. W. Coblentz*
157. The Measurement of Electrical Oscillations in the Receiving Antenna . . . . *L. W. Austin*

158. Some Experiments with Coupled High-Frequency Circuits . . . *L. W. Austin*
159. Some Quantitative Experiments in Long Distance Radiotelegraphy . . . . *L. W. Austin*
160. The Behavior of High-Boiling Oils on Heating in the Air . . . *C. E. Waters*
161. The Determination of Vanadium in Vanadium and Chrome-Vanadium Steels . . . . *J. R. Cain*
162. On the Computation of the Constant C_2 of Planck's Equation by an Extension of Paschen's Method of Equal Ordinates . . . . *Edgar Buckingham and J. H. Dellinger*
163. A Comparison of American Direct Current Switchboard Voltmeters and Ammeters . . . . *T. T. Fitch and C. J. Huber*
164. Study of the Current Transformer with Particular Reference to Iron Loss . . . . *P. G. Agnew*
165. Thermodynamics of Concentration Cells . . . . *Henry S. Carhart*
166. The Capacity and Phase Difference of Paraffined Paper Condensers as Functions of Temperature and Frequency . . . . . . *Frederick W. Grover*
167. The Steam Expansion Line on the Mollier Diagram and a Short Method of Finding the Reheat Factor . . . . *Edgar Buckingham*
168. Radiometric Investigation of Water of Crystallization, Light Filters, and Standard Absorption Bands . . . . *W. W. Coblentz*
169. Formulas and Tables for the Calculation of Mutual and Self Induction (2d edition, revised and enlarged) . *E. B. Rosa and Frederick W. Grover*
170. The Correction for Emergent Stem of a Mercurial Thermometer . . . . *Edgar Buckingham*
171. Thermometric Lag . . . . *D. R. Harper*
172. Deflection Potentiometers for Current and Voltage Measurements . . . . *H. B. Brooks*
173. Outline of Design of Deflection Potentiometers with Notes on the Design of Moving-Coil Galvanometers . . . . *H. B. Brooks*
174. The Determination of Total Sulphur in India Rubber . . . . *C. E. Waters and J. B. Tuttle*
175. The Measurement of the Inductances of Resistance Coils . . . . *Frederick W. Grover and Harvey L. Curtis*
176. Luminous Properties of Electrically Conducting Helium Gas. II. Reproducibility . . . . *P. G. Nutting*
177. Resistance Coils for Alternating Current Work . *H. L. Curtis and F. W. Grover*
178. The Hydrolysis of Sodium Oxalate and Its Influence Upon the Tests for Neutrality . . . . *William Blum*
179. Wave-Lengths of Neon . . . . *Irwin G. Priest*
180. On the Deduction of Wien's Displacement Law . . . . . *Edgar Buckingham*
181. The Four-Terminal Conductor and the Thomson Bridge . . . *Frank Wenner*
182. Standardization of Potassium Permanganate Solution by Sodium Oxalate . . . . *R. S. McBride*
183. Benzoic Acid as an Acidimetric Standard . . . . *G. W. Morey*
184. A Tubular Electrodynamometer for Heavy Currents . . . . . *P. G. Agnew*
185. Thermometric Lag. . . . . *D. R. Harper*
186. Determination of Manganese as Sulphate and by the Sodium Bismuthate Method . . . . *William Blum*
187. A New Precision Colorimeter . . . . *P. G. Nutting*
188. Instruments and Methods Used in Radiometry, II . . . . *W. W. Coblentz*
189. Antenna Resistance . . . . *L. W. Austin*
190. Energy Losses in Some Condensers Used in High-Frequency Circuits . . . . *L. W. Austin*
191. Selective Radiation from Various Substances, IV . . . . . *W. W. Coblentz*
192. On a Modified Form of Stability Test for Smokeless Powder and Similar Materials . . . . *H. C. P. Weber*
193. Atomic Weight of Bromine . . . . *H. C. P. Weber*
194. The Silver Voltameter.—Part I. First Series of Quantitative Experiments . . . . *E. B. Rosa and G. W. Vinal*
195. The Silver Voltameter.—Part II. The Chemistry of the Filter Paper Voltameter and the Explanation of Striations . *E. B. Rosa and G. W. Vinal*
196. The Diffuse Reflecting Power of Various Substances . . . *W. W. Coblentz*

197. Density and Thermal Expansion of Ethyl Alcohol and of Its Mixtures with Water . . . *N. S. Osborne, E. C. McKelvy, and H. W. Bearce*
198. A Micropyrometer . . . . *G. K. Burgess*
199. A Simplified Formula for the Change in Order of Interference Due to Changes in Temperature and Pressure of Air . . . . . . *I. G. Priest*
200. New Calorimetric Resistance Thermometers . *H. C. Dickinson and E. F. Mueller*
201. The Silver Voltameter.—Part III. Second Series of Quantitative Experiments and the Preparation and Testing of Silver Nitrate . . . . *E. B. Rosa, G. W. Vinal, and A. S. McDaniel*
202. Note on Cold-Junction Corrections for Thermocouples . . . . *P. D. Foote*
203. The Analysis of Alternating Current Waves by the Method of Fourier, with Special Reference to Methods of Facilitating the Computations . . . . *F. W. Grover*
204. The Constants of Spectral Radiation of a Uniformly Heated Inclosure or So-Called Black Body, Part I . . . . . . *W. W. Coblentz*
205. Melting Points of the Refractory Elements.—I. Elements of Atomic Weight from 48 to 59 . . . . . . *G. K. Burgess and R. G. Waltenberg*
206. High-Frequency Ammeters . . . . *J. H. Dellinger*
207. A Comparative Study of American Direct-Current Watthour Meters . . . . *T. T. Fitch and C. J. Huber*
208. Windage Resistance of Steam Turbine Wheels . . . . . . *E. Buckingham*
209. Latent Heat of Fusion of Ice . *H. C. Dickinson, D. R. Harper 3d, and N. S. Osborne*
210. Observations on Ocean Temperatures in the Vicinity of Icebergs and in other Parts of the Ocean . *C. W. Waidner, H. C. Dickinson, and J. J. Crowe*
211. Accuracy of the Formulas for the Ratio, Regulation, and Phase Angle of Transformers . . . . *P. G. Agnew and F. B. Silsbee*
212. Melting Points of Some Refractory Oxides . . . . *C. W. Kanolt*
213. The Critical Ranges A2 and A3 of Pure Iron . . *G. K. Burgess and J. J. Crowe*
214. Note on the Setting of a Mercury Surface to a Required Height . *M. H. Stillman*
215. Micrometer Microscopes . . . . *A. W. Gray*
216. The Pentane Lamp as a Working Standard . . *E. C. Crittenden and A. H. Taylor*
217. Testing Potential Transformers . . . . *H. B. Brooks*
218. Comparison of the Silver and Iodine Voltameters and the Determination of the Faraday . . . . *G. W. Vinal and S. J. Bates*
219. Production of Temperature Uniformity in an Electric Furnace . *A. W. Gray*
220. The Silver Voltameter.—Part IV. Third Series of Quantitative Experiments and Special Investigations . *E. B. Rosa, G. W. Vinal, and A. S. McDaniel*
221. Influence of Atmospheric Conditions in the Testing of Sugars . . . . *F. J. Bates and F. P. Phelps*
222. Flame Standards in Photometry . . . . . *E. B. Rosa and E. C. Crittenden*
223. The Testing of Potentiometers . . . . . . *Frank Wenner and Ernest Weibel*
224. The Emissivity of Metals and Oxides.—I. Nickel Oxide (NiO) in the Range 600 to 1300° C . . . . *G. K. Burgess and P. D. Foote*
225. Adjustments of the Thomson Bridge in the Measurement of Very Low Resistances . . . . *Frank Wenner and Ernest Weibel*
226. Quantitative Experiments in Radiotelegraphic Transmission . *L. W. Austin*
227. Measurements on Standards of Radiation in Absolute Value . *W. W. Coblentz*
228. An Experimental Study on the Koepsel Permeameter . . *Charles W. Burrows*
229. Various Modifications of Bismuth-Silver Thermopiles Having a Continuous Absorbing Surface . . . . *W. W. Coblentz*
230. Combustion Calorimetry and the Heats of Combustion of Cane Sugar, Benzoic Acid, and Naphthalene . . . . *H. C. Dickinson*
231. Specific Heat of Copper in the Interval 0° to 50° C.; with a Note on Vacuum-Jacketed Calorimeters . . . . *D. R. Harper 3d*
232. Equilibrium in the System: Lead Acetate, Lead Oxide, and Water, at 25° . . . . *Richard F. Jackson*
233. A Watthour Meter Method of Testing Instrument Transformers . *P. G. Agnew*
234. Insulating Properties of Solid Dielectrics . . . . *Harvey L. Curtis*
235. A Direct-Reading Instrument for Measuring the Logarithmic Decrement and Wave Length of Electromagnetic Waves . . *Frederick A. Kolster*
236. Electrical Resistance and Critical Ranges of Pure Iron . . . . *G. K. Burgess and I. N. Kellberg*
237. Absorption, Reflection, and Dispersion Constants of Quartz . *W. W. Coblentz*

238. Characteristic Equations of Tungsten Filament Lamps and Their Application in Heterochromatic Photometry . *G. W. Middlekauff and J. F. Skogland*
239. A Vibration Electrometer . . . . *W. W. Coblentz*
240. Studies on the Silver Voltameter . . . . . *G. A. Hulett and G. W. Vinal*
241. A Wheatstone Bridge for Resistance Thermometry . . . . *C. W. Waidner, H. C. Dickinson, E. F. Mueller, and D. R. Harper 3d*
242. The Emissivity of Metals and Oxides.—II. Measurements with the Micropyrometer . . . . *G. K. Burgess and R. G. Waltenberg*
243. The Emissivity of Metals and Oxides.—III. The Total Emissivity of Platinum and the Relation Between Total Emissivity and Resistivity . . . . *Paul D. Foote*
244. A Comparison of Stellar Radiometers and Radiometric Measurements on 110 Stars . . . . *W. W. Coblentz*
245. Temperature Coefficient of Magnetic Permeability Within the Working Range . . . . *R. L. Sanford*
246. Methods of Measuring the Inductances of Low Resistance Standards . . . . *Frank Wenner, Ernest Weibel, and F. B. Silsbee*
247. An Aneroid Calorimeter . . . . . . *H. C. Dickinson and N. S. Osborne*
248. Specific Heat and Heat of Fusion of Ice . *H. C. Dickinson and N. S. Osborne*
249. The Emissivity of Metals and Oxides.—IV. Iron Oxide . . . . *George K. Burgess and Paul D. Foote*
250. Characteristics of Radiation Pyrometers . *George K. Burgess and Paul D. Foote*
251. Interference Measurements of Wave Lengths in the Iron Spectrum (2851–3701), with Notes on Comparisons of Lengths of Light Waves by Interference Methods, and Some Wave Lengths in the Spectrum of Neon Gas . . . . *Keivin Burns and W. F. Meggers*
252. Effective Resistance and Inductance of Iron and Bimetallic Wires . . . . *John M. Miller*
253. A Direct-Reading Device for use in Computing Characteristics of Vacuum Tungsten Lamps . . . . *J. F. Skogland*
254. A Study of the Quality of Platinum Ware . *George K. Burgess and P. D. Sale*

CIRCULARS

1. Verification of Standards and Measuring Instruments.
2. Measurements of Length and Area, Including Thermal Expansion.
3. Verification of Standards of Mass.
4. Verification of Standards of Capacity.
5. Testing of Clinical Thermometers.
6. Fees for Electric, Magnetic, and Photometric Testing.
7. Pyrometer Testing and Heat Measurements.
8. Testing of Thermometers.
9. Testing of Glass Volumetric Apparatus.
10. Legal Weights (in pounds) per Bushel of Various Commodities.
11. The Standardization of Bomb Calorimeters.
12. Verification of Polariscopic Apparatus.
13. United States Government Standard Specifications for Incandescent Electric Lamps.
14. Analyzed Irons and Steels—Methods of Analysis.
15. The International Unit of Light.
16. The Testing of Hydrometers.
17. Magnetic Testing.
18. Standard Gauge for Sheet and Plate Iron and Steel.
19. Standard Density and Volumetric Tables.
20. Testing of Electrical Measuring Instruments.
21. Precision Measurements of Resistance and Electromotive Force.
22. Standard Specifications for Transformers, Oil-immersed, Self-cooled, 60-cycle, 2200 Volts.
23. Standardization of Electrical Practice in Mines.
24. Publications of the Bureau of Standards.
25. Standard Analyzed Samples—General Information.
26. Analyzed Iron and Manganese Ores—Methods of Analysis.
27. The Testing and Properties of Optical Instruments.

28. The Determination of the Optical Properties of Materials.
29. Announcement of a Change in the Value of the International Volt.
30. Lime: Its Properties and Uses.
31. Copper Wire Tables.
32. Standards of Gas Service.
33. United States Government Specification for Portland Cement.
34. The Relation of the Horsepower to the Kilowatt.
35. Melting Points of Chemical Elements.
36. The Testing and Properties of Electrical Condensers.
37. Electric Wire and Cable Terminology.
38. The Testing of Mechanical Rubber Goods.
39. Specifications for and Measurement of Standard Sieves.
40. Sodium Oxalate as a Standard in Volumetric Analysis.
41. Testing and Properties of Textile Materials.
42. Metallographic Testing.
43. The Metric Carat.
44. Polarimetry.
45. The Testing of Materials.
46. The Testing of Barometers.
47. Units of Weight and Measure; Definitions and Tables of Equivalents.
48. Standard Methods of Gas Testing.
49. Safety Rules to be Observed in the Operation of Electrical Equipment and Lines.
50. National Standard Hose Couplings and Fittings for Public Fire Service.
51. Measurement of Time and Tests of Timepieces.
52. Regulation of Electrotyping Solutions.
53. The Composition, Properties, and Testing of Printing Inks.
54. Proposed National Electrical Safety Code.
55. Measurements for the Household.
56. Standards for Electric Service.

TECHNOLOGIC PAPERS

1. The Effect of Preliminary Heat Treatment upon the Drying of Clays (53 pp.) . . . *A. V. Bleininger*
2. The Strength of Reinforced Concrete Beams. Results of Tests of 333 Beams (first series) (200 pp.) . . . *R. L. Humphrey and L. H. Losse*
3. Tests of Absorptive and Permeable Properties of Portland Cement Mortars and Concretes, Together with Tests of Damp Proofing and Waterproofing Compounds and Materials (127 pp.) . . . *Rudolph J. Wig*
4. The Effect of Added Fatty and Other Oils upon the Carbonization of Mineral Lubricating Oils (14 pp.) . . . *C. E. Waters*
5. The Effect of High-Pressure Steam on the Crushing Strength of Portland Cement Mortar and Concrete (25 pp.) . . . *R. J. Wig*
6. The Determination of Chromium and Its Separation from Vanadium in Steels (6 pp.) . . . *J. R. Cain*
7. The Testing of Clay Refractories, With Special Reference to Their Load Carrying Capacity at Furnace Temperatures (78 pp.) . . . *A. V. Bleininger and G. H. Brown*
8. A Rapid Method for the Determination of Vanadium in Steels, Ores, etc., Based on Its Quantitative Inclusion by the Phosphomolybdate Precipitate (20 pp.) . . . *J. R. Cain and J. C. Hostetter*
9. The Density and Thermal Expansion of Linseed Oil and Turpentine (27 pp.) . . . *H. W. Bearce*
10. The Melting Points of Fire Bricks (17 pp.) . . . *C. W. Kanolt*
11. Comparison of Five Methods Used to Measure Hardness (27 pp.) . *Ralph P. Devries*
12. Action of the Salts in Alkali Water and Sea Water on Cements (157 pp.) . . . *P. H. Bates, A. J. Phillips, and R. J. Wig*
13. The Evaporation Test for Mineral Lubricating and Transformer Oils (13 pp.) . . . *C. E. Waters*
14. Legal Specifications for Illuminating Gas (31 pp.) . *E. B. Rosa and R. S. McBride*
15. Surface Insulation of Pipes (44 pp.) . . . *Burton McCollum and O. S. Peters*
16. The Manufacture of Lime (130 pp.) . . . *W. E. Emley*
17. The Function of Time in the Vitrification of Clays (26 pp.) . . . *G. H. Brown and G. A. Murray*

18. Electrolysis in Concrete (137 pp.) . *E. B. Rosa, Burton McCollum, and O. S. Peters*
19. Physical Testing of Cotton Yarns (31 pp.) . . . . *W. S. Lewis*
20. Determination of Sulphur in Illuminating Gas (46 pp.) . . . . *R. S. McBride and E. R. Weaver*
21. The Dehydration of Clays (23 pp.) . . . *G. H. Brown and E. T. Montgomery*
22. The Effect of Overfiring Upon the Structure of Clays (23 pp.) . . . . *A. V. Bleininger and E. T. Montgomery*
23. The Technical Control of the Colloidal Matter of Clays (118 pp.) . *H. E. Ashley*
24. The Determination of Phosphorus in Steels Containing Vanadium (11 pp.) . . . . *J. R. Cain and F. H. Tucker*
25. Electrolytic Corrosion of Iron in Soils (69 pp.) . *Burton McCollum and K. H. Logan*
26. Earth Resistance and Its Relation to Electrolysis . . . . *Burton McCollum and K. H. Logan*
27. Special Studies in Electrolysis Mitigation.—I. A Preliminary Study of Conditions in Springfield, Ohio, with Recommendations for Mitigation and Control (55 pp.) . . . . *E. B. Rosa and Burton McCollum*
28. Methods of Making Electrolysis Surveys . *Burton McCollum and A. H. Ahlborn*
29. The Variation in Results of Sieving with Standard Cement Sieves (16 pp.) . . . . *R. J. Wig and J. C. Pearson*
30. The Viscosity of Porcelain Bodies (11 pp.) . *A. V. Bleininger and Paul Teetor*
31. Some Leadless Boro-Silicate Glazes Maturing at about 1100° C. (22 pp.) . . . . *E. T. Montgomery*
32. Special Studies in Electrolysis Mitigation.—II. Electrolysis from Electric Railway Currents and Its Prevention—Experimental Test on a System of Insulated Negative Feeders in St. Louis (34 pp.) . . . . *E. B. Rosa, Burton McCollum, and K. H. Logan*
33. Determination of Carbon in Steel and Iron by the Barium Carbonate Titration Method (12 pp.) . . . . *J. R. Cain*
34. Determination of Ammonia in Illuminating Gas (23 pp.) . . . . *J. D. Edwards*
35. Combustion Method for the Direct Determination of Rubber (11 pp.) . *L. G. Wesson*
36. Industrial Gas Calorimetry (150 pp.) . . . *C. W. Waidner and E. F. Mueller*
37. Iodine Number of Linseed and Petroleum Oils (17 pp.) . . . . *W. H. Smith and J. B. Tuttle*
38. Observations on Finishing Temperatures and Properties of Rails (63 pp.) . . *G. K. Burgess, J. J. Crowe, H. S. Rawdon, and R. G. Waltenberg*
39. Analysis of Printing Inks (20 pp.) . . . . . *J. B. Tuttle and W. H. Smith*
40. The Veritas Firing Rings (10 pp.) . . . . *A. V. Bleininger and G. H. Brown*
41. Lead Acetate Test for Hydrogen Sulphide in Gas (46 pp.) . . . . *R. S. McBride and J. D. Edwards*
42. Standardization of No. 200 Cement Sieves (51 pp.) *R. J. Wig and J. C. Pearson*
43. Hydration of Portland Cement (71 pp.) . . . *A. A. Klein and A. J. Phillips*
44. Investigation of the Durability of Cement Drain Tile and Alkali Soils (56 pp.) . . *R. J. Wig and G. M. Williams, in cooperation with S. H. McCrory, E. C. Bebb, and L. R. Ferguson*
45. A Study of Some Recent Methods for the Determination of Total Sulphur in Rubber (16 pp.) . . . . *J. B. Tuttle and A. Isaacs*
46. A Study of the Atterberg Plasticity Method (18 pp.) . . . *Charles S. Kinnison*
47. Value of the High-Pressure Steam Test of Portland Cements (34 pp.) . . . . *R. J. Wig and H. A. Davis*
48. An Air-Analyzer for Determining the Fineness of Portland Cement (74 pp.) . . . . *H. C. Pearson and W. H. Sligh*
49. Emergent Stem Correction for Thermometers in Creosote Oil Distillation Flasks (21 pp.) . . . . *R. M. Wilhelm*
50. Viscosity of Porcelain Bodies High in Feldspar (7 pp.) . . . . *A. V. Bleininger and C. S. Kinnison*
51. Use of Sodium Salts in the Purification of Clays and in the Casting Process (40 pp.) . . . . *A. V. Bleininger*
52. Electrolysis and Its Mitigation (37 pp.) . . *E. B. Rosa and Burton McCollum*
53. An Investigation of Fusible Tin Boiler Plugs (143 pp.) . . . . *George K. Burgess and P. D. Merica*
54. Special Studies in Electrolysis Mitigation.—III. A Report on Conditions in Springfield, Ohio, with Insulated Feeder System Installed . . . . *Burton McCollum and G. H. Ahlborn*

55. Special Studies in Electrolysis Mitigation.—IV. A Preliminary Study of Electrolysis Conditions in Elyria, Ohio, with Recommendations for Mitigation . . . . *Burton McCollum and K. H. Logan*
56. Protection of Life and Property Against Lightning . . . . . . *O. S. Peters*
57. Difference in Weight Between Raw and Clean Wools . . . . *Walter S. Lewis*
58. Compressive Strength of Portland Cement Mortars and Concretes . . . . *R. J. Wig, G. M. Williams, and E. R. Gates.*

MISCELLANEOUS

International Metric System. (Chart.)
Tables of Equivalents of United States Customary and Metric Weights and Measures. (See Circular 47.)
The International Metric System of Weights and Measures. (Pamphlet.)
First Conference on the Weights and Measures of the United States.
Second Annual Conference on the Weights and Measures of the United States.
Third Annual Conference on the Weights and Measures of the United States.
Fourth Annual Conference on the Weights and Measures of the United States.
Fifth Annual Conference on the Weights and Measures of the United States.
Sixth Annual Conference on the Weights and Measures of the United States.
Seventh Annual Conference on the Weights and Measures of the United States.
Eighth Annual Conference on the Weights and Measures of the United States.
Ninth Annual Conference on the Weights and Measures of the United States.
Tenth Annual Conference on the Weights and Measures of the United States.
State and National Laws Concerning the Weights and Measures of the United States (second edition).
The National Bureau of Standards. (Descriptive pamphlet.)
Report of the International Committee on Electrical Units and Standards.

⁂

Sci 1520.203

DEPARTMENT OF COMMERCE

Vol. 12 No. 3

BULLETIN
OF THE
BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

FEBRUARY 3, 1916

CONTENTS

Page

Calculation of the Maximum Force Between Two Coaxial Circular Currents . . . . *Frederick W. Grover* 317

Construction of Primary Mercurial Resistance Standards . . . . *F. A. Wolff, M. P. Shoemaker, and C. A. Briggs* 375

Note on the Resistance of Radiotelegraphic Antennas . . . . . *L. W. Austin* 465

A Method of Measuring Earth Resistivity . . . . *Frank Wenner* 469

A New Relation Derived from Planck's Law . . . . *Paul D. Foote* 479

WASHINGTON
GOVERNMENT PRINTING OFFICE
1916

DEPARTMENT OF COMMERCE

Vol. 12 No. 3

BULLETIN

OF THE

BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

FEBRUARY 3, 1916

CONTENTS

Page
Calculation of the Maximum Force Between Two Coaxial Circular Currents . . . . *Frederick W. Grover* 317
Construction of Primary Mercurial Resistance Standards . . . . *F. A. Wolff, M. P. Shoemaker, and C. A. Briggs* 375
Note on the Resistance of Radiotelegraphic Antennas . . . . *L. W. Austin* 465
A Method of Measuring Earth Resistivity . . . . *Frank Wenner* 469
A New Relation Derived from Planck's Law . . . . *Paul D. Foote* 479

WASHINGTON
GOVERNMENT PRINTING OFFICE
1916

Harvard College Library
[illegible]
FROM
United States Government.

ADDITIONAL COPIES
OF THIS PUBLICATION MAY BE PROCURED FROM
THE SUPERINTENDENT OF DOCUMENTS
GOVERNMENT PRINTING OFFICE
WASHINGTON, D. C.
AT
25 CENTS PER COPY
Subscription Price, $1 Per Volume of 4 Numbers

CALCULATION OF THE MAXIMUM FORCE BETWEEN TWO COAXIAL CIRCULAR CURRENTS

By Frederick W. Grover

CONTENTS

Page

I. Introduction 318

II. Formulas for the force between two coaxial circular coils 320

  1. Maxwell's formula in zonal harmonics 320
  2. Maxwell's formula in elliptic integrals 321
  3. Nagaoka's formulas in q series 322

III. The maximum force 323

  1. Rayleigh's approximate formula for the distance z_m 324

IV. Derivation of the new formulas 325

V. Use of the formulas 332

  1. Differential formulas for calculating z_m 332
  2. Approximate formulas 335
  3. The calculation of the maximum force 338

VI. Variation formulas for the maximum force F_m and the maximum distance ratio y_m 339

VII. Variation of the force for small displacements of the coils from the position of maximum force 343

VIII. Best values of radii of the coils 345

IX. Examples 347

  Example 1.—Calculation of q and q_1 (formulas 6a) 347
  Example 2.—Illustrating the calculation of y_m by (33) 349
  Example 3.—Calculation of y_m by formula (37) 351
  Example 4.—Illustration of the use of the curve for obtaining a first approximation 352
  Example 5.—Calculation by (37) from a first approximation derived from the curve 353
  Example 6.—Calculation of the factor P 354
  Example 7.—Calculation of the factor X 355
  Example 8.—Use of differential formulas (38) and (39) 357
  Example 9.—Calculation by differential formulas (40), (41), and (42) 357
  Example 10.—Calculation of the maximum force by formulas (34) and (5) 358
  Example 11.—Calculation of the maximum force by (37) and (6) .. 359
  Example 12.—The coefficients η and ϵ 360

IX. Examples—Continued. Page.

Example 13.—Illustrating calculation of maximum force for coils whose radii have a ratio differing slightly from one of the values in Table 4 361
Example 14.—Calculation of the maximum force between the Bureau of Standards current balance coils.......... 362
Table A.—Calculation of maximum force distance z_m for the Bureau of Standards coils.............................. 363
Table B.—Calculation of the maximum force for the Bureau of Standards coils................................ 364
X. Summary.. 365
Appendix.. 368
Table 1.—Values of the Factor P in Equation (33a) 368
Table 2.—Values of the Factor X in Equation (37a)...................... 369
Table 3.—Values of the factors in the differential equations (38) to (42).. 371
Table 4.—Fundamental values of the maximum force ratio y_m and the maximum force F_m...................................... 372
Table 5.—Coefficients η and ϵ in Equations (48), (51), (55), and (58)...... 373

I. INTRODUCTION

Aside from its mathematical interest, the problem of the calculation of the force which exists between two circular currents has received a good deal of attention, owing to its practical importance in the calculation of the constants of apparatus used in absolute measurements. In such practical cases, it is true, the current is not concentrated in a circular filament, but flows in a number of wires, generally so disposed as to form a circular coil of rectangular cross section. The calculation of the mutual inductance of two such coils, as well as the force exerted between them, is, however, most readily based on the ideal case, a correction being applied to take into account the difference between the actual distribution of the current and the ideal distribution. This correction may be made with all the precision necessary, even in the most refined work, by the methods of Rayleigh and Lyle, provided that the dimensions of the cross sections are small with respect to the radii of the coils and the distance between them. In the formula of quadratures[1] (Rayleigh) the coil is replaced by a number of circular filaments, the weighted mean being taken of their separate effects; in the method of Lyle[2] the coil is shown to be equivalent to two circular filaments, whose radii and positions depend on the dimensions of the cross section. For details of such calculations the reader is referred to the original articles and to articles in this Bulletin.[3]

---

[1] Gray, Absolute Meas., Vol. II, Part II, p. 403. This Bulletin, **2**, p. 370; 1906 (Scientific Papers No. 42).

[2] Phil. Mag., **3**, p. 310; 1902. Also, this Bulletin, **2**, p. 374; 1906 (Scientific Papers No. 42).

The mutual energy of two currents is equal to the product of their mutual inductance by the two current strengths. Accordingly, the force exerted by one circular filament on the other, in any specified direction, is equal to the rate of change of the mutual inductance as the filament moves in that direction multiplied by the product of the two current strengths. The force between two circular currents of unit strength may, therefore, be found by differentiating any suitable formula for their mutual inductance. The formulas of Maxwell and Nagaoka derived in this way for the special case of parallel coaxial circles are given for convenience of reference in the following section. These formulas allow of the calculation of the force with an accuracy which is limited only by the precision attainable in the measurement of the dimensions of the coils and the calculation of the correction for the finite cross section.

The present paper has to do with the special case, very important in practice, where the two parallel, coaxial, circular filaments are placed with such a distance between their planes that their mutual attraction or repulsion is a maximum. This is the condition which is realized in the form of current balance, first used by Lord Rayleigh[4] in 1884, and by Rosa, Dorsey, and Miller[5] at the Bureau of Standards. In this form of balance a coil is suspended from one arm of a balance, with its plane midway between those of two horizontal fixed coils, the distances between the planes of the coils being so chosen that the forces exerted by the currents in the fixed coils on the current in the suspended coil are a maximum. This arrangement has the advantage that not only is the force sensibly constant for small displacements of the suspended coil from this ideal position, but the constant of the balance is a function of the *ratio* of the radii of the fixed and movable coils, which ratio can be found with great accuracy by electrical methods, thus rendering unnecessary the measurement of the individual radii. The position of the coils for maximum force can be found by experiment.

---

[3] This Bulletin, **2**, pp. 397-399, 1906 (Scientific Papers No. 42); and **8**, p. 323, 331, 1911 (Scientific Papers No. 169).

[4] Phil. Trans., **175**, pp. 411-460, 1884; Rayleigh's Scientific Papers, **2**, pp. 278-332.

[5] This Bulletin, **8**, pp. 269-393; 1911 (Scientific Papers No. 169).

The value of the maximum force can be calculated from the known ratio of the radii by interpolation between a number of values of the force, calculated by known formulas, for various distances in the neighborhood of the experimentally determined correct position of the coils. As far as is known to the author, this is the only method which has been previously used. The formulas and methods here developed are presented as affording a more direct solution of the problem, and although the amount of numerical work is, in the general case, still appreciable a check is afforded on the methods previously employed. For such cases as occur in practice, the constants may be obtained with very little labor from the tables of fundamental values given below.

The main formulas (33) to (37) were derived by the author in June, 1911. Since then, however, other work has interfered to delay the development of the auxiliary formulas and the preparation of the tables.

II. FORMULAS FOR THE FORCE BETWEEN TWO COAXIAL CIRCULAR COILS.

1. MAXWELL'S FORMULA IN ZONAL HARMONICS.[6]

Maxwell expressed the mutual inductance between two coaxial circular currents, whose radii A and a subtend, respectively, the angles α_1 and α_2 at the origin, and whose planes lie at distances B and b from this point, by a formula in zonal harmonics

$$M = 4\pi^2 c \sin^2\alpha_1 \sin^2\alpha_2 \left\{ \frac{1}{2}\frac{c}{C} P_1'(\alpha_1) P_1'(\alpha_2) + \cdots\cdots \right. \tag{1}$$
$$\left. + \frac{1}{n(n+1)} \frac{c^n}{C^n} P_n'(\alpha_1) P_n'(\alpha_2) + \cdots\cdots \right\}$$

where $c = \sqrt{a^2+b^2}$ and $C = \sqrt{A^2+B^2}$

Substituting for the zonal harmonics their well-known values in terms of sines and cosines, and introducing for the latter their values in terms of the radii and the distances b, B, and c, C, the formula (1) becomes

---

[6] Elect. and Mag., Vol. II, sec. 696.

$$\begin{aligned} M = {} & 1.2\,\pi^2\frac{A^2a^2}{C^3} + 2.3\,\pi^2\frac{A^2Ba^2b}{C^5} \\ & + 3.4\,\pi^2\frac{A^2a^2(B^2-\frac{1}{4}A^2)(b^2-\frac{1}{4}a^2)}{C^7} + 4.5\,\pi^2\frac{Bb(B^2-\frac{3}{4}A^2)(b^2-\frac{3}{4}a^2)A^2a^2}{C^9} \\ & + 5.6\,\pi^2\frac{(B^4-\frac{3}{2}B^2A^2+\frac{1}{8}A^4)(b^4-\frac{3}{2}b^2a^2+\frac{1}{8}a^4)A^2a^2}{C^{11}} \\ & + 6.7\,\pi^2\frac{Bb(B^4-\frac{5}{2}B^2A^2+\frac{5}{8}A^4)(b^4-\frac{5}{2}b^2a^2+\frac{5}{8}a^4)A^2a^2}{C^{13}} + \cdots\cdots \end{aligned} \tag{2}$$

which on differentiation with respect to b gives for the force[7]

$$\begin{aligned} F = \frac{\pi^2A^2a^2}{C^4}\Big\{ & 1.2.3\frac{B}{C} + 2.3.4\frac{b(B^2-\frac{1}{4}A^2)}{C^3} + 3.4.5\frac{B(B^2-\frac{3}{4}A^2)(b^2-\frac{1}{4}a^2)}{C^5} \\ & + 4.5.6\frac{B(B^4-\frac{3}{2}B^2A^2+\frac{1}{8}A^4)\,(b^2-\frac{3}{4}a^2)\,b}{C^7} \\ & + 5.6.7\frac{B(B^4-\frac{5}{2}B^2A^2+\frac{5}{8}A^4)(b^4-\frac{3}{2}b^2a^2+\frac{1}{8}a^4)}{C^9} + \cdots\Big\} \end{aligned} \tag{3}$$

On account of its slow convergence, the formula (3) is not suitable for numerical calculations. It is, however, valuable, in that it gives a clear view of the way the force varies when the distance between the coils is increased.

2. MAXWELL'S FORMULA IN ELLIPTIC INTEGRALS.[8]

An accurate expression for the force was derived by Maxwell by differentiating his well-known formula for the mutual inductance of two coaxial circles.

$$M = -4\pi\sqrt{Aa}\left\{\left(k-\frac{2}{k}\right)K+\frac{2}{k}E\right\}$$

the result being

$$F = -\frac{\pi z\sin\gamma}{\sqrt{Aa}}\{2K-(1+\sec^2\gamma)E\} \tag{4}$$

where A and a are the radii of the two circles, z is the distance between their planes, and K and E are the complete elliptic integrals to modulus $k = \sin\gamma = \dfrac{2\sqrt{Aa}}{\sqrt{(A+a)^2+z^2}}$

---

[7] This Bulletin, 8, p. 325, 1911 (Scientific Papers No. 171).

[8] Elect. and Mag., Vol. II, sec. 701.

A table was prepared by Lord Rayleigh giving the value of log $[\sin\gamma\{2K-(1+\sec^2\gamma)E\}]$ to seven places of decimals.[9] This table was recalculated at the Bureau of Standards, using Legendre's tables of elliptic integrals, the logarithms being carried out to ten places, and some small errors in the seventh place corrected. The table thus corrected was reproduced in the article on the absolute determination of the international ampere, by Rosa, Dorsey, and Miller.

3. NAGAOKA'S FORMULAS

The force between two parallel coaxial circles has also been expressed by Nagaoka[10] in terms of the q series of Jacobi. These expressions have the advantage of rapid convergence and of not requiring the use of tables of elliptic integrals. Nagaoka's formula (29), extended by the author of the present paper to include four additional terms, is

$$F=\frac{192\,\pi^2 z}{\sqrt{Aa}}\,q^{\frac{3}{2}}\Big[1+20\,q^2+225\,q^4+1840q^6 \\ +12120\,q^8+68052\,q^{10} \\ +337465\,q^{12}+1513740\,q^{14} \\ +6247665\,q^{16}+\ldots\ldots\Big] \tag{5}$$

Extending Nagaoka's formula (21), the author obtained the further formula which is equivalent to the formula (8) found by Nagaoka in a more recent article.[11]

$$F=\frac{\pi z}{16q_1\sqrt{Aa}}\Big[(1+12\,q_1-192\,q_1^2+1232\,q_1^3-5634\,q_1^4 \\ +21648q_1^5-73600q_1^6+226944q_1^7-648189q_1^8+\ldots) \\ -12\,q_1\log_e\frac{1}{q_1}\cdot(1-10\,q_1+60q_1^2-300\,q_1^3 \\ +1300\,q_1^4-4884\,q_1^5+16320\,q_1^6 \\ -49920\,q_1^7+142500\,q_1^8-\cdot\cdot\cdot\cdot\cdot)\Big] \tag{6}$$

---

[9] Phil. Trans., 175, pp. 411-460, 1884; Rayleigh's Scientific Papers, 2, 327.

[10] Jour. College of Science, Tokyo, vol. 16, art. 15, p. 12, 1903; Phil. Mag., 6, p. 19, 1903.

[11] Tokyo Math.-Phys. Soc., vol. 6, p. 156, 1911.

where

$$\left.\begin{aligned}
q&=\frac{l}{2}+2\left(\frac{l}{2}\right)^5+15\left(\frac{l}{2}\right)^9+\cdots\cdot\\
\frac{l}{2}&=\frac{1}{2}\,\frac{1-\sqrt{k'}}{1+\sqrt{k'}}=\frac{k^2}{2(1+k')(1+\sqrt{k'})^2}\\
k&=\frac{2\sqrt{Aa}}{\sqrt{(A+a)^2+z^2}}\qquad k'=\sqrt{1-k^2}=\frac{\sqrt{(A-a)^2+z^2}}{\sqrt{(A+a)^2+z^2}}\\
q_1&=\frac{l_1}{2}+2\left(\frac{l_1}{2}\right)^5+15\left(\frac{l_1}{2}\right)^9+\cdots\cdots\\
\frac{l_1}{2}&=\frac{1}{2}\,\frac{1-\sqrt{k}}{1+\sqrt{k}}=\frac{k'^2}{2(1+k)(1+\sqrt{k})^2}
\end{aligned}\right\}\qquad(6a)$$

In most cases only two or three terms need be calculated to give the force with a good deal of precision. The formulas are given at length here so as to give all needed precision, even in the most unfavorable cases. Formula (6) converges most rapidly for circles which are near together. It is not so convenient to use as (5) and will be employed only in those cases where the convergence of (5) is not satisfactory. With the number of terms included here, the force may be calculated by one or the other of the two formulas with an accuracy of better than 1 part in 100 000, even in the most unfavorable case. To facilitate calculations, Nagaoka has published[12] tables giving the values of $\frac{F\sqrt{Aa}}{z}$ for different values of q.

Further formulas for the force have also been given by Nagaoka, for which the original articles already cited should be consulted.

III. THE MAXIMUM FORCE

The foregoing formulas suffice to give the value of the force between the circles with all the precision required in the most careful work.

For the calculation of the constant of the Rayleigh current balance we need, as has already been explained, to obtain either

---

[12] Tokyo Math.-Phys. Soc., vol. 6, p. 154, 1911.

(*a*) an expression giving the maximum value of the force between the circular currents as a function of the ratio of their radii, or (*b*) a formula for calculating the value of the distance z_m which exists between their planes when the force is a maximum. The maximum force F_m may, in the latter case, be calculated by substituting z_m for z in any one of the formulas for the force given above, which converges satisfactorily.

Although more direct, the first method is, in some cases, less accurate than the second, and as a knowledge of the distance z_m is of interest in itself, formulas for both methods have been given below.

1. RAYLEIGH'S APPROXIMATE FORMULA FOR z_m

Obviously z_m may be found by differentiating any suitable expression for the force F with respect to the distance z between the circles, equating the result to zero, and solving for z. The result is the value z_m for which the force is a maximum. Provided the formula for z_m is not too complicated, it gives on substitution in the formula for the force an expression suitable for calculating the maximum force F_m.

The differentiation of Maxwell's formula (3) gives, when the origin is taken at the center of the smaller circle ($b=0$)

$$\left(\frac{dF}{dz}\right)_{b=0}=\frac{\pi^2A^2a^2}{C^4}\left[2\cdot3\cdot4\frac{(z_m^2-\frac{1}{4}A^2)}{C^5}\right.$$
$$\left.-3\cdot5\cdot6\,\frac{a^2(z_m^4-\frac{3}{2}\,z_m^2A^2+\frac{1}{8}A^4)}{C^7}+\cdot\cdot\right]=0 \qquad (7)$$

This may be solved for z_m by a method of approximation. The first term shows that $\frac{z_m}{A}$ is less than unity. Taking as a first approximation that value of $\frac{z_m}{A}$ which makes the first term zero, and substituting this value in the second term, we find as a second approximation Rayleigh's equation[13]

$$\frac{z_m^2}{A^2}=\frac{1}{4}\left(1-\frac{9a^2}{5A^2}\right)$$

or (8)

$$\frac{z_m}{A}=\frac{1}{2}\left(1-\frac{9a^2}{10A^2}-\cdot\cdot\cdot\right)$$

---

[13] Rayleigh: Scientific Papers, vol. 2, p. 284.

It is not difficult, by substituting this value in the second and third terms of (7) to obtain a further approximation

$$\frac{z_m}{A}=\frac{1}{2}\left(1-\frac{9a^2}{10A^2}-\frac{1}{4}\frac{a^4}{A^4}\right) \tag{9}$$

Unfortunately, the slow convergence of these expressions, and the rapid increase in the amount of labor necessary for obtaining each new term, prevent the use of (8) and (9) except for purposes of orientation, unless within the limited range where $\frac{a}{A}$ is small.

The differentiation of Maxwell's equation (4) in elliptic integrals is complicated and does not give a satisfactory formula for z_m.

By following the method of Nagaoka the author of the present paper has obtained expressions in the form of q series, which enable the values of z_m and F_m to be calculated with all the accuracy necessary in refined work, and these formulas have been tested thoroughly as to their applicability for numerical calculations and as to their agreement with the results found by other methods.

IV. DERIVATION OF THE NEW FORMULAS

The mutual inductance of two parallel coaxial circles of radii A and a, the distance between their planes being z, is given by the integral[14]

$$M=4\pi\, Aa\int_0^\pi \frac{\cos\theta\, d\theta}{\sqrt{A^2+a^2+z^2-2\,Aa\cos\theta}} \tag{10}$$

where θ is the angle between two elements ds and ds' of the two circles.

Differentiating this with respect to z, we find for the force

$$F=\frac{dM}{dz}=4\pi\, Aa\, z\int_0^\pi \frac{\cos\theta\, d\theta}{(A^2+a^2+z^2-2\,Aa\cos\theta)^{\frac{3}{2}}} \tag{11}$$

and the condition which must hold when the force between the circles is a maximum is

$$\begin{aligned}\frac{dF}{dz}=\frac{d^2M}{dz^2}=4\pi\, Aa\int_0^\pi \frac{\cos\theta\, d\theta}{(A^2+a^2+z^2-2\,Aa\cos\theta)^{\frac{3}{2}}}\\ +4\pi\, Aa\, z^2\int_0^\pi \frac{3\cos\theta\, d\theta}{(A^2+a^2+z^2-2\,Aa\cos\theta)^{\frac{5}{2}}}=0\end{aligned} \tag{12}$$

---

[14] Maxwell, Elect. & Mag.; Vol. II, sec. 701.

Nagaoka in the first of the papers above cited has shown how to express the elliptic integral (11) in elliptic functions, and by expanding these in terms of q functions has obtained the formulas (5) and (6) (the latter in a different form) for the force, which have been given above.

For the integration of (12) we need evaluate only one new integral—the second. For completeness, however, we will also indicate the solution of the first integral as given by Nagaoka.

Put

$$\cos\theta = \alpha s + \beta \tag{13}$$

then

$$-\sin\theta d\theta = \alpha ds$$

$$\sin^2\theta = 1 - (\alpha s + \beta)^2 = -\alpha^2 s^2 - 2\alpha\beta s + (1 - \beta^2)$$

$$A^2 + a^2 + z^2 - 2Aa\cos\theta = [(a^2 + A^2 + z^2 - 2aA\beta) - 2aA\alpha s]$$

and therefore,

$$\sin\theta\sqrt{A^2 + a^2 + z^2 - 2Aa\cos\theta} =$$

$$\sqrt{[A^2 + a^2 + z^2 - 2aA\beta - 2aA\alpha s][-\alpha^2 s^2 - 2\alpha\beta s + (1 - \beta^2)]}$$

In order that the cubic under the radical may assume the canonical Weierstrassian form

$$\sqrt{4(s - e_1)(s - e_2)(s - e_3)} = \sqrt{4s^3 - g_2 s - g_3} = \sqrt{S} \tag{14}$$

the following conditions must be satisfied

$$2aA\alpha^3 = 4$$

$$e_1 + e_2 + e_3 = 6aA\beta\alpha^2 - \alpha^2(a^2 + A^2 + z^2) = 0$$

which give

$$\alpha = \left(\frac{2}{Aa}\right)^{\frac{1}{3}}, \qquad \beta = \frac{A^2 + a^2 + z^2}{6Aa} \tag{15}$$

$$A^2 + a^2 + z^2 - 2Aa\cos\theta = \frac{4}{\alpha^3}(2\beta - \alpha s)$$

whence

$$\frac{1}{4}g_2 = \frac{3\beta^2 + 1}{\alpha^2} \qquad \frac{1}{4}g_3 = \frac{2\beta(\beta^2 - 1)}{\alpha^3}$$

$$e_1 = \frac{2\beta}{\alpha} \qquad e_2 = \frac{1 - \beta}{\alpha} \qquad e_3 = -\frac{(1 + \beta)}{\alpha} \tag{16}$$

We have accordingly for the integrand of the first integral

$$\frac{\cos\theta\, d\theta}{(A^2+a^2+z^2-2Aa\cos\theta)^{\frac{3}{2}}} = -\frac{\alpha ds(\alpha s+\beta)}{\sqrt{S}\cdot\frac{4}{\alpha^3}(2\beta-\alpha s)}$$

and for the limits we notice that, when

$$\theta=\pi,\ \alpha s+\beta=-1 \text{ and therefore } s=e_3$$
$$\theta=0,\ \alpha s+\beta=\ \ 1 \text{ and therefore } s=e_2$$

Introducing these conditions, the first integral becomes

$$I_1=4\pi Aa\int_{e_2}^{e_3}\left[-\frac{\alpha^4}{4}\frac{(\alpha s+\beta)}{(2\beta-\alpha s)}\cdot\frac{ds}{\sqrt{S}}\right] \tag{18}$$

Introducing the Weierstrassian function, $\wp u$ which is defined by the equations $u=\int_s^\infty \frac{ds}{\sqrt{S}}$ and $s=\wp u$ (19)

and remembering that when $s=e_3$, $u=\omega_3$; and when $s=e_2$, $u=\omega_2$, the integral becomes

$$I_1=-4\pi Aa\int_{\omega_3}^{\omega_2}\frac{\alpha^4}{4}\frac{(\alpha\wp u+\beta)}{(\alpha\wp u-2\beta)}du \tag{20}$$

The integrand may be written

$$1+\frac{3\beta}{\alpha\wp u-2\beta}=1+\frac{3}{2}\frac{e_1}{\wp u-e_1} \tag{21}$$

which, taking into account the relations

$$\left.\begin{aligned}&\frac{1}{\wp u-e_1}=-\frac{e_1}{(e_1-e_2)(e_1-e_3)}+\frac{\wp(u\pm\omega_1)}{(e_1-e_2)(e_1-e_3)}\\&\omega_2-\omega_3=\omega_1\\&\int_{\omega_3}^{\omega_2}\wp(u\pm\omega_1)du=-\eta_1\end{aligned}\right\} \tag{22}$$

gives finally

$$\frac{I_1}{4\pi Aa}=-\frac{\alpha^4}{4}\left[\omega_1-\frac{3}{2}\frac{e_1}{(e_1-e_2)(e_1-e_3)}(\eta_1+e_1\omega_1)\right] \tag{23}$$

From what has gone before, the second integral I_2 can be shown to be equivalent to

$$I_2 = \frac{\alpha^7}{16} \int_{\omega_3}^{\omega_2} \frac{[\alpha \mathrm{p}u + \beta]}{[\alpha \mathrm{p}u - 2\beta]^3} du \tag{24}$$

The integrand may be written

$$\frac{1}{\alpha} \frac{1}{\mathrm{p}u - e_1} + \frac{3}{2} \frac{e_1}{\alpha} \frac{1}{(\mathrm{p}u - e_1)^2}$$

and the first term can be integrated by (22). For the second term we make use of the relations

$$\left.\begin{aligned} \frac{(e_1 - e_2)^2 (e_1 - e_3)^2}{(\mathrm{p}u - e_1)^2} &= \mathrm{p}^2(u \pm \omega_1) - 2e_1 \mathrm{p}(u \pm \omega_1) + e_1^2 \\ \mathrm{p}^2(u + \omega_1) &= \frac{1}{6} \mathrm{p}'' u + \frac{g_2}{12} \end{aligned}\right\} \tag{25}$$

which, remembering that $\mathrm{p}'\omega_3 = 0$, and $\mathrm{p}'\omega_2 = 0$ gives, finally, for the second term

$$(e_1 - e_2)^2 (e_1 - e_3)^2 \int_{\omega_3}^{\omega_2} \frac{du}{(\mathrm{p}u - e_1)^2} = \frac{y_2}{12} \omega_1 + 2e_1 \eta_1 + e_1^2 \omega_1$$

so that the integral I_2 becomes

$$I_2 = \frac{\alpha^6}{16} \left[- \frac{(\eta_1 + e_1 \omega_1)}{(e_1 - e_2)(e_1 - e_3)} + \frac{3}{2} \frac{e_1}{(e_1 - e_2)^2 (e_1 - e_3)^2} \left(\frac{g_2}{12} \omega_1 + 2e_1 \eta_1 + e_1^2 \omega_1 \right) \right] \tag{27}$$

The integrated form of equation (12) is therefore found to be

$$\frac{1}{4\pi A a} \frac{\partial^2 M}{\partial z^2} = - \frac{\alpha^4}{4} \left[\omega_1 - \frac{3}{2} \frac{e_1}{(e_1 - e_2)(e_1 - e_3)} (\eta_1 + e_1 \omega_1) \right] \tag{28}$$

$$- \frac{3}{16} \alpha^6 z^2 \left[- \frac{(\eta_1 + e_1 \omega_1)}{(e_1 - e_2)(e_1 - e_3)} + \frac{3}{2} \frac{e_1}{(e_1 - e_2)^2 (e_1 - e_3)^2} \left(e_1^2 \omega_1 + 2e_1 \eta_1 + \frac{g_2}{12} \omega_1 \right) \right]$$

For numerical calculations we have next to express the formula just derived in terms of the θ functions of Jacoby, from which the transition to the q series may readily be made.

It is convenient to write the last term of (28) in the form

$$\frac{3}{2}\frac{e_1^2}{(e_1-e_2)^2(e_1-e_3)^2}\left[(\eta_1+e_1\omega_1)+\left(\eta_1+\frac{g_2}{12}\frac{\omega_1}{e_1}\right)\right]$$

and then, using the relations,

$$\left.\begin{aligned}&\eta_1+e_1\omega_1=-\frac{1}{4\omega_1}\frac{\theta_2''(0)}{\theta_2(0)},\ e_1-e_3=\frac{\pi^2}{4\omega_1^2}\theta_3^4(0),\ e_1-e_2=\frac{\pi^2}{4\omega_1^2}\theta_0^4(0)\\&e_1=\frac{1}{3}\frac{\pi^2}{4\omega_1^2}[\theta_3^4(0)+\theta_0^4(0)],\ \frac{g_2}{12}=\frac{1}{18}\frac{\pi^4}{16\omega_1^4}[\theta_0^8(0)+\theta_2^8(0)+\theta_3^8(0)]\\&\eta_1\omega_1=-\frac{1}{12}\frac{\theta_1'''(0)}{\theta_1'(0)}=-\frac{1}{12}\left[\frac{\theta_0''(0)}{\theta_0(0)}+\frac{\theta_2''(0)}{\theta_2(0)}+\frac{\theta_3''(0)}{\theta_3(0)}\right]\end{aligned}\right\}\quad(29)$$

we find, finally, that the condition which must be satisfied by z_m becomes

$$\begin{aligned}\frac{3}{16}\alpha^6 z_m^2\Bigg[&\frac{4}{\pi^4}\frac{\theta_2''}{\theta_2}\frac{1}{(\theta_3^4+\theta_0^4)}-\frac{2}{9\pi^4}\left(\frac{1}{\theta_0^4}+\frac{1}{\theta_3^4}\right)\left(\frac{\theta_0''}{\theta_0}+4\frac{\theta_2''}{\theta_2}+\frac{\theta_3''}{\theta_3}\right)\\&-\frac{2}{9\pi^2}\left(1-\frac{\theta_0^4}{\theta_3^4}-\frac{\theta_3^4}{\theta_0^4}\right)\Bigg]=-\frac{\alpha^4}{\omega_1^2}\left[\frac{\theta_3^4\theta_0^4}{(\theta_0^4+\theta_3^4)}+\frac{1}{2\pi^2}\frac{\theta_2''}{\theta_2}\right]\end{aligned}\quad(30)$$

all the θ functions being to argument zero.

To find an expression involving q functions we make use of the well known relations

$$\left.\begin{aligned}&\theta_0=1-2q+2q^4-2q^9+\ldots\ldots+(-1)^n 2q^{n^2}+\ldots\ldots\\&\theta_3=1+2q+2q^4+2q^9+\ldots\ldots+2q^{n^2}+\ldots\ldots\\&\theta_2=2q^{\frac{1}{4}}[1+q^2+q^6+q^{12}+\ldots\ldots+q^{n(n+1)}+\ldots\ldots]\end{aligned}\right.\quad(31)$$

The substitution of these values gives rise to long and tedious reductions and only the final result will be given. This is

$$\begin{aligned}z_m^2=\frac{Aa}{20q}\Big[&1-36q^2+722q^4-11208q^6+155487q^8\\&-2065492q^{10}+27054638q^{12}\\&-353044772q^{14}+4603353515q^{16}-\ \ldots\ldots\Big]\end{aligned}\quad(32)$$

or, if the square root be taken,

$$\begin{aligned}z_m=\frac{\sqrt{Aa}}{2\sqrt{5q}}\Big[&1-18q^2+199q^4-2022q^6+21547q^8\\&-242522q^{10}+2829828q^{12}\\&-33755570q^{14}+408424637q^{16}-\ \ldots\ldots\Big]\end{aligned}\quad(33)$$

Substituting this value of z_m in equation (5) we derive for the maximum force (putting for q the value q_0 corresponding to z_m)

$$\begin{aligned}F_m=\frac{96}{\sqrt{5}}\pi^2 q_0^2\Big[&1+2q_0^2+64q_0^4-252q_0^6+4882q_0^8\\&-50480q_0^{10}+631392q_0^{12}\\&-7604902q_0^{14}+93488433q^{16}-\ \ldots\ldots\Big]\end{aligned}\quad(34)$$

In all the equations thus far given the elliptic functions have had as semiperiods the quantities ω_1 and ω_3, respectively real and pure imaginary, and $q = e^{i\pi\tau}$, where $\tau = \frac{\omega_3}{\omega_1}$ and $i = \sqrt{-1}$. It can, however, be shown that a given expression can be made to depend also on elliptic functions having for periods ω_3 and $-\omega_1$, respectively real and imaginary, if attention be paid to the following points. The invariants g_2 and g_3, and the quantity e_2 are unchanged by the transformation. The remaining quantities e_1, e_3, k, k', τ, $q = e^{i\pi\tau}$ go into the following, respectively, e_3, e_1, k', k, $\tau_1 = -\frac{1}{\tau}$, $q' = e^{i\pi\tau_1}$.

We may, consequently, write (28) in the form

$$\begin{aligned}\frac{1}{4\pi Aa}\frac{\partial^2 M}{\partial z^2} = &- \frac{\alpha^4}{4}\frac{1}{\omega_3}\left[\omega_1\omega_3 - \frac{3e_1}{2\,(e_1-e_2)\,(e_1-e_3)}(\eta_1\omega_3 + e_1\omega_1\omega_3)\right] \\ &- \frac{3\alpha^6}{16}z_m^2\left[-\frac{1}{\omega_3}(\eta_1\omega_3 + e_1\omega_1\omega_3)\right. \\ &\left.+\frac{3}{2}\frac{e_1^2}{(e_1-e_2)^2(e_1-e_3)^2}\frac{1}{\omega_3}\left\{2\eta_1\omega_3 + e_1\omega_1\omega_3 + \frac{g_2}{12}\frac{\omega_1\omega_3}{e_1}\right\}\right]\end{aligned} \tag{34}$$

and for the transformation to θ functions introduce into this the relations

$$\begin{aligned}&\eta_1\omega_3 = \omega_1\eta_3 + i\frac{\pi}{2}\quad, \qquad \frac{\omega_1}{\omega_3} = \frac{1}{\pi i}\log_e\frac{1}{q_1} \\ &\eta_1\omega_3 + e_1\omega_1\omega_3 = \frac{\omega_1}{\omega_3}\left(\omega_3\eta_3 + e_1\omega_3^2\right) + \frac{\pi i}{2} \\ &e_1\omega_3^2 + \omega_3\eta_3 = -\frac{1}{4}\frac{\theta_0''\,(0/\tau_1)}{\theta_0\,(0/\tau_1)} \\ &e_1 - e_2 = -\frac{\pi^2}{4\omega_3^2}\theta_2^4(0/\tau_1)\quad, \qquad e_1 - e_3 = -\frac{\pi^2}{4\omega_3^2}\theta_0^4(0/\tau_1) \\ &e_1 = -\frac{1}{3}\frac{\pi^2}{4\omega_3^2}\left[\theta_2^4(0/\tau_1) + \theta_3^4(0/\tau_1)\right] \\ &g_2 = \frac{2}{3}\left(\frac{\pi^2}{2\omega_3}\right)^4\left[\theta_0^8(0/\tau_1) + \theta_2^8(0/\tau_1) + \theta_3^8(0/\tau_1)\right]\end{aligned} \tag{35}$$

which give

$$\begin{aligned}\frac{3z_m^2}{16A^2a^2}\Bigg[-\log_e\frac{1}{q_1}\Bigg\{\frac{4}{\pi^4}\frac{\theta_0''}{\theta_0}\frac{1}{\theta_2^4\theta_3^4}-\frac{2}{9}\left(\frac{1}{\theta_3^4}+\frac{1}{\theta_2^4}\right)^2\left(\frac{\theta_2''}{\theta_2}+\frac{\theta_3''}{\theta_3}+4\frac{\theta_0''}{\theta_0}\right)\\ +\frac{2}{9\pi^2}\left(\frac{1}{\theta_3^4}+\frac{1}{\theta_2^4}\right)\left(1-\frac{\theta_2^4}{\theta_3^4}-\frac{\theta_3^4}{\theta_2^4}\right)\Bigg\} \qquad (36)\\ -\frac{8}{\pi^2}\frac{1}{\theta_3^4\theta_2^4}+\frac{8}{3\pi^2}\left(\frac{1}{\theta_3^4}+\frac{1}{\theta_2^4}\right)^2\Bigg]\\ =\frac{1}{\pi^2Aa\theta_0^4}\left[\left(\frac{1}{\theta_3^4}+\frac{1}{\theta_2^4}\right)-\log_e\frac{1}{q_1}\left(1-\frac{1}{2\pi^2}\frac{\theta_0''}{\theta_0}\left(\frac{1}{\theta_3^4}+\frac{1}{\theta_2^4}\right)\right)\right]\end{aligned}$$

and on substituting for these θ functions, whose argument is zero, and whose periods have the ratio τ_1, expansions exactly like those of (31) excepting that q_1 appears in place of q, we obtain a second expression for the distance z_m corresponding to the maximum force.

$$\begin{aligned}z_m^2[(1-16q_1+376q_1^2-4672q_1^3+38948q_1^4-252192q_1^5\\ +1365888q_1^6-6463360q_1^7+27500946q_1^8-\ldots\ldots)\\ (37)\\ -120q_1^2\log_e\frac{1}{q_1}\cdot(1-16q_1+154q_1^2-1120q_1^3+6680q_1^4\\ -34272q_1^5+156268q_1^6-\ldots\ldots)]\\ =32aAq_1[(1+24q_1+36q_1^2+384q_1^3+402q_1^4+3456q_1^5\\ +3064q_1^6+23040q_1^7+18351q_1^8+\ldots\ldots)\\ -12q_1\log_e\frac{1}{q_1}\cdot(1+2q_1+24q_1^2+28q_1^3+264q_1^4\\ +252q_1^5+2016q_1^6+1696q_1^7+12264q_1^8+\ldots)]\end{aligned}$$

It does not seem advisable to substitute this equation in (6) to obtain an expression for the maximum force F_m, but, rather, to calculate *numerical* values of z_m from (37) and to substitute this numerical value of z_m in (6).

V. USE OF THE FORMULAS

It is convenient to write the two formulas (33) and (37) just derived in the abbreviated forms

$$y_m = \frac{z_m}{A} = \sqrt{\frac{a}{A} \cdot \frac{1}{20q}} \cdot P = \sqrt{\frac{\alpha}{20q}} \cdot P \tag{33a}$$

and

$$\left(\frac{z_m}{A}\right)^2 \left[S - 120 q_1^2 \log_e \frac{1}{q_1} \cdot T\right] = 32 \frac{a}{A} q_1 \left[Q - 12 q_1 \log_e \frac{1}{q_1} \cdot R\right]$$

or

$$y_m^2 = \left(\frac{z_m}{A}\right)^2 = 32 \frac{a}{A} q_1 X = 32 \alpha q_1 X \tag{37a}$$

respectively. Tables of values of the quantities P and X have been calculated to aid in using these formulas, and will be found in the appendix. Examples illustrating the use of these tables are given below.

In employing the formulas just derived, we are met by the difficulty that the quantity z_m depends on either q or q_1, which, in their turn, depend on the moduli k' and k, and the latter, as (6a) shows us, involve the very quantity z_m which we wish to calculate. Since, however, the variations of q and q_1 with small changes of z_m are not important, we may solve our equations by successive approximation, and fortunately, if we know at the start, the *approximate* value of z_m, it is not a difficult matter to obtain the true value with a great deal of precision, after only a few trials. This process may be expedited by the use of suitable differential formulas, or by use of the tables of values of P and X.

1. DIFFERENTIAL FORMULAS FOR CALCULATING z_m

For small changes in z due to a given change in q, we have, in the neighborhood of $(z = z_m)$, with α constant,

$$\frac{\Delta y}{y} = \frac{\Delta z}{z} = \frac{1}{2} \frac{\Delta q}{q} (1 + 72q^2 - 296q^4 + 4608q^6 - 57320q^8 + \cdot \cdot \cdot \cdot) \tag{38}$$

as can be verified by differentiating (32).

Similarly, by differentiating (6a), and putting for q its approximate value $\frac{l}{2}$, we find the general expression (α constant)

$$\frac{\Delta q}{q} = -\frac{\Delta z}{z}\,\frac{k^4(1+10q^4)}{4\frac{a}{A}k'^{\frac{1}{2}}(1-k')}\cdot\frac{z^3}{A^3} = -\frac{\Delta z}{z}\cdot\frac{k^2(1+k')}{4\frac{a}{A}k'^{\frac{1}{2}}}\cdot\frac{z^3}{A^3}\cdot(1+10q^4) \tag{39}$$

$$= -\frac{\Delta y}{y}\cdot\frac{k^2(1+k')}{4\alpha k'^{\frac{1}{2}}}y^3(1+10q^4)$$

Differentiating (37) we may also find an expression corresponding to (38). The result is not, however, simple. Writing (37) in the form $z_m{}^3 C_1 = 32 A\alpha q_1 C_2$, we find (for constant α)

$$\frac{\Delta y}{y} = \frac{\Delta z}{z} = \frac{1}{2}\,\frac{\Delta q_1}{q_1}\left(1 - \frac{q_1}{C_1}\frac{dC_1}{dq_1} + \frac{q_1}{C_2}\frac{dC_2}{dq_1}\right) \tag{40}$$

$$\begin{aligned}\frac{dC_1}{dq_1} = &-16[1 - \frac{109}{2}q_1 + 996q_1^2 - 10892q_1^3 + 87210q_1^4 \\ &\qquad - 562308q_1^5 + \cdots\cdots] \\ &- 240q_1 \log_e \frac{1}{q_1}[1 - 24q_1 + 308q_1^2 - 2800q_1^3 + 20040q_1^4 \\ &\qquad - 119952q_1^5 + 625072q_1^6 - \cdots\cdots]\end{aligned}$$

$$\begin{aligned}\frac{dC_2}{dq_1} = &\,36[1 + \frac{8}{3}q_1 + 40q_1^2 + 54q_1^3 + 568q_1^4 + \frac{1784}{3}q_1^5 \\ &\qquad + 5152q_1^6 + \frac{13930}{3}q_1^7 + \cdots\cdots] \\ &- 12 \log_e \frac{1}{q_1}[1 + 4q_1 + 72q_1^2 + 113q_1^3 + 1320q_1^4 + 1512q_1^5 \\ &\qquad + 14112q_1^6 + 13568q_1^7 + 110376q_1^8 + \cdots\cdots]\end{aligned}$$

$$\begin{aligned}C_1 = &[1 - 16q_1 + 376q_1^2 - 4672q_1^3 + 38948q_1^4 - 252192q_1^5 \\ &\qquad + 1365888q_1^6 - \cdots\cdots] \\ &- 120q_1^2 \log_e \frac{1}{q_1}[1 - 16q_1 + 154q_1^2 - 1120q_1^3 + 6680q_1^4 \\ &\qquad - 34272q_1^5 + 156268q_1^6 - \cdots\cdots]\end{aligned} \tag{41}$$

$$\begin{aligned}C_2 = &[1 + 24q_1 + 36q_1^2 + 384q_1^3 + 402q_1^4 + 3456q_1^5 + 3064q_1^6 \\ &\qquad + 23040q_1^7 + 18351q_1^8 + \cdots\cdots] \\ &- 12q_1 \log_e \frac{1}{q_1}[1 + 2q_1 + 24q_1^2 + 28q_1^3 + 264q_1^4 + 252q_1^5 \\ &\qquad + 2016q_1^6 + 1696q_1^7 + 12264q_1^8 + \cdots\cdots]\end{aligned}$$

and the equation corresponding to (39) is (α constant)

$$\frac{\Delta q_1}{q_1}=\frac{\Delta z}{z}\cdot\frac{k^{\frac{1}{2}}}{4\frac{a}{A}(1-k)}\cdot\frac{z^2}{A^2}=\frac{\Delta z}{z}\frac{k^{\frac{1}{2}}(1+k)}{4\frac{a}{A}k'^2}\cdot\frac{z^2}{A^2}=\frac{\Delta y}{y}\frac{k^{\frac{1}{2}}(1+k)y^2}{4\alpha k'^2} \qquad (42)$$

Table 3 (appendix) gives the values of $\frac{\Delta z}{z}$ and $\frac{\Delta q}{q}$ or $\frac{\Delta q_1}{q_1}$ for a number of values of α, and will be found useful in making calculations by this method.

In making a calculation of the value of z_m, using these differential formulas, the procedure is as follows: Having obtained an approximate value of z_m by methods to be described below, the calculation of q or q_1 is made (see Example 1), and the values thus found used in (33) or (37). In general, a somewhat more accurate value of z_m will thus be obtained, which will differ from the original by a certain fractional part of the whole, and this difference is then to be taken as $\frac{\Delta z}{z}$ in formula (39) or (42), according as (33) or (37) is being used. We obtain thus the fractional change in q or q_1 corresponding to the second approximation of z_m. With this value of $\frac{\Delta q}{q}$ or $\frac{\Delta q_1}{q_1}$ we can at once compute from (38) or (40) a third approximation of z_m, and by repeating this process we rapidly approximate the true value of z_m. For this process we need calculate the coefficients in the differential formulas *once only*, since they do not need to be known with any great accuracy, and in general the values interpolated from Table 3 will be sufficient.

From an examination of this process it will be evident that each successive approximation of z_m differs from the preceding by a fractional amount which is related to the corresponding fractional difference immediately preceding by a constant factor f, which is less than unity. Or, otherwise expressed, if z_1, z_2, $\cdots\cdots z_m$ are the successive approximations and

$\frac{\Delta z}{z}=\frac{z_2-z_1}{z_1}$ then $\frac{z_m-z_1}{z_1}=\frac{\Delta z}{z}[1+f+f^2+\cdots\cdots+f_m]=\frac{\Delta z}{z}\frac{1}{1-f}$ in its limit. The factor f may be taken from Table 3.

Having obtained the final value of z_m, a check calculation should be made to see whether this value satisfies the main formula (33) or (37), which by the procedure just outlined does not therefore need to be used but twice, including the check calculation.

2. APPROXIMATE FORMULAS

Obviously the amount of time and labor necessary to obtain a precise value of z_m by the preceding formulas depends, in large measure, on the closeness with which the first approximation, which is made the basis of the calculation, approaches the true value. It is, therefore, very desirable that, before using formulas (33) or (37), a value of z_m may be found which is within, say, one part in a thousand of the truth.

The Rayleigh formula (8) or formula (9) is very satisfactory for this purpose for small values of $\frac{a}{A}$; for the important case of $\frac{a}{A}=\frac{1}{2}$, it is, however, one per cent in error, and for still larger values of $\frac{a}{A}$ its insufficiency becomes such as to render its use entirely unsatisfactory.

For values of $\frac{a}{A}$ for which a is nearly equal to A, the following development of (37) is of assistance.

Putting $\frac{a}{A}=1-\delta$, $\frac{z}{A}=y$, we find from (6a) the approximation (holding for small values of δ)

$$q_1=\frac{(\delta^2+y^2)}{64}\left[1+\delta+\tfrac{1}{8}(\delta^2+y^2)\right] \tag{43}$$

and (37) may be written with sufficient accuracy

$$y_m^2=\left(\frac{z_m}{A}\right)^2=32\,\frac{a}{A}\,q_1\,\frac{\left[1+24q_1-12q_1\log_e\frac{1}{q_1}\right]}{1-16q_1} \tag{44}$$

To obtain a starting point for the use of (43) and (44), we neglect for the moment the last term in (43). The main term in (44) then becomes $y_m^2=32\,(1-\delta)\frac{(\delta^2+y_m^2)}{64}(1+\delta)$, whence

$$y_m=\delta\sqrt{\frac{1-\delta^2}{1+\delta^2}} \tag{45}$$

Starting with this value in (43), we calculate a value of q_1 to be used in (44), and thus a second approximation of y_m, and so on. Very little work is necessary to obtain a very satisfactory approximation to y_m, since the correction term in (44) need be calculated but once, and a very moderate accuracy in the value of $\log_e \frac{1}{q_1}$ will suffice.

The range of usefulness of (43) and (44) is not, however, great, and does not include those cases which are most important in practice. For the especially interesting case, where $\frac{a}{A}$ is neither small nor near to unity, the following empirical method has proved very useful. Having, so far as the author has been able to determine, no simple physical significance, it is to be regarded as merely a useful coincidence.

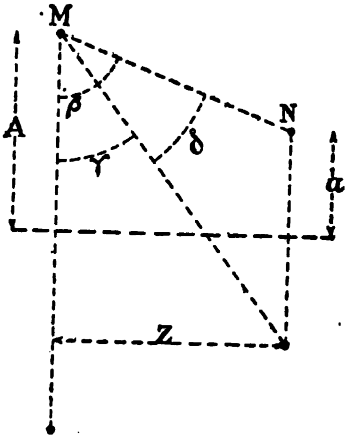


FIG. 1.—*Definition of the angle δ*

If in Fig. 1 we pass a plane through the common axis of the two circles, and calculate the angle δ, subtended by that diameter NP of the small circle which lies in this plane (as viewed from the point M of the larger circle, lying in the same plane), it will be found that the critical distance z_m has such a value as to make the angle $\psi = \frac{A}{a}\,\delta$ roughly the same, whatever the value of $\frac{a}{A}$.

This angle ψ has a value of 45° for $\alpha = \frac{a}{A} = 1$, increases to a value of about 46° 30′ for $\alpha = 0.7$, and falls with decreasing values of α to about 45° 50′ for $\alpha = 0$.

To aid in finding this angle for any given value of $\frac{a}{A}$, Table 4 will be useful. In this the angle ψ has been calculated for a number of values of $\frac{a}{A} = \alpha$, for which the distance z_m was calculated to within a part in a million by the methods already described. To aid in interpolation the curve, Fig. 2, connecting the values in this table, has been plotted.

Having taken from the curve, or by interpolation from the table the value of the angle ψ for any other value of α not included in the table, we derive the approximate value of $y_m = \frac{z_m}{A}$ desired, by means of the following equations.

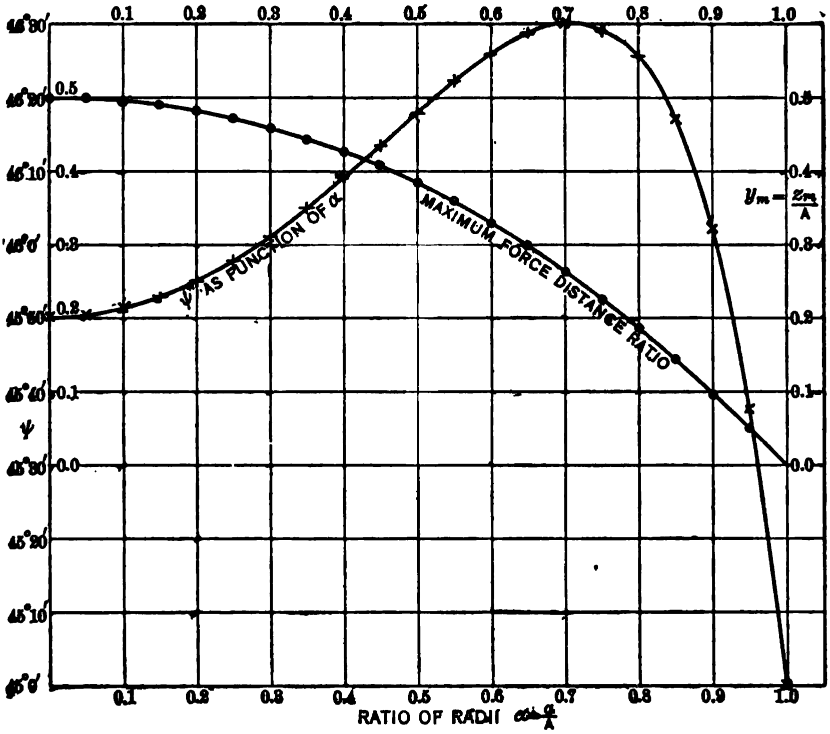


FIG. 2.—*Giving the angle ψ as a function of the ratio of the radii. Useful in obtaining a first approximation to y_m*

Putting

$$\alpha = \frac{a}{A}, \psi = \frac{\delta}{\alpha}, M = \tan \delta, \frac{z}{A} = y$$

$$M = \tan (\beta - \gamma) = \frac{2\,\alpha y}{(1-\alpha^2)+y^2} \tag{46}$$

$$\therefore y_m = \frac{\alpha}{M} - \sqrt{\frac{\alpha^2}{M^2} - (1-\alpha^2)} = \frac{\alpha}{M}\left[1 - \sqrt{1 - \frac{M^2}{\alpha^2}(1-\alpha^2)}\right]$$

the angles β and γ being those shown in Fig. 1.

A study of the equation

$$\frac{\Delta y}{y}=\frac{\Delta M}{M}\left[\frac{1}{\sqrt{1-\frac{M^2}{\alpha^2}(1-\alpha^2)}}\right] \qquad (y=y_m)$$

shows, that in the most unfavorable case, to obtain y_m correct to one part in 1,000, the angle ψ taken from the curve must be correct to within about 30″. This accuracy can be obtained from the curve without serious difficulty, even in the worst case, and in general this accuracy can be exceeded.

This method of obtaining a first approximation to y_m is illustrated in examples 4 and 5 below.

3. THE CALCULATION OF THE MAXIMUM FORCE

The distance z_m between the planes of the circular currents, corresponding to the condition that the force between them is a maximum, having been calculated by the formulas already given, the value of the maximum force itself may be computed by substituting the value of z_m in formula 5 or 6. The former will be used principally for values of α (the ratio of the radii) lying between zero and 0.5. The latter holds for circles of more nearly equal radii, although the two formulas overlap somewhat in their ranges.

Formula (34), which was derived by substituting in (5) the value of y_m given by (33), is useful for values of α to which (5) applies. In (34) the quantity q_0 is that value of q which corresponds to the distance z_m. It must be emphasized, however, that (34) is not a *general* formula for the force, as are (5) and (6).

The method of calculation of the maximum force from these three formulas is illustrated in examples 10 and 11, such a value of α having been chosen that all three formulas are applicable to the same case, thus giving a check on the accuracy of the three formulas.

Table 4 gives the values of y_m and the maximum force F_m for values of α from 0.05 to 0.95 in steps of 0.05. In computing these values, seven place logarithms were used so that the values given should not be in error by more than a part in a million.

Since, in a practical case, the coils would be designed so that their radii should approximate some round value, the data in Table 4 may be regarded as fundamental, and the usefulness of this table is further enhanced by the fact that the values of y_m and F_m for values of α differing slightly from these fundamental values, may be obtained very simply from the data in the table without recourse to the main formulas (33) and (37).

For example, if a pair of coils, constructed to have radii in the ratio of 1 to 2, are found, as a result of measurement, to have an actual ratio of the radii greater than one-half by 0.005, the maximum force for this pair of coils can be found by applying to the value given in Table 4 for $\alpha = 0.5$, a factor, which is a function both of the difference 0.005 and of α.

To calculate this factor it is necessary to determine what relation exists between a small change in α and the changes in y_m and F_m corresponding. This matter is taken up in the next section.

VI. VARIATION FORMULAS FOR THE MAXIMUM FORCE F_m AND THE MAXIMUM DISTANCE RATIO y_m

To determine the change in y_m or F_m for a small fractional change $\frac{\Delta\alpha}{\alpha}$ in the ratio of the radii, we differentiate equations (33a) and (37a).

Equation (33a) gives

$$\frac{\Delta y_m}{y_m} = \frac{1}{2}\frac{\Delta\alpha}{\alpha} - \frac{1}{2}\frac{\Delta q}{q} + \frac{\Delta P}{P} \tag{46}$$

Writing

$$\Delta q = \frac{\partial q}{\partial\alpha}\Delta\alpha + \frac{\partial q}{\partial y_m}\Delta y_m$$

$$\frac{\Delta P}{P} = \frac{1}{P}\frac{\partial P}{\partial q}\Delta q, \quad \frac{\partial q}{\partial\alpha} = \frac{\partial q}{\partial k'}\cdot\frac{\partial k'}{\partial\alpha}, \quad \frac{\partial q}{\partial y_m} = \frac{\partial q}{\partial k'}\cdot\frac{\partial k'}{\partial y_m}$$

and using (6a) to evaluate the differential coefficients, we have

$$\Delta q = \frac{4K'\alpha}{Z}\left[y_m^2\cdot\frac{\Delta y_m}{y_m} - \frac{\lambda}{2}\cdot\frac{\Delta\alpha}{\alpha}\right]$$

$$\frac{1}{P}\frac{\partial P}{\partial q} = -36q\left[1 - \frac{37}{9}q^2 + 64q^4 - \frac{7165}{9}q^6 + \frac{93974}{9}q^8 - \cdots\right] \tag{47}$$

which, substitued in (46) give the relation

$$\frac{\Delta y_m}{y_m}\left[1+\frac{4K'\alpha}{Z}y_m^2\left(\frac{1}{2q}-\frac{1}{P}\frac{dP}{dq}\right)\right]=\frac{1}{2}\frac{\Delta\alpha}{\alpha}\left[1+\frac{4K'\alpha}{Z}\lambda\left(\frac{1}{2q}-\frac{1}{P}\frac{dP}{dq}\right)\right] \tag{48}$$

where

$$K'=-\frac{(1+10q^4)}{2\sqrt{k'}(1+\sqrt{k'})^2},\ \lambda=1-\alpha^2+y_m^2 \tag{49}$$

$$Z=[(1+\alpha)^2+y_m^2]^{\frac{1}{2}}\sqrt{(1-\alpha)^2+y_m^2}$$

For the change in the maximum force

$$\frac{\Delta F_m}{F_m}=\frac{1}{F_m}\cdot\frac{\partial F_m}{\partial q}\cdot\Delta q \tag{50}$$

Writing $\frac{\Delta y_m}{y_m}=\frac{H}{2}\frac{\Delta\alpha}{\alpha}$ in the equation for Δq previously derived, and differentiating (34) to obtain $\frac{1}{F_m}\frac{\partial F_m}{\partial q}$, the final expression becomes

$$\frac{\Delta F_m}{F_m}=\frac{4K'\alpha}{Zq}(y_m^2H-\lambda)\ (1+2q^2+124q^4-1132q^6+14360q^8-\cdots)\cdot\frac{\Delta\alpha}{\alpha} \tag{51}$$

in which the factor H is to be calculated by formula (48).

The expressions (48) and (51) are to be used for values of α up to about 0.5. To cover the range of larger values, we start from (37a), which gives on differentiation

$$\frac{\Delta y_m}{y_m}=\frac{1}{2}\left[\frac{\Delta C_2}{C_2}-\frac{\Delta C_1}{C_1}+\frac{\Delta\alpha}{\alpha}+\frac{\Delta q_1}{q_1}\right] \tag{52}$$

Writing

$$\Delta q_1=\frac{\partial q_1}{\partial k}\cdot\frac{\partial k}{\partial\alpha}\cdot\Delta\alpha+\frac{\partial q_1}{\partial k}\cdot\frac{\partial k}{\partial y_m}\cdot\Delta y_m$$

$$K=-\frac{1}{2\sqrt{k}(1+\sqrt{k})^2},\ N=\frac{\Delta C_1}{C_1}=\frac{1}{C_1}\cdot\frac{\partial C_1}{\partial q_1}\cdot\Delta q_1 \tag{53}$$

$$Y=[(1+\alpha)^2+y_m^2]^{\frac{3}{2}},\ M=\frac{\Delta C_2}{C_2}=\frac{1}{C_2}\cdot\frac{\partial C_2}{\partial q_1}\cdot\Delta q_1$$

and using (6a) to carry out the differentiations indicated in (53),

$$\Delta q_1 = \frac{K\sqrt{\alpha}}{Y}\left[\lambda\frac{\Delta\alpha}{\alpha} - 2y_m^2 \cdot \frac{\Delta y_m}{y_m}\right] \tag{54}$$

from whence we find immediately

$$\frac{\Delta y_m}{y_m}\left[1 + \frac{K\sqrt{\alpha}}{Y}y_m^2\left(M - N + \frac{1}{q_1}\right)\right] = \frac{1}{2}\frac{\Delta\alpha}{\alpha}\left[1 + \frac{K\sqrt{\alpha}}{Y}\lambda\left(M - N + \frac{1}{q_1}\right)\right] \tag{55}$$

in which the quantities M and N are to be calculated from (53) and (41).

Equation (6) for the maximum force may be written $F_m = \frac{\pi y_m \Phi}{16 q_1 \sqrt{\alpha}}$ which gives on differentiation

$$\frac{\Delta F_m}{F_m} = \frac{\Delta y_m}{y_m} - \frac{1}{2}\frac{\Delta\alpha}{\alpha} - \frac{\Delta q_1}{q_1} + \frac{\Delta\Phi}{\Phi} \tag{56}$$

Putting

$$\frac{\Delta\Phi}{\Phi} = \frac{1}{\Phi}\frac{d\Phi}{dq_1} \cdot \Delta q_1 = \Pi\, \Delta q_1 \tag{57}$$

and writing as before, $\frac{\Delta y_m}{y_m} = \frac{H}{2} \cdot \frac{\Delta\alpha}{\alpha}$, equations (56) and (57) become

$$\begin{aligned}\frac{\Delta F_m}{F_m} &= \frac{1}{2}(H - 1)\frac{\Delta\alpha}{\alpha} + \left(\Pi - \frac{1}{q_1}\right)\Delta q_1 \\ &= \frac{\Delta\alpha}{\alpha}\left[\frac{1}{2}(H - 1) + \left(\Pi - \frac{1}{q_1}\right)\frac{K\sqrt{\alpha}}{Y}(\lambda - H y_m^2)\right]\end{aligned} \tag{58}$$

To calculate Π we have

$$\begin{aligned}\frac{d\Phi}{dq_1} &= 24(1 - 21q_1 + 184q_1^2 - 1089q_1^3 + 5160q_1^4 - \cdots) \\ &\quad - 12 \log_e \frac{1}{q_1} \cdot (1 - 20q_1 + 180q_1^2 - 1200q_1^3 + 6500q_1^4 - \cdots) \\ \Phi &= (1 + 12q_1 - 192q_1^2 + 1232q_1^3 - 5634q_1^4 + \cdots) \\ &\quad - 12q_1 \log_e \frac{1}{q_1} \cdot (1 - 10q_1 + 60q_1^2 - 300q_1^3 + 1300q_1^4 - \cdots)\end{aligned} \tag{59}$$

These formulas, although somewhat complicated in appearance, are not difficult to use, especially since most of the quantities involved enter also in the main formulas.

The factors η and ϵ, defined by the equations

$$\frac{\Delta y_m}{y_m} = \eta \frac{\Delta \alpha}{\alpha} \qquad (60)$$

$$\frac{\Delta F_m}{F_m} = \epsilon \frac{\Delta \alpha}{\alpha}$$

have been calculated by means of the equations just developed, for the same values of α as in Table 4, and are given in Table 5. Values of η and ϵ for other values of α may be derived by interpolation. From the method of derivation of the formulas for η and ϵ, it is evident that these values hold only for very small values of $\frac{\Delta \alpha}{\alpha}$. For differences, $\frac{\Delta \alpha}{\alpha}$, greater than about 0.001, the values taken from the table must be corrected by a factor whose value is determined from the following considerations.

Table 4 gives the value F_0 of the maximum force for a certain value α_0 of the ratio of the radii. It is desired to find the value of the maximum force $F_1 = F_0\left(1 + \frac{\Delta F}{F_0}\right)$ corresponding to a slightly different ratio of the radii α_1, connected with the former ratio by the relation $\alpha_1 = \alpha_0\left(1 + \frac{\Delta \alpha}{\alpha_0}\right)$.

For an infinitely small increment of α_0, $d\alpha$, the fractional change in the value of the maximum force is $\frac{dF}{F_0} = \epsilon_0 \frac{d\alpha}{\alpha_0}$, the value of ϵ_0 being that taken from Table 5 for the argument α_0. Similarly, in the neighborhood of α_1, $\frac{dF}{F_1} = \epsilon_1 \frac{d\alpha}{\alpha_1}$, ϵ_1 being obtained from Table 5 by direct interpolation, using the argument α_1. For the *finite* difference $\Delta \alpha = \alpha_1 - \alpha_0$, the corresponding difference ΔF is

$$\frac{\Delta F}{F_0} = \frac{1}{F_0}\int_{\alpha=\alpha_0}^{\alpha=\alpha_1} dF = \frac{1}{\alpha_0}\int_{\alpha_0}^{\alpha_1} \epsilon \, d\alpha$$

where the coefficient ϵ is a function of α. We may approximate the value of this integral closely, if we use the average value of ϵ over the interval $\Delta\alpha$. This average value is, however, not the simple mean of ϵ_0 and ϵ_1, since $\epsilon_1 = \frac{\alpha_1}{F_1}\frac{dF}{d\alpha}$ and that portion of the integral contributed by the element $d\alpha$ in the neighborhood of α_1 is not $\epsilon_1 \frac{d\alpha}{\alpha_1}$, but $\epsilon_1 \frac{\alpha_0}{\alpha_1}\frac{F_1}{F_0}$ because in the original statement of the problem, fractional increments of F_m and α are to be referred to F_0 and α_0, the fundamental values given in Table 4.

Taking these facts into account in finding the average value of ϵ, we find as an approximation to the integral the expression

$$\frac{\Delta F}{F_0} = \frac{1}{2}\frac{\Delta\alpha}{\alpha_0}\left[\epsilon_0 + \frac{F_1}{F_0}\cdot\frac{\alpha_0}{\alpha_1}\cdot\epsilon_1\right] \tag{61}$$

which is a very close approximation to the truth for values of $\frac{\Delta\alpha}{\alpha_0}$ as great as 0.01, over the range of values of α for which interpolation in Table 5 may be accurately made.

Entirely similar considerations enable to be found for the change in y_m corresponding to a small change in α, the equation

$$\frac{\Delta y}{y_0} = \frac{1}{2}\frac{\Delta\alpha}{\alpha_0}\left[\eta_0 + \frac{y_1}{y_0}\cdot\frac{\alpha_0}{\alpha_1}\cdot\eta_1\right] \tag{62}$$

These formulas are illustrated in examples 13 and 14, where the constants are calculated for the coils of the Bureau of Standards current balance, and the results compared with the values calculated by the methods described in the original article.

VII. VARIATION OF THE FORCE FOR SMALL DISPLACEMENTS OF THE COILS FROM THE POSITION OF MAXIMUM FORCE

From Maxwell's formula (3) it is evident that the force corresponding to positions of the coils in the neighborhood of the critical distance bears to the maximum value of the force a relation which may be expressed by the power series

$$F = F_m[1 - \gamma(z_m - z)^2 + \delta(z_m - z)^3 - \cdot\cdot\quad] \tag{63}$$

in which z is the actual distance between the planes of the coils, z_m the distance for which the force has its maximum value F_m, and γ and δ are constants. The term in the first power of the displacement $(z_m - z)$ is lacking.

Remembering that the force is a function of the ratio of the radii alone, we may write

$$\frac{\Delta F}{F_m} = -c\left(\frac{\Delta y}{y_m}\right)^2 + d\left(\frac{\Delta y}{y_m}\right)^3 - \cdots \qquad (64)$$

where c and d are positive constants depending on α alone. This formula holds with a good deal of accuracy for values of $\frac{\Delta y}{y_m}$ as great as 0.04.

The following table will allow of the interpolation of the value of the constant c with an accuracy sufficient in practically all cases:

Values of the Constant c as a Function of α

α	c
0.1	0.805
0.3	0.755
0.5	0.690
0.7	0.610
0.9	0.510

The constant d is sensibly equal to 0.5 for values of α ranging from 0.1 to 0.9. These values of c and d were found by actually calculating the force, by the methods already developed, for various values of α, and for differences $\frac{\Delta y}{y_m}$ of ± 0.01 and ± 0.04. The value of $\frac{\Delta F}{F_m}$ corresponding to a negative displacement $\frac{-\Delta y}{y_m}$ differs from that produced by an equal positive displacement $\frac{\Delta y}{y_m}$ by an amount which depends on d alone. The average value of $\frac{\Delta F}{F_m}$ corresponding to equal positive and negative displacements $\frac{\Delta y}{y_m}$ and $\frac{-\Delta y}{y_m}$ is a measure of the value of c alone. The values of c and d here given show how slowly the force varies in the neighborhood of the position of maximum force.

VIII. BEST VALUES OF RADII OF THE COILS

The formulas and methods above developed furnish a solution of the general problem of finding the maximum force between two circular currents with any given ratio of their radii. The practically important question, What ratio of the radii is best in designing a Rayleigh current balance? may also be answered in a general way from a consideration of the numerical values in the tables here presented.

The conditions which should be fulfilled by a current balance are broadly as follows:

1. The force should be large enough to be accurately measurable.
2. The number of turns on the coils should be kept as small as possible in order to provide that the resistance of the coils may be small. Otherwise the heating of the coils may cause considerable changes in the dimensions of the coils, as well as air currents which will affect the accuracy of the weighing.
3. The cross section of the coils should be made small relatively to the distance between the windings of the fixed and movable coils.

Naturally the best design will be one of compromise, since these conditions are to some extent incompatible; and, as is usual, quite a latitude in the design is allowable without materially detracting from the performance of the balance.

Large values of α (radius of the moving coil nearly equal to that of the fixed coil) give large values of the force per turn per unit current. This advantage, however, is counterbalanced by the necessity of keeping the cross section of the coils small; for, as may be seen from Table 4, the distance z_m is, in the cases supposed, so small that very moderate values of the area of cross section will give rise to large corrections for the latter, with consequent difficulty in determining its value. Further, the change in the force for small changes in the radii and for small displacements of the coils from the position of maximum force are much larger with coils of nearly equal radii (α large) than for coils of quite different radii (α small).

These disadvantages, combined with the undesirability of using such large moving coils as postulate a long arm balance, render impracticable the employment of moving coils with radii nearly equal to the radius of the fixed coil.

The opposite case of a balance whose moving coil is small relatively to the fixed coil, although free from the difficulties of the previous case, suffers under the disadvantage of a small value of the force per unit current per turn, so that either the number of turns on the coils or else the value of the currents must be increased to compensate for this difficulty. This renders probable in the former case a troublesome correction for the area of cross section of the coils, and in the latter difficulties in the manipulation of the balance, as a result of air currents or, at least, considerable changes in the dimensions of the coils as a result of undue heating of the coils.

From these considerations one is lead to expect that a suitable design will avoid either extreme, and this is confirmed by the numerical results given in Table 4. Starting with very small values of the maximum force (per unit current per turn in each coil) for values of α about 0.1, the value of F_m increases rapidly with increase of α, so that it is not difficult to obtain a sufficiently large value of the force with a moving coil having a radius about one-half as large as that of the fixed coils, and without encountering serious difficulty from the disadvantages which beset the use of relatively small moving coils on the one hand and relatively large moving coils on the other.

These points are illustrated in the following example, in which are tabulated the principal constants of five current balances, each of which is supposed to have a fixed coil of 25 cm radius, the radii of the moving coils in the different cases being taken of various values. The nomenclature is the same as in the preceding discussion, except that there are introduced the additional quantities d_m, the distance between the windings of the coils in a radial plane and Δ the fractional change in the value of the force for a displacement of 1 mm from the position of maximum force.

$\alpha=$	0.1	0.3	0.5	0.7	0.9
a	2.5	7.5	12.5	17.5	22.5
z_m	12.39	11.47	9.59	6.61	2.446
d_m	25.68	20.92	15.75	10.00	3.50
F_m	0.1709	1.646	5.352	14.507	58.06
ϵ	2.016	2.160	2.549	3.649	9.934
Δ	53×10^{-6}	60×10^{-6}	75×10^{-6}	137×10^{-6}	868×10^{-6}

An examination of these values shows that it will not be advisable to use a value of α greater than about 0.7 nor less than about 0.3. The coils of the Bureau of Standards coils were designed to have ratios of 0.4 and 0.5.

IX. EXAMPLES

EXAMPLE 1. CALCULATION OF q AND q_1 (FORMULAE 6a)

One or both of these quantities has to be calculated when the majority of the formulas of this paper are to be used. The following arrangement of the calculations has been found convenient:

Let us obtain the values of q and q_1 corresponding to the case $\alpha=\frac{a}{A}=0.5$, $y=\frac{z}{A}=0.3875$. The formulas (6a) for k and k' may be conveniently written in the form

$$k^2=\frac{4\frac{a}{A}}{\left(1+\frac{a}{A}\right)^2+\left(\frac{z}{A}\right)^2}=\frac{4\frac{a}{A}}{4\frac{a}{A}+\left[\left(1-\frac{a}{A}\right)^2+\frac{z^2}{A^2}\right]}=\frac{4\alpha}{4\alpha+[(1-\alpha)^2+y^2]}$$

$$k'^2=\frac{\left(1-\frac{a}{A}\right)^2+\left(\frac{z}{A}\right)^2}{\left(1+\frac{a}{A}\right)^2+\left(\frac{z}{A}\right)^2}=\frac{\left(1-\frac{a}{A}\right)^2+\left(\frac{z}{A}\right)^2}{4\frac{a}{A}+\left[\left(1-\frac{a}{A}\right)^2+\frac{z^2}{A^2}\right]}=\frac{[(1-\alpha)^2+y^2]}{4\alpha+[(1-\alpha)^2+y^2]}$$

Here $4\alpha=2$, $\quad(1-\alpha)^2=0.25$

$$\begin{aligned} \log y &= \bar{1}.5882717\\ 2\log y &= \underline{\bar{1}.1765434}\\ y^2 &= 0.15015624\\ (1-\alpha)^2 &= \underline{0.25}\\ \text{Sum} &= 0.40015624 \end{aligned}$$

$$k^2=\frac{2}{2.40015624} \qquad k'^2=\frac{0.40015624}{2.40015624}$$

$\log \text{numerator} = 0.3010300$
$\log \text{denominator} = \underline{0.3802395}$
$\log k^2 = \bar{1}.9207905$
$\log M = \underline{0.8788851}$
$\text{Diff.} = \quad \log \frac{l}{2} = \bar{1}.0419054$

$5 \log \frac{l}{2} = \bar{5}.20952$
$\log 2 = \underline{0.30103}$
$\text{Sum} = \bar{5}.51055$
$2\left(\frac{l}{2}\right)^5 = 0.00003240$
$\frac{l}{2} = \underline{0.11012995}$
$q = 0.11016235$
$\log q = \bar{1}.0420332$

$\log \text{numerator} = \bar{1}.6022296$
$\log \text{denominator} = \underline{0.3802395}$
$\log k'^2 = \bar{1}.2219901$
$\log k' = \bar{1}.6109950$
$\log \sqrt{k'} = \underline{\bar{1}.8054975}$

$(1 + \sqrt{k'}) = 1.6389950$
$(1 + k') = 1.4083147$
$\log (1 + \sqrt{k'}) = 0.2145777$
$\log (1 + k') = 0.1486997$

$\log 2 = \underline{0.3010300}$
$\log 2(1 + k') = 0.4497297$
$2 \log (1 + \sqrt{k'}) = \underline{0.4291554}$
$\log M = \text{Sum} = 0.8788851$

Check.

$\sqrt{k'} = 0.6389950$
$1 - \sqrt{k'} = 0.3610050$
$2(1 + \sqrt{k'}) = 3.2779900$

$\log \; (1 - \sqrt{k'}) = \bar{1}.5575132$
$\log 2(1 + \sqrt{k'}) = \underline{0.5156076}$
$\text{Diff.} = \log \frac{l}{2} = \bar{1}.0419056$

In the above calculation q has been obtained by the use of the second of the formulas under (6a). The first relation, that involving $1 - \sqrt{k'}$, may, when $\sqrt{k'}$ is nearly equal to unity, give an accuracy much inferior to that obtained by the other formula. It is, however, useful as a check, and is so used in the calculation just given.

The procedure to be followed in calculating q_1 is, in every respect, similar to that for q, except that k takes the place of k' throughout, and that the difference between q_1 and $\frac{l_1}{2}$ is, on account of the smallness of q_1, generally negligible.

$$\begin{aligned}
\log k^2 &= \bar{1}.9207905\\
\log k &= \bar{1}.9603952\\
\log \sqrt{k} &= \underline{\bar{1}.9801976}\\
1+\sqrt{k} &= 1.9554272\\
1+k &= 1.9128411\\
\log(1+\sqrt{k}) &= 0.2912416\\
\log(1+k) &= 0.2816789\\
\log 2 &= \underline{0.3010300}\\
\log 2\,(1+k) &= 0.5827089\\
2\log(1+\sqrt{k}) &= \underline{0.5824832}\\
\log M = \text{Sum} &= 1.1651921
\end{aligned}$$

$$\begin{aligned}
\log k'^2 &= \bar{1}.2219901\\
\log M &= \underline{1.1651921}\\
\log \frac{l_1}{2} = \log q &= \bar{2}.0567980\\
\text{since } 5 \log \frac{l_1}{2} &= \overline{10}.28399\\
q_1 &= 0.011397195
\end{aligned}$$

Check.

$$\begin{aligned}
\sqrt{k} &= 0.9554272\\
1-\sqrt{k} &= 0.0445728\\
2(1+\sqrt{k}) &= \underline{3.9108544}\\
\log(1-\sqrt{k}) &= \bar{2}.6490699\\
\log 2(1+\sqrt{k}) &= \underline{0.5922716}\\
\text{Sum} = \log \frac{l_1}{2} &= \bar{2}.0567983
\end{aligned}$$

EXAMPLE 2.—ILLUSTRATING THE CALCULATION OF $y_m = \frac{z_m}{A}$ BY (33), STARTING FROM THE APPROXIMATION BY RAYLEIGH'S FORMULA (8)

For the case $\alpha = \frac{a}{A} = 0.5$ we obtain as a first approximation $y = 0.3875$. The value of q corresponding to this value of y has been calculated in the previous example.

From Table 1 we find by interpolation that the value of P corresponding to $q = 0.110016235$ is $P = 0.807662$. The calculation of a second approximation to y by equation (33) is as follows:

$$\begin{aligned}
\log q &= \bar{1}.0420332\\
\log 20 &= \underline{1.3010300}\\
\text{Sum} &= 0.3430632\\
\log \alpha &= \underline{\bar{1}.6989700}\\
\log \frac{\alpha}{20q} &= \bar{1}.3559068
\end{aligned}$$

$$\begin{aligned}
\tfrac{1}{2} \log \frac{\alpha}{20q} &= \bar{1}.6779534\\
\log P &= \underline{\bar{1}.9072297}\\
\text{Sum} = \log y &= \bar{1}.5851831\\
y &= 0.384754 = \\
&\text{second approximation}
\end{aligned}$$

The calculation may now be repeated, using this second approximation for y_m to find more accurate values of k, k', and q, and thence a third approximation to y_m, and so on till successive approximations do not differ sensibly from the preceding. The successive approximations in the present case are, to six places of logarithms,

Approximation	log y
1	$\bar{1}.588272$
2	$\bar{1}.585183$
3	$\bar{1}.584228$
4	$\bar{1}.583933$
5	$\bar{1}.583843$
6	$\bar{1}.583816$
7	$\bar{1}.583807$
8	$\bar{1}.583804$
9	$\bar{1}.583803$

The necessary labor may, however, be very much reduced by the use of the differential formulas (38) and (39). Using in (39) the second approximation of y_m already found, and the values of k, k', and q already calculated above, there results

$$\frac{\Delta q}{q} = -0.3330\frac{\Delta z}{z} \quad , \quad \frac{\Delta z}{z} = -0.9215\frac{\Delta q}{q}$$

Since we are concerned with fairly small differences, we may, without serious error, use in place of $\frac{\Delta q}{q}$ and $\frac{\Delta z}{z}$ the differences in the logarithms of the different values of q and z or y. The difference of the logarithms of the first and second approximation to y_m is 0.0030866. The formulas (38) and (39) show that the logarithm of the third approximation to y_m will differ from that of the second approximation by 0.3330 of 0.9215 of 0.0030886, and the difference between the logarithms of the fourth and third approximations will be less than this value in the ratio of the same factor, and so on for each successive approximation. Denoting this factor by f, we may find at once the factor F by which we must multiply the difference between the first and second approximations, in order to obtain the amount by which the final value differs from the first approximation. It is $F = 1 + f + f^2 + f^3 + \ldots + f^n$, which gives, when n is taken indefinitely great, the value $F = \frac{1}{1-f}$. Using here the value $f = 0.30685$, and the difference 0.0030866, the final difference 0.0044559 is found, which subtracted from the logarithm of the first approximation gives log $y_m = \bar{1}.5838158$. This value differs slightly from the true value, for the reason that the data used in calculating the factor f does not quite apply to such a large interval as that here in question. We may, however, use this value as the starting point in obtaining the true value. Using the value $\bar{1}.5838158$ as a first approximation, formula (33) gives as a second approximation $\bar{1}.5838076$ (a difference in the logarithm of -82 in the seventh place), which, using the same value of the factor f as before, indicates that the correction to be applied is -118 in the seventh place, and that the final value is $\bar{1}.5838040$. Substituting this value in formula (33) the result obtained does not differ from this by as much as a unit in the seventh place, showing that equation (33) is satisfied and that the correct value has been found.

This method was used in obtaining the data in Table 4. In most cases, however, the first approximation used did not differ by more than one or two parts in a thousand from the true value, and for such cases a single application of formula (33) was necessary, excepting as a check on the results by the differential formula. To aid in further calculations, the data of Table 3 have been calculated, and by interpolation from this table it should be possible to derive the proper value of f without recourse to formulas (38) and (39).

EXAMPLE 3.—CALCULATION OF y_m BY FORMULA (37)

To illustrate the use of formula (37) we may take the same case as that in the preceding example, but will start from the final value found in the latter, and show that it is consistent with (37). The mode of procedure is, however, the same, in case only a first approximation is available, except that the factor f is to be calculated from the differential formulas (40) and (42).

Starting from the value log $y = \bar{1}.5838040$, we find for q_1 the value 0.01131756, to which corresponds the value $X = 0.812335$, which is obtained by interpolation in Table 2. Direct calculation of X from equation (37) gives the value 0.8123349 (see example 7).

The calculation of y_m by formula (37) is then as follows:

$$
\begin{array}{rl}
\log 32 = & 1.5051500 \\
\text{"} \quad q_1 = & \bar{2}.0537527 \\
\text{"} \quad \alpha = & \bar{1}.6989700 \\
\text{"} \quad X = & \underline{\bar{1}.9097351} \\
2 \log y_m = & \bar{1}.1676078 \\
\log y_m = & \bar{1}.5838039 \qquad y_m = 0.3835340
\end{array}
$$

The value of the logarithm of y_m just found agrees as closely with that found by (33) as the accuracy of the logarithms will allow. The results by the two formulas agree within, at most, one part in four million. This case must be regarded as giving a very searching test of the accuracy of the coefficients in the two formulas, since terms of the eighth order have to be taken into account in each of the formulas.

EXAMPLE 4.—ILLUSTRATION OF THE USE OF CURVE (FIG. 2) FOR OBTAINING A FIRST APPROXIMATION

Let us take the value $\alpha = 0.45$. For this value of α the curve, Fig. 2, gives

$$
\begin{array}{l}
\psi = \dfrac{\delta}{\alpha} = 46^\circ\ 13'.6 \text{ or } 46^\circ\ 13'\ 36'' \\
\delta = 46^\circ\ 13'\ 36'' \times 0.45 = 20^\circ\ 48'\ 7''.2
\end{array}
$$

$$
\begin{array}{rll}
\log \alpha = \log 0.45 & = \bar{1}.6532125 & 1 - \alpha^2 = 1 - 0.2025 \\
\text{" } M = \text{" } \tan \delta & = \bar{1}.5796743 & \qquad = 0.7975 \\
\text{" } \dfrac{\alpha}{M} & = 0.0735382 & \\
2 \text{ " } \quad \text{"} & = 0.1470764 & \\
\text{"} (1 - \alpha^2) & = \bar{1}.9017307 & \\
\therefore \text{"} \dfrac{M^2}{\alpha^2}(1 - \alpha^2) & = \bar{1}.7546543 & \dfrac{M^2}{\alpha^2}(1 - \alpha^2) = 0.5684003 \\
\log \left[1 - \dfrac{M^2}{\alpha^2}(1 - \alpha^2) \right] & = \bar{1}.6350811 & \\
& & 1 - \dfrac{M^2}{\alpha^2}(1 - \alpha^2) = 0.4315997 \\
\tfrac{1}{2} \text{ " } \quad \text{"} & = \underline{\bar{1}.8175406} & \\
& & \sqrt{1 - \dfrac{M^2}{\alpha^2}(1 - \alpha^2)} = 0.6569626
\end{array}
$$

$$\log\left[1-\sqrt{1-\frac{M^2}{\alpha^2}(1-\alpha^2)}\right]=\bar{1}.5353416$$

$$1-\sqrt{1-\frac{M^2}{\alpha^2}(1-\alpha^2)}=0.3430374$$

$$\log\frac{\alpha}{M}=\underline{0.0735382}$$

$$\text{sum}=\text{“}\quad y_m=\bar{1}.6088798=\log\text{ first approximation.}$$

Carrying out the calculation by means of (33) we find log $q=\bar{2}.9887127$, log $P=\bar{1}.9271036$ (Table 1) and, as a second approximation log $y_m=\bar{1}.6088385$, which is a difference in the logarithm of -413 in the seventh place. The factor f for this case is 0.2874 (Table 3), which gives a final difference of -580. Subtracting this from the logarithm of the first approximation, we have the final value log $y_m=\bar{1}.6088218$. Using this value of log y_m in (33) the value found is log $y_m=\bar{1}.6088217$, which is sensibly the same, showing that the correct value is, in this case, found with only two applications of the main formula, and of these the last is necessary as a check only.

The value of the angle ψ corresponding to the correct value of y_m is $46^\circ\ 13'\ 22''.62$, which is only 0.2 of a minute less than the value taken from the curve. This corresponds to about 1 part in 10 000 difference between the correct value and the first approximation.

EXAMPLE 5.—FORMULA (37), USED IN CONNECTION WITH A FIRST APPROXIMATION TAKEN FROM THE CURVE (FIG. 2)

For $\alpha=0.65$ the curve (Fig. 2) gives $\psi=46^\circ\ 28'.5$, and a calculation exactly like that in the preceding example yields, as a first approximation, log $y_m=\bar{1}.4750232$. Substituting this value in (37), we find log $q_1=\bar{3}.6893759$, or $q_1=0.004890755$, to which corresponds log $X=\bar{1}.9426711$ (Table 2), and the second approximation log $y_m=\bar{1}.4750552$. The logarithms of the first and second approximations here differ by 320 in the seventh place. For this case Table 3 gives $f=0.3725$, and the final correction is 1.594 times 320, or 510, which, added to the first approximation, yields the value log $y_m=\bar{1}.4750742$. A check calculation using this value in (37) gives a value of the logarithm greater by one unit in the seventh place. The true value of ψ is about 0.15 of a minute larger than the value taken from the curve.

EXAMPLE 6.—CALCULATION OF THE FACTOR P IN EQUATION (33a)

In the preceding examples the factor P in equation (33a) has been obtained from Table 1 (see Appendix) by interpolation, using second and third differences. For this interpolation we use the formula

$$P = P_0 + h\Delta_1 + \frac{h(h-1)}{\lfloor 2}\Delta_2 + \frac{h(h-1)(h-2)}{\lfloor 3}\Delta_3 + \ldots .$$

where P_0 is the value of P corresponding to the nearest tabular value q_0 of q, lower than that for which P is desired, h is the fraction of the tabular interval by which the actual value of q exceeds q_0, and Δ_1, Δ_2, Δ_3 are the first, second, and third differences, respectively, given in the table.

Assuming $q = 0.11016235$, the value in example 1, we find

$$\begin{array}{rcrl} P_0 & = & 0.808160 & \Delta_1 = -3074.5 \quad \Delta_2 = -14 \\ h\Delta_1 & = & -\ 499 & h = 0.16235 \\ \frac{h(h-1)}{2}\Delta_2 & = & +\ \ 1 & \\ \hline P & = & .807662 & \end{array}$$

In the relatively rare cases where the accuracy of the values of P, derived by interpolation from the Table 1, is not sufficient, we have to calculate the values of the terms of the series in (33) as was done in forming Table 1, retaining enough terms in the series and enough figures in the values of these terms to give the accuracy desired.

Thus for the value of q above, we have $\log q = \bar{1}.0420332$, and the calculation of the terms may be arranged as below. In the first column are given the logarithms of the successive powers of q required in (33). In the second column appear the logarithms of the terms of the series. These are obtained with little trouble if the logarithms of the constant coefficients in the series are written along the edge of a piece of paper, which may be placed under the numbers of the first column in such a way that the logarithm of any required coefficient may be brought directly under the logarithm of the corresponding power of q, and thus the addition of logarithms necessary may be readily carried out. In the third column are grouped the values of the terms of the series, the positive terms being grouped together first and under them the negative terms. The value of P found shows that its value as interpolated from Table 1 is accurate to the number of places given in the table.

Logarithms of powers of q	Logarithms of terms of series	Terms of series
$\bar{1}.0420332$		
$\bar{2}.0840664$	$\bar{1}.3393389n$	1.0293080
$\bar{4}.1681328$	$\bar{2}.4669854$	.0004674
$\bar{6}.2521992$	$\bar{3}.5579804n$	.0000090
$\bar{8}.3362656$	$\bar{4}.6696524$	.0000002
$\overline{10}.4203320$	$\bar{5}.8050832n$	1.0297846 = Sum of positive terms.
$\overline{12}.5043984$	$\bar{6}.956158$	−0.2184434
$\overline{14}.5884648$	$\bar{6}.1116810n$	.0036139
$\overline{16}.6725312$	$\bar{7}.283643$	.0000638
		.0000013
		0.2221224 = Sum of negative terms.
		P = 0.8076622

EXAMPLE 7.—CALCULATION OF THE FACTOR X IN EQUATIONS (37) AND (37a)

To illustrate the use of these formulas let us take the case assumed in example 3, where we found $\log q_1 = \bar{2}.0537527$.

The formula requires that the quantity $\log_e \frac{1}{q_1}$ shall be calculated. This may be quickly and accurately obtained by first writing $\log_{10} \frac{1}{q_1}$, and then performing the multiplication by the modulus ($\log_e 10 = 2.3025851$) by the aid of the special table provided for this purpose in collections of six-place and seven-place logarithms. The table referred to contains all the multiples of the modulus up 100 times this quantity. By breaking up the desired logarithm into parts of two figures each the successive partial products may be written down directly from the table, the decimal point being properly placed in each case, and the sum of these products is the result desired.

Thus in the present case

$$\log_{10} q_1 = \bar{2}.0537527$$

$$\log_{10} \frac{1}{q_1} = 1.9462473$$

$$\begin{array}{rl} \text{Tabular value for } 1.9 & = 4.37491168 \\ \text{`` `` `` } .046 & = 0.10591891 \\ .00024 & = 0.00055262 \\ .0000073 & = \underline{0.00001681} \\ \text{Sum} & = 4.4814000 \quad = \log_e \frac{1}{q_1} \end{array}$$

$$\log_{10} \log_e \frac{1}{q_1} = 0.6514137$$

$$\log_{10} \left[12 q_1 \log_e \frac{1}{q_1} \right] = 1.7843476$$

$$\log_{10} \left[120 q_1^2 \log_e \frac{1}{q_1} \right] = 2.8381004$$

The calculation of the series Q, R, S, and T in equation (37) may be carried out by the same method as that used for the series in (33) in the preceding example. The logarithms of the ascending powers of q_1 enter into each of these series. Seven terms are necessary in order to obtain Q to seven places, six terms for R, and only five each for S and T.

Thus we find

$$Q = 0.8609019 \qquad S = 1.2767962$$

$$R = 0.8371244 \qquad T = 1.0257542$$

$$120 q_1^2 R \log_e \frac{1}{q_1} = \underline{0.0576621} \qquad 12\, q_1\, T \log_e \frac{1}{q_1} = \underline{0.6242965}$$

$$C_1 = 0.8032398 \qquad C_2 = 0.6524997$$

$$\log C_2 = \bar{1}.8145803$$

$$\text{``}\ C_1 = \underline{\bar{1}.9048452}$$

$$\text{``}\ X = \bar{1}.9097351$$

$$X = 0.8123349$$

The value of X found by interpolation from Table 2, using the interpolation formula given in example 6 is $X = 0.812335$. The amount of labor in a direct calculation of X may be materially reduced if, in the computation of the smaller terms of the series, five-place logarithms are used.

EXAMPLE 8.—USE OF EQUATIONS (38) AND (39)

Table 3, which is based on these equations, will suffice in the usual case, the required values of f being found by interpolation. In cases where it may be necessary to calculate f directly, the terms of the series in (38) may be obtained by the method of example 6.

For the case $\alpha=0.5$ the terms in (38) are 1.87982, 0.00841, and 0.00020, while the negative terms are 0.04420, and 0.00125, giving the result $f_2=0.9215$, for the factor in (38).

For the use of (39) the results of the main calculation by (33) will usually be available. In the present case

$$\begin{aligned} \log 10\,q^4 &= \bar{3}.174128 \\ (1+10\,q^4) &= 1.001493 \\ \hline \log(1+10q^4) &= 0.000648 \\ \log y_m^2 &= \bar{1}.167608 \\ ``\ k^2 &= \bar{1}.921344 \\ ``\ (1+k') &= 0.148298 \\ \hline \text{Sum} &= \bar{1}.237898 \\ &\ \bar{1}.715439 \end{aligned}$$

$$\begin{aligned} \log k' &= \bar{1}.609606 \\ \tfrac{1}{2}\ ``\ \ k' &= \bar{1}.804803 \\ \hline \tfrac{3}{2}\ ``\ \ k' &= \bar{1}.414409 = \text{Sum} \\ ``\ \ 4\alpha &= 0.301030 \\ \hline &\ \bar{1}.715439 = \text{Sum} \end{aligned}$$

$$\begin{aligned} \text{Diff.} &= \bar{1}.522459 = \log f_1, \text{ equation (39)} \\ &\ \bar{1}.964491 = \ ``\ \text{factor of (38)} \\ \hline \text{Sum} &= \bar{1}.486950 = \log f \\ &\ 0.30687\ \ = f \end{aligned}$$

EXAMPLE 9.—CALCULATION BY FORMULAS (40), (41), AND (42)

Let us consider the same case as in the preceding example, $\alpha=0.5$.

$$\log q_1=\bar{2}.053753$$

$$\log_e \frac{1}{q_1}=4.481400 \text{ (see example 7).}$$

The calculation of the terms in the series for $\frac{dC_1}{dq_1}$ and $\frac{dC_2}{dq_1}$ in (40) offer nothing new, and will be found by the same method as in the preceding examples. The values found are $\frac{dC_1}{dq_1}=-17.2414$, and $\frac{dC_2}{dq_1}=-19.443$.

Making use of the values of C_1 and C_2 found in example 7, the rest of the calculation of the factor f follows readily.

$$\frac{1}{C_2}\frac{dC_2}{dq_1} = -29.798$$

$$\frac{1}{C_1}\frac{dC_1}{dq_1} = -21.465$$

$$\text{Diff.} = -\ 8.333$$

$$\log \text{Diff.} = 0.92080$$

$$\text{"}\ \ q_1 = \bar{2}.05375$$

$$\text{Sum} = \bar{2}.97455$$

$$-0.09431$$

$$1.00000$$

$$\text{Sum} = 0.90569$$

$$\tfrac{1}{2}\ \text{"} = 0.45284$$

factor in (40)

$$f = 0.30677$$

$$\log k^2 = \bar{1}.921344$$

$$\text{"}\ \ \sqrt{k} = \bar{1}.980336$$

$$\text{"}\ \ k^{\frac{5}{2}} = \bar{1}.901680 = \text{Sum}$$

$$\text{"}\ \ y_m^2 = \bar{1}.167608$$

$$\text{"}\ \ (1+k) = 0.281811$$

$$\text{Sum} = \text{A} = \bar{1}.351099$$

$$\log 4\alpha = 0.301030$$

$$\text{"}\ \ k'^2 = \bar{1}.219213$$

$$\text{Sum} = \text{B} = \bar{1}.520243$$

$$\text{Diff.} = (\text{A}-\text{B}) = \bar{1}.830856 = \log \text{factor } (42)$$

$$\log 0.45284 = \bar{1}.655950$$

$$\log f = \bar{1}.486806$$

EXAMPLE 10.—CALCULATION OF THE MAXIMUM FORCE BY FORMULAS (34) AND (5)

We have found in examples 2 and 3 for the case $\alpha = 0.5$ the value of $\log y_m = \bar{1}.5838040$, to which corresponds $\log q = \bar{1}.0435322$. This value of q is the quantity which is designated by q_0 in equation (34).

In making the substitution of this value in (34) the method of example 6 gives for the terms in the parenthesis the following values:

$$\begin{array}{rl}
1.0244396 & \\
0.0095567 & \\
0.0001089 & \\
0.0000021 & \\
\hline
1.0341073 & = \text{Sum of positive terms.} \\
-0.0004598 & \\
0.0000138 & \\
0.0000003 & \\
\hline
0.0004739 & = \text{Sum of negative terms.} \\
1.0336334 & = \text{series in parenthesis.}
\end{array}$$

The remainder of the calculation is then

$$\begin{array}{rl}
\log \text{series} & = 0.0143666 \\
\text{“} \quad \dfrac{96\pi^2}{\sqrt{5}} & = 2.6270859 \\
\text{“} \quad q_0^2 & = \underline{\bar{2}.0870644} \\
\text{“} \quad F_m & = 0.7285169 \\
F_m & = 5.352010
\end{array}$$

If, instead of using (34), we substitute the value of y_m found previously in the general equation for the force, we should, of course, arrive at the same result, since equation (34) was derived by just this process. The value of q will be the same as that in the preceding calculation. The terms in the series are found to be

$$\begin{array}{rl}
 & 1.2443963 \\
 & 0.0335978 \\
 & 0.0033575 \\
 & 0.0002702 \\
 & 0.0000185 \\
 & \underline{0.0000011} \\
\text{Sum} = & 1.2816414
\end{array}$$

$$\begin{array}{rl}
\log \text{sum} & = 0.1077666 \\
\text{“} \quad y_m & = \bar{1}.5838040 \\
\text{“} \quad 192\pi^2 & = \underline{3.2776009} \\
 & 2.9691715 \\
 & \underline{\bar{3}.7593455} \\
\log F_m & = 0.7285170
\end{array}
\qquad
\begin{array}{rl}
\log q^2 & = \bar{2}.0870644 \\
\text{“} \quad \sqrt{q} & = \underline{\bar{1}.5217661} \\
\text{“} \quad q^{\frac{5}{2}} & = \bar{3}.6088305 \\
\text{“} \quad \sqrt{\alpha} & = \underline{\bar{1}.8494850} \\
 & \bar{3}.7593455
\end{array}$$

which gives a value of F_m in agreement with the result by the preceding calculation, thus checking (34).

EXAMPLE 11.—CALCULATION OF THE MAXIMUM FORCE, FORMULAS (37) AND (6)

The maximum force will be calculated in this example for the same case as in the preceding example to show the agreement of the formulas. In both cases the higher order terms of the series are important. The value of q_1, corresponding to the position of the coils for maximum force, has already been derived in example 3. It is $\log q_1 = \bar{2}.0537527$.

The two sets of terms in the parentheses in (6) are then as follows:

	1.1358107	1.0076852
	0.0017854	0.0000213
	0.0000040	1.0077065
	1.1376006	
	−0.0245927	− 0.1131756
	0.0000924	0.0004349
	0.0000001	0.0000009
	−0.0246852	− 0.1136114
Series =	1.1129154	0.8940951 = series.
Second term =	0.5441659	10.729141 = multiplied by 12.
Parenthesis =	0.5687495	

$$\log = 1.0305649$$

$$\log_{10} \log_e \frac{1}{q_1} = 0.6514137$$

$$\log_{10} q_1 = \bar{2}.0537527$$

$$\log \sqrt{\alpha} = \bar{1}.8494853$$

$$\text{"} \quad q_1 = \bar{2}.0537527$$

$$\bar{3}.9032377$$

$$\text{log second term} = \bar{1}.7357313$$

$$\text{log parenthesis} = \bar{1}.7549210$$

$$\log \tfrac{\pi}{16} = \bar{1}.2930299$$

$$\text{"} \quad y_m = \bar{1}.5838040$$

$$\bar{2}.6317549$$

$$\bar{3}.9032377$$

$$\log F_m = 0.7285172$$

$$F_m = 5.352014$$

This result differs from that by the other independent formula (5) by no more than the errors of calculation with seven-place logarithms.

EXAMPLE 12.—THE COEFFICIENTS η AND ϵ

These coefficients have been calculated for several values of α and are given in Table 5. To show the agreement between the two sets of formulas, in the range where both may be used, the following values may be cited for the case $\alpha = 0.5$.

	By (48) and (51)	By (55) and (58)	B.S. current balance
$\alpha = 0.5$	$\eta = -0.62813$	$\eta = -0.62808$	$\eta = -0.62$
	$\epsilon = 2.5494$	$\epsilon = 2.5493$	$\epsilon = 2.555$

For the case $\alpha=0.4$, the agreement of the two formulas is as good, the mean results being $\eta=-0.35298$ and $\epsilon=2.3102$. The values found experimentally with the B. S. current balance were, for $\alpha=0.4$, $\eta=-0.35$ and $\epsilon=2.317$. The agreement is to be regarded as satisfactory when it is remembered that the values calculated by the formulas are differential coefficients, whereas the experimental values refer to finite displacements.

EXAMPLE 13.—ILLUSTRATING THE METHOD FOR CALCULATING THE MAXIMUM FORCE FOR COILS WHOSE RADII HAVE A RATIO DIFFERING SLIGHTLY FROM ONE OF THE VALUES OF TABLE 4

Consider the case of a pair of coils whose radii were intended to have a ratio of 1:2, but whose actual values have a ratio of $\alpha=0.5(1.01)=0.505$.

Here $\frac{\Delta\alpha}{\alpha_0}=0.01$, if we take $\alpha_0=0.5$. For the value $\alpha_0=0.5$, Table 4 gives $\log F_m=0.7285170$, and Table 5 gives for $\alpha=0.5$, $\epsilon=2.5493$; for $\alpha=0.55$, $\epsilon=2.7205$, with the tabular differences $\Delta_1=0.1712$, $\Delta_2=0.0507$, and $\Delta_3=0.0240$.

To obtain ϵ_1, the value corresponding to $\alpha=0.505$, we have to interpolate for an interval of 0.1 of that used in the table, and we find the result $\epsilon_1=2.5493+0.0171-0.0023=2.5641$.

This gives, therefore, as a first approximation, $\frac{\Delta F}{F_0}=0.01\times2.56$ and, consequently, $\frac{F_1\alpha_0}{F_0\alpha_1}=\frac{1.0256}{1.01}=1.01545$, and $\epsilon_1'=2.5641\times1.01545=2.6037$. As a first approximation, therefore, the mean value over the interval is $\frac{1}{2}(2.5493+2.6037)=2.5765$. Using this value, we have, as a second approximation, $\frac{F_1\alpha_0}{F_0\alpha_1}=\frac{1.025765}{1.01}=1.01561$, which gives $\epsilon_1'=2.5641\times1.01561=2.6041$, and the second approximation to the mean value is $\epsilon=2.5767$, which is very little different from the first approximation.

We have, therefore, $\frac{\Delta F}{F_0}=0.025767$, and the required value of F_m for $\alpha=0.505$ is found by multiplying the tabular value of F_m for $\alpha_0=0.5$ by the factor 1.025767.

$$\begin{aligned} \log\,(1.025767) &= 0.0110487 \\ \text{“}\ F_0 &= \underline{0.7285170} \\ \log F_m &= 0.7395657 \quad F_m = 5.489916 \text{ for } \alpha = 0.505. \end{aligned}$$

To check this calculation, we may find the value of η for this same interval. Table 5 gives

$$\eta_0 = -0.62810 \text{ for } \alpha_0 = 0.5$$

$$\eta_1 = -0.64591 \text{ interpolated for } \alpha_1 = 0.505$$

$$\eta_1 \frac{\alpha_0}{\alpha_1} \frac{y_1}{y_0} = -0.64591 \left(\frac{1 - .00636}{1.01} \right) = -0.63545$$

$$\eta_1{}' = -0.63178$$

A second approximation gives $\eta_1{}' = -0.63179$. Using the value $\frac{\Delta y}{y_0} = -0.0063179$, we find $\bar{1}.5810514$ as the value of $\log y_m$ derived from the tabular value for $\alpha_0 = 0.5$. Using this as a first approximation, we find by use of the main equation (33) the slightly more accurate value $\log y_m = \bar{1}.5810517$ for $\alpha = 0.505$, and substituting this result in (34) the force is found to have a value of 5.489926, a value only two parts in a million different from the value found by the interpolation process. This difference is not large when we consider the largeness of the interval over which the interpolation was made and the very considerable magnitude of the second and third differences of the tabulated values of ϵ. The amount of work necessary for finding the value of the maximum force by use of the interpolated value of ϵ is, of course, vastly less than that required for the application of the exact formulas.

EXAMPLE 14.—CALCULATION OF THE MAXIMUM FORCE BETWEEN THE BUREAU OF STANDARDS CURRENT BALANCE COILS AND THE DISTANCES BETWEEN THE COILS

To illustrate the use of the formulas of this article for actual coils, the complete calculations are given of the constants of the coils of the Bureau of Standards current balance. The work is arranged for easier following in tabular form (Tables A and B); all the steps, except some of minor importance, are given in the compass of the table. For comparison, the published results given in Tables IX and XI of the article on the determination of the international ampere are appended. The latter values, as has already been explained, were obtained by interpolation between the values of the force calculated by formula (4) for a number of distances of the coils in the neighborhood of the position corresponding to maximum force.

TABLE A

Calculation of Maximum Distance z_m for Bureau of Standards Coils

Coils	$\frac{\Delta\alpha}{\Delta}$	Interpolated value η_1	η_1' $-\eta_1 \frac{\alpha_0}{\alpha_1} \frac{y_m}{y_0}$	η mean	$\frac{\Delta y}{y_0}$	log y_m	z_m	Values current balance
M_2L_1	−0.0010800	−0.62620	−0.62715	−0.62755	+.0006778	$\bar{1}.5840983$	9.60946	9.6092
M_2L_2	− .0010566	.62625	.62733	.62772	6632	.5840919	9.60910	9.6089
M_2L_3	+ .0002793	.62859	.62831	.62822	−.0001755	.5837278	9.58822	9.5882
M_2L_4	.0000908	.62826	.62817	.62813	− 570	.5837792	9.59117	9.5912
M_2S_1	+ .0047609	−0.63652	−0.63158	−0.62984	−.0029986	$\bar{1}.5824998$	7.64009	7.6400
M_2S_2	.0052139	.63731	.63190	.63000	32848	.5823750	7.63445	7.6344
M_3L_1	+ .0022365	−0.35492	−0.35384	−0.35341	−.0007904	$\bar{1}.6294549$	10.66733	10.6673
M_3L_2	.0022600	.35503	.35394	.35346	7981	.6294515	10.66700	10.6669
M_3L_3	.0036000	.35611	.35438	.35368	−.0012732	.6292449	10.64769	10.6476
M_3L_4	.0034113	.35595	.35431	.35364	12064	.6292741	10.65041	10.6502
M_4L_1	− .0038325	−0.62133	−0.62520	−0.62665	+.0024016	$\bar{1}.5848458$	9.62602	9.6256
M_4L_2	− .0038091	.62127	.62512	.62661	23868	.5848394	9.62565	9.6253
M_4L_3	− .0024768	.62365	.62617	.62714	15533	.5844781	9.60480	9.6047
M_4L_4	− .0026649	.62341	.62611	.62711	16712	.5845292	9.60775	9.6077

It must be remembered that the maximum force here calculated is not quite equal to the maximum force between the actual coils, but is the value between the equivalent filaments by which the coils may be replaced, on the assumption that their cross sections are square. It is the force F_1 in the article referred to, and the correction for the actual deviation from a square cross section must be applied as there shown (pp. 329 to 334).

The equivalent radii of the coils, derived by Lyle's Method, are given there in Table IX. They are:

Coil	Equivalent radius	Coil	Equivalent radius
M_2	12.50552	S_2	19.97113
M_3	10.03763	L_1	25.03808
M_4	12.47106	L_2	25.03749
S_1	19.98014	L_3	25.00405
		L_4	25.00877

Table A covers the calculation of the distance z_m for maximum force. Column 2 gives the fractional differences between the actual ratios of the radii and the nominal values α_0. The latter is $\alpha_0 = 0.4$ for the movable coil M_3, when used with the fixed coils L_1, L_2, L_3, and L_4: for the other combinations it is 0.5. Column 3 gives the interpolated values of η_1, and column 4 the same quantity multiplied by the correction factor $\frac{\alpha_0}{\alpha_1} \frac{y_1}{y_0}$.

In columns 5 and 6 appears the mean value η and the change in y_0, $\frac{\Delta y}{y_0} = \eta \frac{\Delta \alpha}{\alpha_0}$ for the actual fractional deviations of the coils from the nominal ratio of the radii. The values of column 7 are found by adding the logarithms of $\left(1 + \frac{\Delta y}{y_0}\right)$ to the values of log y_0 for the nominal ratio taken from Table 4, viz:

$$\text{For } \alpha_0 = 0.5 \quad \log y_0 = \bar{1}.5838040$$
$$\text{"} \quad \alpha_0 = 0.4 \quad \text{"} \quad y_0 = \bar{1}.6297983$$

TABLE B

Calculation of the Maximum Force for the Bureau of Standards Coils

Coils	$\frac{\Delta\alpha}{\alpha_0}$	Interpolated value η_1	$\eta_1' = \eta_1 \frac{\alpha_0 F_m}{\alpha_1 F_0}$	η mean	$\frac{\Delta F}{F_0}$	$\log\left(1+\frac{\Delta F}{F_0}\right)$	log F_1	F_1	F_1 values current balance
M_2L_3	+0.0002793	2.5493	2.5510	2.5502	+0.0007123	0.0003092	0.7288262	5.355823	5.355834
M_2L_4	+ .0000908	2.5493	2.5499	2.5496	.0002315	.0001006	0.7286176	5.353251	5.353251
M_2S_1	+0.0047609	2.5563	2.5754	2.5623	+0.0121987	0.0052658	0.7337828	5.417298	5.417298
M_2S_2	.0052139	2.5569	2.5776	2.5635	.0133658	.0057662	0.7342832	5.423544	5.423552
M_3L_1	+0.0022365	2.3118	2.3186	2.3144	+0.0051761	0.0022422	0.4966225	3.137780	3.137781
M_3L_2	.0022600	2.3119	2.3187	2.3145	.0052308	.0022658	.4966461	3.137951	3.137953
M_3L_3	.0036000	2.3130	2.3239	2.3171	.0083416	.0036077	.4979880	3.147662	3.147665
M_3L_4	.0034113	2.3129	2.3232	2.3167	.0079030	.0034187	.4977990	3.146292	3.146292
M_4L_3	−0.0024768	2.5458	2.5360	2.5426	−0.0062974	$\bar{1}$.9972564	0.7257734	5.318308	5.318310
M_4L_4	− .0026649	2.5454	2.5349	2.5421	− .0067744	.9970479	.7255649	5.315754	5.315754

The values of the distance z_m then follow on multiplying the values in column 7 by the radius A of the larger coil. The values published in Table IX appear in the last column. The differences between the latter and the values here calculated are very small and are not systematic.

In Table B appear the calculations of the maximum force. These should be clear from the description of the preceding table. The values of the maximum force F_0, corresponding to the nominal ratios 0.5 and 0.4, are taken from Table 4, and form the basis of the calculations. In the last column are given the values of F_m from Table XI of the article on the current balance. The differences between these values and those here calculated are, on the average, less than a part in a million. In fact, the agreement is better than for the distances z_m in the preceding table. This is explained when it is remembered that the force changes very slowly in the neighborhood of the critical distance, and thus makes it difficult to obtain the actual position of the coil for maximum force by interpolation.

X. SUMMARY

1. The force of attraction or repulsion between two parallel, coaxial circular currents has its maximum value for a distance z_m, between their planes, such that the ratio $\frac{z_m}{A}$ is a function of the ratio of the radii $\frac{a}{A}$ alone.

2. The value of the force between the two circular filaments, when one ampere flows in each, is a function of the ratio of the radii alone. A knowledge of the maximum value of this force is of practical importance in the absolute measurement of current, since the Rayleigh current balance employs two coils placed experimentally at the distance which gives the maximum force between them. The calculation of the constant of this current balance postulates a knowledge of the maximum value of the force in terms of the dimensions of the coils.

4. Formulas have been developed by Maxwell and Nagaoka which allow the force to be calculated between any two parallel, coaxial currents whose planes are at any desired distance z apart. Previously the maximum value of the force has been derived by interpolation, graphically or otherwise, between values of the force calculated for various positions of the coils in the region where the force is known to be a maximum.

5. In this paper, by a method which is an extension of that used by Nagaoka in deriving his formulas for the force between the coils, two equations are derived, which give the distance between the coils corresponding to maximum force, as a function of the ratio of the radii, α. These relations are in the form of q series, one depending on a modulus which is complementary to that of the other. One or the other of these formulas will be found to converge well for any desired value of α.

6. Unfortunately, these relations do not, apparently, allow of being put in a form such as to allow the distance z_m to be calculated directly. They may, however, be solved for z_m by successive approximation without serious difficulty.

7. Starting from Rayleigh's approximation formula for z_m, which gives precise values over only a limited range of values of the ratio of the radii, methods are here worked out for facilitating the process of approximation, together with an empirical method for obtaining a more accurate first approximation than can, in general, be obtained by the formula of Rayleigh.

8. Tables are given of the distance z_m and the maximum force F_m for values of the ratio of the radii advancing in steps of 0.05, as well as a number of tables of auxiliary quantities, which materially reduce the labor of calculation of the distance z_m and the maximum force for such values of α as are not included in the tables.

9. A method is also given for easily obtaining the values of the desired quantities from the fundamental values in the tables for coils having radii whose ratio differs only slightly from one of the nominal values included in the tables. This covers the case of current balance coils in practice.

10. The use of the formulas and tables is thoroughly illustrated by examples so designed as to show the agreement of the different methods. Finally, a comparison is given of the results of calculations made for the coils of the Bureau of Standards current balance, by the method of this paper, with the published values obtained by the interpolation process. The agreement is, on the average, closer than one part in a million. For practical cases like this the use of the tables given here enables the constants to

be found with vastly less labor than in a general case not covered by the fundamental values in the tables. Even in the general case, however, the method of this paper is likely to be less time consuming than the interpolation method, and has the advantage of furnishing a check on the latter.

WASHINGTON, January 1, 1915.

APPENDIX

TABLE 1

Table of Values of the Quantity in Brackets in Equation 33 (Factor P in Equation (33a))

q	P	Δ_1	Δ_2	q	P	Δ_1	Δ_2
0	1.000000	− 18	−36	0.040	0.971701	−1406	−32.5
0.001	0.999982	54	36	.041	.970295	1438.5	32
.002	.999928	90	36	.042	.968856	1470.5	31.5
.003	.999838	126	36	.043	.967386	1502	32
.004	.999712	162	36	.044	.9658835	1534	31
.005	.999550	198	36	.045	.964350	1565	31
0.006	0.999352	− 234	−36	0.046	0.962784	−1596	−31.5
.007	.9991185	270	35.5	.047	.961188	1627.5	31
.008	.998849	305.5	35.5	.048	.959560	1658	31
.009	.998543	341	36	.049	.957902	1689	30
.010	.998202	377	36	.050	.956213	1719	30
0.011	0.997825	− 413	−35	0.051	0.954494	−1749.5	−30
.012	.997412	448	36	.052	.952744	1779.5	30
.013	.996964	484	36	.053	.950965	1809	29.5
.014	.996480	520	35	.054	.9491555	1839	29
.015	.995960	555	35	.055	.947317	1868	29
0.016	0.995405	− 590	−36	0.056	0.945449	−1897	−29
.017	.994815	626	35	.057	.943552	1926	29
.018	.994189	661	35	.058	.941626	1954.5	28.5
.019	.993528	696	35	.059	.939671	1983	28
.020	.992832	731	35	.060	.937688	2011	28
0.021	0.992101	− 766	−35	0.061	0.935677	−2039	−28
.022	.991334	801	35	.062	.933638	2067	27
.023	.990533	836	34	.063	.9315715	2094	27
.024	.989698	870	35	.064	.929478	2121	27
.025	.988827	905	34	.065	.927356	2148	27
0.026	0.987922	− 939	−35	0.066	0.925208	−2175	−26
.027	.986983	974	34	.067	.9230335	2201	26
.028	.986009	1007	34	.068	.920832	2227	26
.029	.985002	1042	34	.069	.918605	2253	26
.030	.983960	1076	−33.5	.070	.916352	2279	25
0.031	0.982884	−1109.5	−33.5	0.071	0.914073	−2304	−25
.032	.9817745	1143	34	.072	.911769	2329	25
.033	.980631	1177	33	.073	.909440	2354	24
.034	.979455	1210	33	.074	.9070855	2379	24
.035	.978245	1243	33	.075	.904707	2403	24
0.036	0.977002	−1276	−33	0.076	0.902304	−2427	−24
.037	.975726	1309	32.5	.077	.899877	2451	23.5
.038	.974417	1341.5	32.5	.078	.897426	2474	23
.039	.973075	1374	32	.079	.894952	2497	23
.040	.971701	1406	32.5	.080	.892455	2520	23

TABLE 1—Continued

Table of Values of the Quantity in Brackets in Equation 33 (Factor P in Equation (33a))—Continued

q	P	Δ^1	Δ^2	q	P	Δ^1	Δ^2
0.080	0.892455	−2520	−23	0.100	0.838072	−2919	−17
.081	.889934	2543	22.5	.101	.835153	2935	17
.082	.8873915	2565	22	.102	.8322175	2952	16
.083	.884826	2587	22	.103	.829265	2968	16
.084	.882239	2609	22	.104	.826297	2985	15
.085	.879630	2631	21.5	.105	.823312	3000	15
0.086	0.876999	−2652	−21	0.106	0.820312	−3015.5	−15
.087	.874347	2673	21	.107	.8172965	3031	15
.088	.871674	2694	20.5	.108	.814266	3046	14
.089	.868981	2713.5	20	.109	.811220	3060	14
.090	.866267	2734	19.5	.110	.808160	3074.5	14
0.091	0.863533	−2754	−19	0.111	0.805086	−3088	−14
.092	.860779	2773	19.5	.112	.801998	3102	13.5
.093	.858006	2793	19	.113	.798896	3115	13
.094	.855213	2811	19	.114	.795781	3128	13
.095	.852402	2830	18	.115	.792652	3141	13
0.096	0.849572	−2848	−18	0.116	0.789511	−3154	−12
.097	.846724	2867	17.5	.117	.786358	3166	12
.098	.843857	2884	17.5	.118	.783192	3178	12
.099	.840973	2901.5	17	.119	.780014	3190	12
.100	.838072	2919	17	.120	.776825	3202	12

TABLE 2.

Table of Values of Factor X in Equation (37′a.)

q_1	X	Δ_1	Δ_2	q_1	X	Δ_1	Δ_2
0	1.000000	−7058	+1654	0.0015	0.942127	−2667	+69.5
0.0001	0.992942	5404	620	.0016	.939461	2597	65.5
.0002	.9875375	4784	400	.0017	.9368635	2532	62
.0003	.982753	4384	296	.0018	.934332	2470	58
.0004	.978369	4088	235	.0019	.931862	2412	54
.0005	.974281	3853	195	.0020	.929450	2357	52
0.0006	0.970428	−3658	+166	0.0021	0.927093	−2305	+50
.0007	.966770	3493	144	.0022	.9247875	2256	47
.0008	.963277	3348	128	.0023	.922532	2208	45
.0009	.959929	3221	114	.0024	.920324	2163	43
.0010	.956708	3107	104	.0025	.918160	2120	41
0.0011	0.953601	−3002.5	+94	0.0026	0.916040	−2079	+39
.0012	.950599	2909	87	.0027	.913962	2039	38
.0013	.947690	2822	80.5	.0028	.911922	2001	37
.0014	.944869	2741	74.5	.0029	.909921	1964	35
.0015	.942127	2667	69.5	.0030	.907957	1929	34

TABLE 2—Continued

Table of Values of Factor X in Equation (37'a.)—Continued

q_1	X	Δ_1	Δ_2	q_1	X	Δ_1	Δ_2
0.0030	0.907957	−1929	+34	0.0075	0.844435	−1029	+11
.0031	.906028	1895	33	.0076	.843406	1018	12
.0032	.904133	1862	32	.0077	.842388	1006	12
.0033	.902271	1830.5	31	.0078	.8413825	994	11
.0034	.900440	1800	30	.0079	.8403885	983	12
.0035	.898640	1770	29	.0080	.839406	971	11
0.0036	0.896870	−1741	+28	0.0081	0.838435	−960	+11
.0037	.895129	1713	27	.0082	.837474	949	11
.0038	.893416	1686	26	.0083	.836525	938	10
.0039	.891730	1660	25	.0084	.835587	928	10.5
.0040	.890070	1634	25	.0085	.834659	917.5	10.5
0.0041	0.888436	−1610	+25	0.0086	0.833742	−907	+10
.0042	.886826	1585	24	.0087	.832835	897	10
.0043	.885241	1561.5	23	.0088	.831938	887	10
.0044	.883679	1539	22	.0089	.831051	877	10.5
.0045	.882141	1516.5	22	.0090	.830174	867.5	9.5
0.0046	0.880624	−1495	+21	0.0091	0.829307	−858	+10
.0047	.8791295	1473.5	21	.0092	.828449	848	9
.0048	.877656	1453	21	.0093	.827601	839	10
.0049	.876203	1432	19	.0094	.826763	829	8
.0050	.874771	1413	19.5	.0095	.825933	821	10
0.0051	0.873359	−1393	+19	0.0096	0.825113	−811	+8
.0052	.871965	1374	18	.0097	.8243015	803	10
.0053	.870591	1356	18	.0098	.823499	793	8
.0054	.869235	1338	17.5	.0099	.822705	785	9
.0055	.867898	1320	17	.0100	.821920	776	8
0.0056	0.8665775	−1303	+17	0.0101	0.821144	−768	+8
.0057	.865275	1286	16.5	.0102	.820375	760	8
.0058	.863989	1269	16	.0103	.819616	752	9
.0059	.862720	1253	16	.0104	.818864	743	7
.0060	.861467	1236.5	15.5	.0105	.818121	736	8
0.0061	0.8602305	−1221	+15	0.0106	0.817385	−728	+8
.0062	.859009	1206	15	.0107	.816657	720	8
.0063	.857803	1191	15	.0108	.815938	712	7
.0064	.856613	1176	14.5	.0109	.815226	705	9
.0065	.8554365	1161	14	.0110	.814521	696	7
0.0066	0.8542755	−1147	+13.5	0.0111	0.813825	−689	+7
.0067	.853128	1133.5	13	.0112	.813136	682	7
.0068	.851995	1119	13	.0113	.812454	675	8
.0069	.850876	1106	13	.0114	.811779	667	7
.0070	.849770	1093	13	.0115	.811112	660	6.5
0.0071	0.848677	−1080	+13	0.0116	0.810452	−653.5	+7.5
.0072	.847597	1066	13	.0117	.8097985	646	7
.0073	.846531	1054	12	.0118	.809153	639	6.5
.0074	.845477	1042	13	.0119	.8085135	632.5	6.5
.0075	.844435	1029	11	.0120	.807881	626	6

TABLE 3

Values of Factors in Differential Equations (38) to (42)

$\alpha=\frac{a}{A}$	logarithms of factors in (38)	logarithms of factors in (39)	log f	f	log F $-\log\frac{1}{1-f}$	F	Δ_1	Δ_2
0	1.69897n	$\bar{1}$.60206n	$\bar{1}$.30103	0.20000	0.09691	1.2500	+ 18	+ 34
0.05	.70209	.60136	.30345	.20112	.09752	.2518	52	37
0.10	.71137	.59926	.31063	.20447	.09934	.2570	89	36
0.15	.72657	.59573	.32231	.21004	.10240	.2659	125	39
0.20	.74735	.59072	.33807	.21780	.10668	.2784	164	41
0.25	.77328	.58406	.35734	.22769	.11221	.2948	205	42
0.30	$\bar{1}$.80383	$\bar{1}$.57587	$\bar{1}$.37969	0.23971	0.11902	1.3153	+247	+ 46
0.35	.83860	.56579	.40438	.25374	.12711	.3400	293	48
0.40	.87716	.55370	.43086	.26969	.13649	.3693	341	52
0.45	.91919	.53936	.45855	.28744	.14718	.4034	393	55
0.50	$\bar{1}$.96448	.52246	.48695	.30686	.15918	.4427	448	58
0.55	0.01292	$\bar{1}$.50259	.51551	.32773	0.17246	1.4875	506	

α	logarithms of factors in 40	logarithms of factors in 42	log f	f	log F	F	Δ_1	Δ_2
0.45	$\bar{1}$.65906	$\bar{1}$.79941	$\bar{1}$.45847	0.28739	0.14715	1.4033	+392	+ 55
0.50	.65595	.83086	.48681	.30677	.15912	.4425	447	56
0.55	1.65597	1.85936	.51534	0.32759	0.17237	1.4872	+503	+ 58
0.60	.65834	.88523	.54357	.34960	.18682	.5375	561	55
0.65	.66246	.90864	.57110	.37248	.20237	.5936	616	47
0.70	.66782	.92970	.59755	.39584	.21885	.6552	663	30
0.75	.67391	.94841	.62232	.41910	.23590	.7215	693	+ 1
0.80	$\bar{1}$.68030	1.96471	$\bar{1}$.64501	0.44159	0.25305	1.7908	694	− 50
0.85	.68659	.97845	.66504	.46241	.26955	.8602	644	−132
0.90	.69229	.98932	.68161	.48041	.28434	.9246	512	−270
0.95	.69678	$\bar{1}$.99684	.69362	.49388	.29575	1.9758	242	
1.00	$\bar{1}$.69897	0.00000	$\bar{1}$.69897	0.50000	0.30103	2.0000		

TABLE 4

Fundamental Values of the Maximum Force Ratio $y_m = \frac{z_m}{A}$ and the Maximum Force F_m

$\alpha = \frac{a}{A}$	$\log y_m$	y_m	ϕ			$\log F_m$	F_m
			°	′	″		
0	$\bar{1}.6989700$	0.5000000	45	50	11.8		0
0.05	.6979913	.4988745	45	50	31.04	$\bar{2}.6279557$	0.04245763
0.10	.6950381	.4954936	45	51	28.52	$\bar{1}.2326395$	0.1708596
0.15	.6900572	.4898434	45	53	3.47	$\bar{1}.5892443$	0.3883688
0.20	.6829567	.4818998	45	55	14.40	$\bar{1}.8454176$	0.7005153
0.25	.6736010	.4716296	45	57	59.40	0.0475214	1.1156332
0.30	$\bar{1}.6618033$	0.4589901	46	01	15.60	0.2163112	1.6455504
0.35	.6473125	.4439279	46	04	58.77	0.3629849	2.306667
0.40	.6297983	.4263815	46	09	3.72	0.4943803	3.121622
0.45	.6088217	.4062765	46	13	22.62	0.6150973	4.121898
0.50	.5838040	.3835341	46	17	45.84	0.7285170	5.352011
0.55	$\bar{1}.5539627$	0.3580657	46	21	59.60	0.8373702	6.876544
0.60	.5182254	.3297808	46	25	45.82	0.9441070	8.792392
0.65	.4750743	.2985893	46	28	39.62	1.0512140	11.25159
0.70	.4222883	.2644163	46	30	8.59	1.1615723	14.50682
0.75	.3564223	.2272073	46	29	26.32	1.2790169	19.01152
0.80	$\bar{1}.2717509$	0.1869610	46	25	30.92	1.4094251	25.66995
0.85	$\bar{1}.1576668$	.1437695	46	17	1.94	1.5633218	36.58658
0.90	$\bar{2}.9905951$	.0978577	46	02	2.59	1.7638911	58.06188
0.95	$\bar{2}.6960993$	.0496706	45	37	59.40	2.0847542	121.54978
1.00		0	45	0	0.00		

TABLE 5

Coefficients η and ϵ in Equations (48), (51), (55), and (58)

$\alpha=\frac{a}{A}$	η	Δ_1	Δ_2	Δ_3	ϵ	Δ_1	Δ_2	Δ_3
0	. 0	− .00451	− .00919	− .00050	2.0000	+ .0040	+ .0082	+ .0003
0.05	− 0.00451	1370	969	87	.0040	122	85	8
0.10	.01821	2339	1056	129	.0162	207	93	12
0.15	.04160	3395	1185	193	.0369	300	105	14
0.20	.07555	4580	1378	263	.0669	405	119	22
0.25	.12135	5958	.01641	366	2.1074	524	141	33
0.30	− 0.18093	− .07599	− .02007	− .00516	2.1598	+ .0665	+ .0174	+ .0043
0.35	.25692	.09606	2523	731	.2263	839	217	62
0.40	.35298	.12129	3254	1083	.3102	.1056	279	98
0.45	.47427	.15383	4337	− .01573	.4158	.1335	377	130
0.50	.62810	.19720	− .0591	− .0262	2.5493	.1712	507	237
0.55	− 0.82530	− .2563	− .0853	− .0395	2.7205	+ .2219	+ .0744	+ .0380
0.60	1.0816	.3416	.1248	.0763	2.9424	.2963	.1124	690
0.65	1.4232	.4664	.2011	.1466	3.2387	.4087	.1814	.1206
0.70	1.8896	.6675	.3477	.3391	3.6474	.5901	.3020	.3790
0.75	2.5571	1.0152	.6868	−1.0101	4.2375	.8921	.6810	.9679
0.80	− 3.5723	−1.7020	−1.6969	−5.0143	5.1496	+1.5731	+1.6489	+4.9548
0.85	5.2743	3.3989	6.7112		6.7127	3.2220	6.6037	
0.90	8.6722	10.1101			9.9347	9.8257		
0.95	−18.7823				19.7604			

CONSTRUCTION OF PRIMARY MERCURIAL RESISTANCE STANDARDS

By F. A. Wolff, M. P. Shoemaker, and C. A. Briggs

CONTENTS

Page
Introduction.......... 375
Part I. Theoretical.......... 377
  A. The method of calculating the resistance of a given mercurial column.. 377
  B. The determination of the caliber corrections.......... 382
    1. Methods of calibration.......... 383
    2. Transformation of caliber corrections from one system into another.......... 388
  C. The corrections for the terminal bulbs.......... 389
Part II. Experimental details.......... 391
  A. Selection of the tubes.......... 392
  B. Calibration of the tubes.......... 392
  C. Determination of the approximate ohm lengths.......... 407
  D. Location of cuts.......... 407
  E. Cutting, grinding, and polishing.......... 408
  F. Calculation of the caliber factors for the finished tubes.......... 410
  G. Determination of M_0.......... 413
  H. Determination of L_0.......... 426
  I. Terminal bulbs.......... 440
  J. Theoretical resistances of the finished tubes.......... 442
  K. The electrical measurements.......... 443
  L. Results of electrical comparisons.......... 457
Part III. Supplementary measurements.......... 459
  Comparison of ohm tubes of several countries.......... 462

INTRODUCTION

The absolute unit of resistance is defined as 10^9 units of the c. g. s. electromagnetic system. Although depending solely on the measurement of length and time, it can not, at the present time, be determined to a higher degree of accuracy than the international unit of resistance can be realized by primary mercurial resistance standards constructed in accordance with the specifications of the London Conference on Electrical Units and Standards (1908).

Since the international unit is in almost exclusive use, concrete standards will continue to be employed for work of the highest precision, even though the unit thus defined should be found to differ appreciably from the absolute unit, as it is of the greatest importance, for purposes of comparison, that the same units be employed everywhere.

There are two possible methods of defining the international unit of resistance—(1) by adopting a particular standard, as has been done in the case of length and mass, and (2) by defining it in terms of the resistance, at a specified temperature, of a pure material of specified dimensions. If suitable materials were available, the first method would offer an ideal solution; but, while considerable improvements have been made in the construction of resistance standards and in the production of specially adapted resistance materials, much remains to be accomplished before such standards can be safely trusted to maintain their values indefinitely. The remaining alternative, best realized through the mercury unit proposed by Siemens in 1860, has therefore been internationally adopted. The dimensions of the mercury column have been twice modified so as to bring the mercury unit into closer agreement with the absolute ohm; but further changes, except possibly in the adoption of more rigorous specifications, are not likely to be made until the accuracy attainable in absolute measurement equals or exceeds that realized in the reproducibility of the mercury unit.

The London International Electrical Conference of 1908 was the first to draw a sharp distinction between the absolute and international units. The ohm was defined as having the value of 1,000,000,000 (10^9) in terms of the centimeter, and second, the ampere, volt, and watt being also defined in absolute units, but at the same time the conference recommended for adoption "a system of *international* units representing the above and sufficiently near for the purpose of electrical measurements, and as a basis for legislation." The unit of resistance, the realization of which is the object of this investigation, was defined as follows:

> The *international ohm* is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and of a length of 106.300 centimeters.

Certain details of the procedure for the construction of mercurial resistance standards, discussed elsewhere in this paper, were also adopted, in the form of specifications. The above value for the mass of mercury contained in a column 106.300 cm long corresponds to a cross section of 1 sq. mm if the density of mercury is taken as 13.5956 grams per cubic centimeter. This method of definition, first adopted by the Chicago congress, was chosen in order to eliminate the uncertainty due to experimental errors in the determination of absolute density, including, as it does, the error in the determination of the relation between the liter and cubic decimeter.

PART I. THEORETICAL

In practice the cross section and length of the mercury column are given by the dimensions of glass tubing, employed to confine the mercury, which can not be expected to agree exactly with the dimensions specified either in magnitude or in uniformity of cross section. The manner in which such variations are taken into account will be briefly described below.

A. THE METHOD OF CALCULATING THE RESISTANCE OF A GIVEN MERCURIAL COLUMN

The resistance and mass of a column of mercury of uniform cross section s_0 and of length L_0 are given by the formulæ:

$$R_0 = \rho_0 \frac{L_0}{s_0} \qquad (1)$$

$$M_0 = D_0 L_0 s_0$$

in which ρ_0 and D_0 denote, respectively, the resistivity and density of mercury.

Multiplying, we have

$$R_0 M_0 = \rho_0 D_0 L_0^2$$

or

$$R_0 = \rho_0 D_0 \frac{L_0^2}{M_0} \qquad (2)$$

But from the definition of the international ohm, $R=1$ when $L_0=106.300$ cm and $M_0=14.4521$ g.

Hence,

$$\rho_0 D_0=\frac{14.4521}{(106.300)^2}$$

and consequently,

$$R_0=\frac{14.4521}{(106.300)^2}\frac{L_0^2}{M_0} \tag{3}$$

This formula gives the resistance, in international ohms, of a column of mercury of uniform cross section, L_0 centimeters in length and M_0 grams in mass. Hence any departure from the mean cross section specified in the definition can be allowed for.

The above formula may also be written

$$R_0=\frac{14.4521}{(106.300)^2}\frac{L_0}{\frac{M_0}{L_0}} \tag{4}$$

in which the term $\frac{M_0}{L_0}$ denotes the mass of mercury per unit length of the tube. For a 1-ohm tube the ohm length is therefore given by the formula:

$$L_0=\frac{(106.300)^2}{14.4521}\frac{M_0}{L_0} \tag{5}$$

But $\frac{M_0}{L_0}$ is given by ratio of the mass in grams to the observed length in centimeters of *any* mercury column employed, if the tube be uniform in cross section (a correction being applied for the menisci at the ends of the column).

Hence,

$$L_0=\frac{(106.300)^2}{14.4521}\frac{M'_0}{L'_0} \tag{6}$$

in which L'_0 denotes the length in centimeters, and M'_0 the mass in grams, of any mercury column in a tube of uniform cross section.

Variable Cross Section.—The correction factor which must be applied for variations in cross section throughout the length of a given tube may be deduced as follows:

$$R'_o = \rho_o \int_o^{L_o} \frac{dl}{s}$$

$$M_o = D_o \int_o^{L_o} s\, dl \qquad (7)$$

$$R'_o M_o = \rho_o D_o \int_o^{L_o} s\, dl \int_o^{L_o} \frac{dl}{s}$$

and hence

$$R'_o = \rho_o \frac{D_o}{M_o} \int_o^{L_o} s\, dl \int_o^{L_o} \frac{dl}{s} \qquad (8)$$

By comparison with formula (2) for a tube of uniform cross section it will be seen that

$$R'_o = R_o \left\{ \frac{1}{L_o^2} \int_o^{L_o} s\, dl \int_o^{L_o} \frac{dl}{s} \right\} \qquad (9)$$

The factor by which R_o, the resistance of a uniform column of the length L_o and mass M_o, must be multiplied to obtain the resistance of a column of the same length and mass but of variable cross section, is known as the caliber factor. Denoting this by $(1+C)$, we have

$$1 + C = \frac{1}{L_o^2} \int_o^{L_o} s\, dl \int_o^{L_o} \frac{dl}{s} \qquad (10)$$

Since it is impossible to directly determine the cross section of a capillary tube from point to point to a sufficient degree of accuracy for the purpose in hand, we must content ourselves with an approximation based on a calibration of the tube. This will give the mean cross section of finite elements into which the tube may be divided. If the resistance of the whole column be computed as the sum of the resistances of n cylindrical elements, each of length l, formula (9) becomes

$$R'_0 = R_0 \frac{1}{L_0^2}\left\{\Sigma_1^n \, sl \, \Sigma_1^n \frac{1}{s} \, l\right\} \tag{11}$$

or, since $nl = L_0$,

$$R'_0 = R_0 \frac{1}{n^2} \Sigma_1^n s \, \Sigma_1^n \frac{1}{s} \tag{12}$$

Hence,

$$1 + C = \left(\frac{1}{n} \Sigma_1^n s\right)\left(\frac{1}{n} \Sigma_1^n \frac{1}{s}\right) \tag{13}$$

If, as is usually the case, the particular length of tube having unit resistance and for which the caliber factor is sought, consists of n elements of length l and two fractional elements of length αl and βl and cross sections s_α and s_β, at the ends of the tube,

$$L_0 = (n + \alpha + \beta) l$$

$$\Sigma \, sl = l\left\{\Sigma_1^n s + \alpha s_\alpha + \beta s_\beta\right\}$$

$$\Sigma \frac{1}{s} l = l\left\{\Sigma_1^n \frac{1}{s} + \frac{\alpha}{s_\alpha} + \frac{\beta}{s_\beta}\right\}$$

and

$$1 + C = \left\{\frac{1}{n + \alpha + \beta}\left(\Sigma_1^n s + \alpha s_\alpha + \beta s_\beta\right)\right\}\left\{\frac{1}{n + \alpha + \beta}\left(\Sigma_1^n \frac{1}{s} + \frac{\alpha}{s_\alpha} + \frac{\beta}{s_\beta}\right)\right\} \tag{14}$$

The caliber factor is therefore given by the product of the mean cross section by the mean reciprocal cross section, each element being weighted in proportion to its length.

Since the s's enter into the numerator of the first series and into the denominator of the second series, the result is obviously independent, as it should be, of the unit in terms of which the cross sections are expressed.

The labor of calculation of the caliber factor can be simplified by expressing the cross sections of the separate elements as follows:

Let

$$\left.\begin{aligned} s_m &= s(1 + k_m) \\ s_\alpha &= s(1 + k_\alpha) \\ s_\beta &= s(1 + k_\beta) \end{aligned}\right\} \tag{15}$$

Then

$$\Sigma_1^n s+\alpha s_\alpha+\beta s_\beta=(n+\alpha+\beta)s\left\{1+\frac{\Sigma_1^n k+\alpha k_\alpha+\beta k_\beta}{n+\alpha+\beta}\right\}$$

and since $\frac{1}{1+k}=1-k+k^2-k^3$, etc.,

$$\Sigma_1^n\frac{1}{s}+\frac{\alpha}{s_\alpha}+\frac{\beta}{s_\beta}=(n+\alpha+\beta)\frac{1}{s}\left\{1-\frac{\Sigma_1^n k+\alpha k_\alpha+\beta k_\beta}{n+\alpha+\beta}+\frac{\Sigma_1^n k^2+\alpha k_\alpha^2+\beta k_\beta^2}{n+\alpha+\beta}-\frac{\Sigma_1^n k^3+\alpha k_\alpha^3+\beta k_\beta^3}{n+\alpha+\beta}+\text{etc.}\right\}$$

Hence

$$1+C=\left\{1+\frac{\Sigma_1^n k+\alpha k_\alpha+\beta k_\beta}{n+\alpha+\beta}\right\}\left\{1-\frac{\Sigma_1^n k+\alpha k_\alpha+\beta k_\beta}{n+\alpha+\beta}+\frac{\Sigma_1^n k^2+\alpha k_\alpha^2+\beta k_\beta^2}{n+\alpha+\beta}-\frac{\Sigma_1^n k^3+\alpha k_\alpha^3+\beta k_\beta^3}{n+\alpha+\beta}+\text{etc.}\right\} \tag{16}$$

Since $\frac{\Sigma_1^n k+\alpha k_\alpha+\beta k_\beta}{n+\alpha+\beta}s$ represents the average difference between s and the mean cross section of the particular portion of the tube to which the caliber factor is referred, it can be made zero or so small in comparison with unity by a suitable choice of s that the cross products (each including another very small term) obtained by multiplying the two factors are negligible. Moreover, the first-mentioned term enters with opposite signs in the two factors, so that it disappears from the product.

Hence,

$$1+C=1+\frac{\Sigma_1^n k^2+\alpha k_\alpha^2+\beta k_\beta^2}{n+\alpha+\beta}-\frac{\Sigma_1^n k^3+\alpha k_\alpha^3+\beta k_\beta^3}{n+\alpha+\beta}+\text{etc.} \tag{17}$$

The accuracy of the caliber factor will depend not only on the accuracy of the observations, but also on the magnitude of the range and the freedom from abrupt discontinuities in the cross section of the tube, since in the calculation each of the elements is supposed to be of uniform cross section throughout its length. The length of the elements must therefore be so short that this must apply to a sufficient degree of approximation. In cases where the cross-section curve deduced from the observations is sufficiently continuous, a further approximation can be made by increasing the number of elements, each of the original elements being considered as made up of two or more having the mean cross sections given by the curve, or more simply by computing the ratio of the sum of the resistances of the separate elements, regarding each as uniformly conical, with the terminal cross sections given by the curve, to their resistance computed as uniform cylinders, the ratio being applied as a correction to the caliber factor.

It must, however, be pointed out that variations within the elements, not following the general variations from element to element throughout the length of the tube, are not disclosed by the calibration, and, moreover, that the likelihood of the presence of such variations decreases as the range of the cross section of the tube decreases and as its general degree of uniformity increases.

B. THE DETERMINATION OF THE CALIBER CORRECTIONS

For the sake of completeness the following discussion of the methods of calibration and the reduction of the results is given, although they have been described a number of times, both in connection with the study of primary thermometers and of ohm tubes.

For simplicity consider a tube provided with an accurate graduation in the interval op which is to be calibrated.

Let v = mean volume per division between o and p.
v_{ab} = mean volume per division between a and b.
V_{ab} = volume between a and b.
s = mean cross section between o and p.
s_{ab} = mean cross section between a and b.

Then

$$v = cs \text{ and } v_{ab} = cs_{ab},$$

c depending upon the units in terms of which v and s are expressed.

If we put

$$V_{ab} = v\{(b-a) + (x_b - x_a)\}$$

Then

$$V_{op} = v\{p + (x_p - x_o)\}$$

But

$$V_{op} = v_p$$

Hence

$$x_p = x_o$$

If we put

$$x_o = o \text{ then } x_p = o$$

and

$$V_{oa} = v(a + x_a)$$
$$V_{ob} = v(b + x_b)$$

Obviously, vx_a, vx_b, and $v(x_b - x_a)$ represent the excess in volume of the respective intervals of the given tube over the corresponding intervals of a strictly uniform tube of the same mean cross section.

In addition,

$$V_{ab} = cs_{ab}(b-a) = v\{(b-a) + (x_b - x_a)\}$$

and since

$$v = cs$$

$$s_{ab} = s\frac{(b-a) + (x_b - x_a)}{(b-a)}$$

or

$$s_{ab} = s\left\{1 + \frac{x_b - x_a}{b-a}\right\} \tag{18}$$

The quantities x_a, x_b, etc., are called the caliber corrections of the tube referred to the interval op, and if determined for a number of equidistant points the relative mean cross sections are given bv the formula

$$s_m = s\left\{1 + \frac{x_{m+1} - x_m}{l}\right\}$$

If

$$s_m = s(1 + k_m),$$

$$k_m = \frac{x_{m+1} - x_m}{l} \tag{19}$$

From either of these the caliber factor of the tube can be calculated subject to the limitations pointed out below.

1. METHODS OF CALIBRATION

Various methods have been devised for calibrating capillary tubing, principally in connection with thermometers. These are, of course, directly applicable to ohm tubes. The following brief description of the theory is based on the appendix to Benoit's[1] paper dealing with the construction of mercury resistance standards for the use of the French post office.

The methods of calibration may be classified as: Simple; complete; abbreviated; combinations of the foregoing.

In the simple calibration of a tube it is considered to consist of n sections of length l. The length of a mercury column, having approximately the length l, is determined when placed in each of the sections. Correcting for the volumes of the two menisci, the relative cross sections are given approximately by the reciprocals of the corrected thread lengths.

---

[1] Construction des Étalons Prototypes de Résistance Électrique, R. Benoit, Paris, Gauthier-Villars, 1885.

In the complete calibration $(n-1)$ calibrating threads of lengths $\frac{1}{n}L$, $\frac{2}{n}L$ . . . . $\frac{n-1}{n}L$ are employed. The length of each thread is determined in all the possible positions, the ends being at or near the $n+1$ points for which the caliber corrections are sought. The number of observations is given by the formula $\frac{(n-1)(n+2)}{2}$, so that when n is large the process is long and tedious.

The abbreviated calibration, devised to minimize the labor involved in a complete calibration, consists in the employment of three or more threads of the lengths given above, each of which is measured in the same positions as in the complete calibration.

Combinations of the above methods are resorted to to further minimize the work, a calibration being first made by subdividing the tube into r sections each of which is further subdivided into $\frac{n}{r}$ sections by any of the methods above described.

The Simple Calibration.—Let x_1, x_2 . . . . x_{n+1} be the caliber corrections for $n+1$ equidistant points, p_1, p_2 . . . . p_{n+1} and let $p_1p_2 = l$.

Let $(m+1)\ l+\Delta_{m+1}$ and $ml+\Delta_m$ be the observed positions of the ends of the thread in the position $p_m\ p_{m+1}$ and let $l+\lambda$ be the length which the calibrating thread would have if measured in a uniform tube of the same mean cross section as that of the given tube between the points p_1 and p_{n+1}.

Then

$$V_{o\,m+1} - V_{o\,m} = v\ (l+\lambda)$$

However,

$$V_{o\,m+1} - V_{o\,m} = v\{(m+1)\ l+x_{m+1}+\Delta_{m+1}\} - v\,\{ml+x_m+\Delta_m\}$$

Hence

$$x_{m+1} - x_m + \Delta_{m+1} - \Delta_m = \lambda$$

Or

$$x_m - x_{m+1} + \lambda = \Delta_{m+1} - \Delta_m = a_m \tag{20}$$

a_m is obviously the excess of the thread length in the position $p_m\,p_{m+1}$ over the length of the subinterval l, and the x's, Δ's and λ must be expressed in terms of the same units.

The resulting n equations, employing the customary notation, are as follows:

$$\left.\begin{array}{l} x_1 - x_2 + \lambda \quad = a_1 \\ x_2 - x_3 + \lambda \quad = a_2 \\ \vdots \quad \vdots \quad \vdots \quad\quad \vdots \\ \vdots \quad \vdots \quad \vdots \quad\quad \vdots \\ x_n - x_{n+1} + \lambda = a_n \end{array}\right\} \tag{21}$$

These are sufficient to determine the x's and λ, since two of the x's can be arbitrarily fixed (which is equivalent to fixing v). The usual practice is to make x_1 and x_{n+1} zero, in which case the caliber corrections are referred to the interval $p_1\,p_{n+1}$.

Making $x_1 = x_{n+1} = o$ and adding the equations we obtain

$$n\lambda = \Sigma_1^n a \text{ or } \lambda = \frac{1}{n}\Sigma_1^n a \tag{22}$$

Substituting in succession in the observational equations the values of the x's are found.

Second Approximation.—In equation (20) the caliber corrections for the ends of the calibrating thread are taken as being equal to the correction at the precise points for which they are sought. This equation should have been written

$$x_{m+\Delta_m} - x_{m+1+\Delta_{m+1}} + \lambda = a_m$$

so that if the caliber corrections are changing appreciably allowance must be made as follows:

$$x_{m+\Delta_m} = x_m + \left(\frac{dx}{dl}\right)_m \Delta_m$$

$$x_{m+1+\Delta_{m+1}} = x_{m+1} + \left(\frac{dx}{dl}\right)_{m+1} \Delta_{m+1}$$

Substituting

$$x_m - x_{m+1} + \lambda = a_m - \left\{\left(\frac{dx}{dl}\right)_m \Delta_m - \left(\frac{dx}{dl}\right)_{m+1} \Delta_{m+1}\right\} \tag{23}$$

By plotting as ordinates the values of the x's first found a first approximation caliber curve is obtained. From this the approximate values of $\left(\frac{dx}{dl}\right)$ can be obtained for the points $p_1 \ldots . p_{n+1}$, so that the necessary correction can be applied to the a's in equations (21), and thus a second approximation to the x's and λ found. If the tube is very irregular, a third approximation, made in the same manner, may be required.

The Complete Calibration.—In the complete calibration as stated above, $n-1$ threads of lengths $\frac{1}{n}L$, $\frac{2}{n}L \ldots . \frac{n-1}{n}L$ are employed, each thread being measured in all possible positions, the ends of the thread being, however, at or near the points $p_1\ p_2 \ldots . p_{n+1}$. In this manner the following observational equations, expressed in the customary notation, are obtained:

$$\left|\begin{array}{l|l|l|l|l}
x_1-x_2\ +\lambda_n=a_{1\,2} & x_1\ -x_3\ +\lambda_{n-1}=a_{1\,3} & \ldots\ldots & x_1-x_{n-1}+\lambda_3=a_{1\,n-1} & x_1-x_n\ +\lambda_2=a_{1\ n} \\
x_2-x_3\ +\lambda_n=a_{2\,3} & x_2\ -x_4\ +\lambda_{n-1}=a_{2\,4} & \ldots\ldots & x_2-x_n\ +\lambda_3=a_{2\,n} & x_2-x_{n+1}+\lambda_2=a_{2\,n+1} \\
\cdot\quad\cdot\quad\cdot\quad\cdot & \cdot\quad\cdot\quad\cdot\quad\cdot & \ldots\ldots & x_3-x_{n+1}+\lambda_3=a_{3\,n+1} & \\
\cdot\quad\cdot\quad\cdot\quad\cdot & \cdot\quad\cdot\quad\cdot\quad\cdot & \ldots. & & \\
\cdot\quad\cdot\quad\cdot\quad\cdot & \cdot\quad\cdot\quad\cdot\quad\cdot & \ldots & & \\
\cdot\quad\cdot\quad\cdot\quad\cdot & \cdot\quad\cdot\quad\cdot\quad\cdot & .. & & \\
\cdot\quad\cdot\quad\cdot\quad\cdot & \cdot\quad\cdot\quad\cdot\quad\cdot & . & & \\
\cdot\quad\cdot\quad\cdot\quad\cdot & x_{n-1}-x_{n+1}+\lambda_{n-1}=a_{n-1\ n+1} & & & \\
x_n-x_{n+1}+\lambda_n=a_{n\ n+1} & & & &
\end{array}\right| \qquad (24)$$

There are thus $\frac{1}{2}(n-1)(n+2)$ equations for the determination of $(n-1)$ x's (2 being arbitrary) and the $(n-1)$ λ's. Methods for the solution of these equations according to the principles of least squares have been given. After finding the values of the x's and λ's, the computed values of the a's can be determined. Denoting the difference between the observed and computed values by Δ, an amount less than the error of observation on account of the least square adjustment of the results, we find as the most probable error of a single observation, for a calibration into tenths,

$$r = .6745\sqrt{\frac{\Sigma\Delta^2}{36}} \qquad (25)$$

The probable errors of the x's, expressed in terms of r, are as follows:

$$\left.\begin{array}{l} r\ (x_1) = r\ (x_{11}) = 0 \\ r\ (x_2) = r\ (x_{10}) = .416\ r \\ r\ (x_3) = r\ (x_9) = .401\ r \\ r\ (x_4) = r\ (x_8) = .387\ r \\ r\ (x_5) = r\ (x_7) = .376\ r \\ \qquad\quad r\ (x_6) = .369\ r \end{array}\right\} \qquad (26)$$

The Abbreviated Calibration into Tenths.—One of the most common of the many possible abbreviated methods of calibration and the principal method employed in the calibration of the four tubes here dealt with consists in limiting the number of threads to three, approximately four-tenths, five-tenths, and six-tenths of the length of the interval to be calibrated. The resulting observational equations are as follows:

$$\begin{array}{|c|c||c|} \hline x_1-x_5+\lambda_7=a_{1\ 5} & x_1-x_6+\lambda_6=a_{1\ 6} & x_1-x_7+\lambda_5=a_{1\ 7} \\ x_2-x_6+\lambda_7=a_{2\ 6} & x_2-x_7+\lambda_6=a_{2\ 7} & x_2-x_8+\lambda_5=a_{2\ 8} \\ \cdot\quad\cdot\quad\cdot\quad\cdot & \cdot\quad\cdot\quad\cdot\quad\cdot & \cdot \\ \cdot\quad\cdot\quad\cdot\quad\cdot & \cdot\quad\cdot\quad\cdot\quad\cdot & \cdot \\ \cdot\quad\cdot\quad\cdot\quad\cdot & \cdot\quad\cdot\quad\cdot\quad\cdot & x_5-x_{11}+\lambda_5=a_{5\ 11} \\ \cdot\quad\cdot\quad\cdot\quad\cdot & x_6-x_{11}+\lambda_6=a_{6\ 11} & \\ x_7-x_{11}+\lambda_7=a_{7\ 11} & & \\ \hline \end{array} \qquad (27)$$

The 18 equations are solved by the method of least squares for the nine independent x's and the three λ's. The method of computation, which has been reduced to its simplest form, is given by by Broch and Guilleaume.[3]

In the abbreviated calibration into tenths

$$r = .6745\sqrt{\frac{\Sigma\Delta^2}{6}} \qquad (28)$$

and the probable errors of the x's are as follows:

$$\left.\begin{array}{l} r\ (x_1) = r\ (x_{11}) = 0 \\ r\ (x_2) = r\ (x_{10}) = .836\ r \\ r\ (x_3) = r\ (x_9) = .864\ r \\ r\ (x_4) = r\ (x_8) = .840\ r \\ r\ (x_5) = r\ (x_7) = .616\ r \\ r\ (x_6) \qquad\quad = .416\ r \end{array}\right\} \qquad (29)$$

---

[3] Guillaume: Traité Pratique de la Thermométrie de Précision, p. 75.

Since the probable error of a single observation, r, is fixed by the accuracy of the observations, it should be the same for the abbreviated and complete calibrations. The mean probable error of the nine x's (excluding x_1 and x_{11}) is 0.773 r, while for the complete calibration into tenths it is 0.392 r, being reduced in a somewhat greater ratio than computed from the number of observations.

The most economical method of calibration, which needs no further description, consists, as stated above, in making a complete, abbreviated or simple calibration of the entire length into a suitable number of sections, each of which is in turn calibrated into sufficiently short elements by one of the above-described methods. This introduces the problem of expressing the caliber corrections in each section, and given in terms of its mean cross section, in terms of the mean cross section of the entire interval.

2. TRANSFORMATION OF CALIBER CORRECTIONS FROM ONE SYSTEM INTO ANOTHER

Let y_1 y_2, etc., denote the caliber corrections in the first system and x_1 x_2, etc., the caliber corrections in the second system.

Denoting the points at the ends of the smaller section by a and b, the caliber corrections in the "x" system by x_a and x_b and in the "y" system by y_a and y_b and the mean cross sections in the two systems by s_x and s_y we have

$$s_x\{(b+x_b)-(a+x_a)\}=s_y\{(b+y_b)-(a+y_a)\} \tag{30}$$

Hence

$$\frac{s_y}{s_x}=\frac{(b+x_b)-(a+x_a)}{(b+y_b)-(a+y_a)}\equiv B \tag{31}$$

To find the caliber correction for any point in terms of the "y" system if it is given in terms of the "x" system, we have

$$\begin{aligned}
y_m&=y_a+\int_a^m \frac{s-s_y}{s_y}\,dl\\
&=y_a-(m-a)+\frac{1}{s_y}\int_a^m s\,dl\\
&=y_a-(m-a)+\frac{s_x}{s_y}\{(m+x_m)-(a+x_a)\}\\
&=y_a+\frac{x_m-x_a+(1-B)\ (m-a)}{B}
\end{aligned} \tag{32}$$

The cross section of the tube at a given point must of course be the same in the two systems

$$s_m = s_x\left(1 + \frac{dx_m}{dl}\right) = s_y\left(1 + \frac{dy_m}{dl}\right) \tag{33}$$

By means of the transformation formula (32) the caliber corrections in the "y" system can therefore readily be calculated from those on the "x" system (or vice versa), if the caliber corrections for the same two points are known in both systems. In most cases the "x" corrections are taken to represent those obtained from the calibration of the separate sections and the "y" corrections those referred to the whole length of the calibrated portion of the tube. The same method can be applied to the calculation of a new system of corrections referred to the particular portion of the tube selected as representing unit resistance. In most cases a simpler calculation will suffice for the latter purpose.

C. THE CORRECTIONS FOR THE TERMINAL BULBS

In accordance with the London specifications, connections for electrical comparisons must be made by the aid of spherical terminal bulbs 4 cm in internal diameter. In addition, the London conference adopted as the correction for the added resistance introduced by the terminal bulbs the formula:

$$R_{T_1} + R_{T_2} = \frac{0.80}{\pi 1063}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

ohm, r_1 and r_2 being expressed in millimeters. In this it is assumed that the correction factor is 0.80 and that the terminal cross section of the ohm tube is circular, the tube cylindrical, and its ends forming continuations of the internal surface of the terminal bulb, all of which may be closely approximated by careful selection of tubes and by proper construction.

The factor 0.80 seems to agree better with the results of actual measurement than 0.82, which was fixed as the most probable value by Lord Rayleigh. As far as reproducibility is concerned, any uncertainty in the value employed will affect the results by the same amount so long as the end correction bears an approximately constant ratio to the resistance contributed by the tube proper, the dimensions of the terminal bulb being specified. The possible variation due to an uncertainty of 0.01 in the factor is shown, for 1-ohm tubes, in Table 1.

TABLE 1

Ohm length	Correction for terminal bulbs	Error due to uncertainty of 0.01 in correction factor	Ohm length	Correction for terminal bulbs	Error due to uncertainty of 0.01 in correction factor
Centimeters	Microhms	Microhms	Centimeters	Microhms	Microhms
120	799.3	10.0	80	978.9	12.2
115	816.4	10.2	75	1011.0	12.6
110	834.8	10.4	70	1046.5	13.1
105	854.4	10.7	65	1086.0	13.6
100	875.5	10.9	60	1130.3	14.1
95	898.3	11.2	55	1180.6	14.8
90	922.9	11.5	50	1238.2	15.5
85	949.7	11.9			

Thus, an uncertainty of 0.01 in the assumed value of the correction factor would in itself introduce a discrepancy of five parts in one million between two tubes having lengths of 50 and 120 cm. Such differences in length are found among tubes of the same set, and might have to be taken into account in comparing the results of different investigators. In the interest of the highest accuracy of reproduction it would, therefore, seem desirable to specify that the mean length of tubes of a set should not depart from 100 cm. by more than would correspond to an uncertainty of one part in one million. This would allow a range of 15 cm and would not be too difficult to realize. A redetermination of the value of the correction factor seems, however, highly desirable for the particular type of terminal bulb adopted and should form the subject of a further investigation.

It also seems desirable, in the absence of any experimental data and in order to eliminate a further possible source of discrepancy, to more exactly specify the dimensions of the end surface of the tube. Since the principal part of the resistance is found close to the end of the capillary, it is evident that the effect of imperfections in the end bulb, particularly adjoining the end surface of the tube, will be lessened as the area of the latter is increased up to a certain limit.

The data for the calculation of the correction for the terminal bulbs are obtained from the relative cross-section curve deduced from the caliber corrections and from the mean cross section of the interval to which the calibration is referred. No error is introduced if the density of the mercury is taken as 13.5956, from which the mass given in the definition of the ohm is derived.

PART II.—EXPERIMENTAL DETAILS

The formula for calculating the resistance of the finished tube being given in terms of L_0, M_0, the caliber factor, and the correction for the terminal bulbs as above described, the construction of mercurial resistance standards may be regarded as comprising the following steps:

A. The selection of a suitable number of tubes of sufficiently uniform cross section, their straightening and annealing.

B. The calibration of the tubes (also the calibration of the graduations, if the measurements of thread length are referred to graduations on the tube).

C. The determination of the approximate ohm lengths, allowance being made for the corrections for terminal bulbs and caliber factor.

D. The selection of the points at which the tubes are to be cut, so that the terminal planes will lie where the cross sections can be most accurately interpolated.

E. The cutting of the tubes, grinding to approximate length, and polishing of the end surfaces. (Also tapering for the terminal bulbs).

F. The calculation of the caliber factors for the finished tubes.

G. The determination of M_0.

H. The determination of L_0.

I. The calculation of the corrections for the terminal bulbs.

J. The calculation of the resistance of the finished tubes.

K. The electrical intercomparison with secondary standards.

L. Results of electrical comparisons.

A. SELECTION OF THE TUBES

A number of lots of tubing were procured from the Jena Glass Works and subjected to a preliminary calibration. Some very good tubes were found, but as the probable error of the caliber factor decreases as the caliber factor itself decreases, and, as pointed out above, it was found desirable to have the ohm length approximately one meter, a special drawing of 59^{III} borosilicate tubing was made for the Bureau by Schott and Genossen, for which we hereby express our appreciation. As the tubes were to be provided with a millimeter graduation, 10 kilos were sent to M. Baudin, the Paris thermometer manufacturer, with instructions to select by preliminary calibration the four tubes most nearly answering specifications furnished, to anneal and straighten them, and to provide each with a millimeter graduation extending over an interval of 110 cm.

Four tubes of "Verre Dur" were also obtained through M. Baudin, who in this case superintended the drawing of the tubing himself. These have not yet been used, principally on account of their small external diameter, although having caliber factors somewhat less than the Jena tubes. The graduation of all the tubes was exceptionally uniform and the lines very sharp, their width not exceeding 50 microns, a result particularly difficult to accomplish with Jena 59^{III} glass.

B. CALIBRATION OF THE TUBES

The comparator used in the calibration of the tubes was an old one which, by slight alterations, was adapted to the purpose. It consists of a massive iron base supporting two short stout angle beams, carrying a pair of parallel horizontal brass bars 105 cm long on which are mounted two micrometer microscopes. The minimum distance between the microscopes is 7 cm, so that no direct observations on shorter threads could be made. The microscopes have an approximate focal length of 20 cm, and when focused on the base of the meniscus of a mercury column in the tube, the tip of the meniscus and the graduations of the tube were also in sufficiently good focus, so that observations could be made with the required accuracy without refocusing.

The micrometer value, checked frequently by comparison with a subdivided scale, was approximately 3.7μ per division of the head, or 370μ per turn. The tubes were mounted in five V-grooved supports aligned 22 cm apart on a steel T beam, and held in position by springs directly over each support. They were thus rigidly supported, and any residual curvature was reduced.

The tubes were cleaned and dried for each calibrating thread employed. The method consisted in first filling the tube with fuming sulphuric acid. After five minutes the acid was removed by suction and the tube washed with several hundred cc of double distilled water. This method was used throughout in the calibration of tube No. 1 and in the calibrations of the remaining tubes with 20-cm threads. In the subcalibration of tubes No. 2, No. 3, and No. 4 the above treatment was followed by washing with a 20 per cent solution of sodium hydroxide and again with double distilled water.

Drying was effected by drawing air, first passed through a calcium chloride bulb, through the tube for a sufficient length of time.

The calibrating threads of double distilled mercury were introduced by means of a glass tube drawn out to a capillary sufficiently small to permit its insertion into the bore. After adjusting the thread length, so as to minimize the second approximation correction, calcium chloride bulbs were attached to either end of the tube to prevent condensation of moisture within. The threads were brought into the desired positions by pressure applied to the rubber tubing by which the drying bulbs were attached.

A simple 20-cm calibration of each tube with two different threads was first made for the interval 5–105 cm. Readings were made for the successive positions at each end of the thread on both the tip and base of the meniscus as well as on the two adjacent millimeter divisions of the graduations; the observations were then repeated in the reverse order. The graduation corrections were applied to all 20-cm threads of the four tubes and also to all threads used in the calibration of tube No. 1.

The subcalibration of the 20-cm sections into tenths was made by taking 8, 10, and 12 cm threads and displacing them 2 cm at a time over the 20-cm section, the observations being repeated in the reverse order. For the end sections the calibration was extended so as to include the 1, 3, 107, and 109 cm points. In the calibration of tube No. 1 the thread lengths were referred to the graduations as described above, but with tubes Nos. 2, 3, and 4, the microscopes were set at a fixed distance for a given thread and the slightly varying lengths expressed in terms of this fixed distance and the micrometer settings. The end sections, 0–25 and 85–110 cm, were also calibrated into tenths by using threads of 10, 12.5, and 15 cm in length. This was done to gain additional information about those portions of the tubes in which the cuts were to be located. The tubes were always recleaned and dried after a thread had been used in calibrating a single section.

To secure a suitable accuracy of the observations on the tip of the meniscus particular attention had to be given to the question of illumination. The desired result was obtained by the aid of an evenly illuminated ground glass plate, extending over the full width of the comparator, and white diffusing screens above, below, and behind the tubes. By cutting off the lateral illumination by means of two blackened metal disks about 4 cm in diameter, held 3 mm apart and each joined to a sleeve 1 cm long and of a diameter slightly larger than that of the ohm tubes, the meniscus appeared black and sharply defined against a white background, thus greatly increasing the ease and accuracy of the observations.

The correction for the menisci was calculated on the assumption that the bounding surfaces were spherical. The length of cylindrical columns having the same volume as a meniscus of height h in a tube of radius r, is given by the formula:

$$\Delta l = \frac{h}{2}\left(1 + \frac{1}{3}\frac{h^2}{r^2}\right)$$

For the mean meniscus height found, the term $\frac{1}{3}\frac{h^2}{r^2}$ amounts to approximately 0.04, and since the maximum variation did not exceed 0.1 mm, the average variation amounting to less than 0.03 mm, the factor was regarded as constant and the corrected thread length was calculated by adding to the length of the cylindrical portion 4 per cent of the mean observed meniscus height. The latter was particularly uniform when the 20 per cent sodium hydroxide solution was used in cleaning the tube.

The temperature adjacent to the thread under measurement was always taken at the beginning and end of each set of observations for each position, and the whole set of observations on a given thread length in the several positions throughout a section reduced to a common temperature.

The variation in temperature of a thread throughout a complete set of observations was slight. The effect of the small progressive changes was, moreover, eliminated by the method of procedure, the observations being always repeated in the reverse order. The above remarks also apply to the variations in the temperature of the bar supporting the miscroscopes. From the results obtained the caliber corrections for the 25, 45, 65, and 85 cm points were first calculated, the corrections for the 5 and 105 cm points, limiting the interval to which the calibration was in each case referred, being put equal to zero.

The observations made for the subcalibration of these 20-cm sections and the 0–25 and 85–110 cm end sections were reduced by the least squares method as given by Guillaume.[8] The caliber corrections thus calculated referred to the particular sections of the tube. From these the corresponding caliber corrections in the interval 5–105 cm were determined. In each case a second approximation correction was applied and the caliber corrections recalculated.

The corrections for the 1 and 3 cm points were determined by the method given in Guillaume, the three threads employed in the calibration of the 5–25 cm section being also measured in the 1–9, 3–11, 1–11, 3–13, 1–13, and 3–15 positions. This enabled the corrections at the 1 and 3 cm points to be determined in terms of the x's in the 5–25 cm interval and the three λ's computed as indicated above, the general formula applicable being $x_m - x_n + \lambda = a$. The same procedure was followed for the determination of the 107 and 109 cm points.

The results are given in Tables 2 to 5 and the four curves following. The quantity, $\frac{s-s_m}{s_m}=\frac{y_{m+1}-y_m}{l}$ tabulated in the last column is denoted by k_m in formula (19), page 383, from which the excess of the caliber factor above unity for the separate sections of each of the four tubes is calculated. The close agreement of the results obtained by the first and second approximations as shown by Table 6 is to be attributed in part to the uniformity of the tubes, but mainly to the careful adjustment of the thread lengths employed, the same thread being used in only one section in the subcalibration into tenths. Attention is also called to the agreement between the 20 and 25 mm calibrations of the end sections. While at first sight it might appear that the values given by the former should *always* exceed those given by the latter, excluding experimental errors, it is obvious that the sign of the difference will depend in part on the location of the irregularities in the curves with respect to the particular subdivisions selected.

---

[8] Guillaume, Traité Pratique de la Thermométrie de Précision, p. 75 et seq.

TABLE 2

Tube No. 1

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm	
1	−0.409	−0.220	+0.00375
3	− .240	− .145	+ .00725
5	.000	.000	+ .00300
7	+ .154	+ .060	− .00355
9	+ .177	− .011	− .00995
11	+ .072	− .210	− .01885
13	− .213	− .587	− .01505
15	− .421	− .888	− .00665
17	− .461	−1.021	− .00155
19	− .387	−1.052	+ .00220
21	− .259	−1.008	+ .00340
23	− .096	− .940	+ .00010
25	.000	− .938	
25	.000	− .938	− .00740
27	− .069	−1.086	− .01280
29	− .247	−1.342	− .00850
31	− .337	−1.512	− .01095
33	−0.478	−1.731	−0.01065
35	− .612	−1.944	− .00675
37	− .669	−2.079	+ .00130
39	− .562	−2.053	+ .00125
41	− .458	−2.028	+ .01095
43	− .158	−1.809	+ .00395
45	.000	−1.730	
45	.000	−1.730	− .00570
47	− .254	−1.844	− .00530
49	− .500	−1.950	+ .00220
51	− .598	−1.906	+ .00820
53	− .576	−1.742	+ .01145
55	− .490	−1.513	+ .01355
57	− .361	−1.242	+ .01080
59	− .288	−1.026	+ .02145
61	− .008	− .597	+ .01080
63	+ .070	− .381	+ .00855
65	.000	− .310	

TABLE 2—Continued

Tube No. 1—Continued

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
65	0.000	−0.310	+0.00060	0	0.000	−0.186	+0.00036
67	+ .132	− .298	+ .00360	2.5	+ .085	− .177	+ .00708
69	+ .324	− .226	+ .00285	5	+ .338	.000	+ .00268
71	+ .501	− .169	− .00435	7.5	+ .480	+ .067	− .00512
73	+ .533	− .256	− .00555	10	+. 428	− .061	− .01612
75	+ .542	− .367	− .01750	12.5	+ .099	− .464	− .01660
77	+ .309	− .717	− .01535	15	− .243	− .879	− .00512
79	+ .120	−1.024	− .01015	17.5	− .295	−1.007	+ .00004
81	+ .036	−1.227	− .00445	20	− .219	−1.006	+ .00196
83	+ .065	−1.316	− .00920	22.5	− .095	− .957	+ .00076
85	.000	−1.500		25	.000	− .938	
85	.000	−1.500	− .00645	85	.000	−1.500	− .00524
87	− .277	−1.629	+ .00025	87.5	− .397	−1.631	+ .00168
89	− .421	−1.624	+ .00360	90	− .623	−1.589	+ .00248
91	− .499	−1.552	+ .00045	92.5	− .829	−1.527	− .00032
93	− .639	−1.543	− .00030	95	−1.104	−1.535	+ .01316
95	− .793	−1.549	+ .01200	97.5	−1.046	−1.206	+ .01704
97	− .703	−1.309	+ .01905	100	− .892	− .780	+ .01148
99	− .474	− .928	+ .01290	102.5	− .876	− .493	+ .01972
101	− .379	− .682	+ .01355	105	− .655	.000	+ .02472
103	− .259	− .411	+ .02055	107.5	− .311	+ .618	+ .02380
105	.000	− .000	+ .02375	110	.000	+1.203	
107	+ .322	+ .475	+ .02355				
109	+ .641	+ .946					

TABLE 3

Tube No. 2

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
1	+0.227	+0.068	−0.00285	7	−0.096	−0.016	−0.00065
3	+ .091	+ .011	− .00060	9	− .188	− .029	+ .00180
5	.000	.000	− .00080	11	− .232	+ .007	+ .00365

TABLE 3—Continued

Tube No. 2—Continued

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm	
13	−0.239	+0.080	+0.00295
15	− .260	+ .139	+ .00460
17	− .248	+ .231	+ .00805
19	− .168	+ .392	+ .00830
21	− .081	+ .558	+ .00720
23	− .018	+ .702	+ .00490
25	.000	+ .800	
25	.000	+ .800	+ .00170
27	+ .091	+ .834	− .00830
29	− .018	+ .668	− .00135
31	+ .011	+ .641	+ .00145
33	+ .097	+ .670	− .00200
35	+ .114	+ .630	− .00410
37	+ .088	+ .548	− .00735
39	− .003	+ .401	− .00510
41	− .048	+ .299	− .00215
43	− .035	+ .256	− .00110
45	.000	+ .234	
45	.000	+ .234	+ .00395
47	+ .101	+ .313	+ .00210
49	+ .165	+ .355	+ .00015
51	+ .189	+ .358	− .00110
53	+ .189	+ .336	− .00135
55	+ .184	+ .309	− .00745
57	+ .057	+ .160	− .01565
59	− .234	− .153	− .00395
61	− .292	− .232	+ .00340
63	− .202	+ .164	+ .00900
65	.000	+ .016	
65	.000	+ .016	+ .00920
67	− .002	+ .200	+ .01230
69	+ .059	+ .446	+ .01475
71	+ .168	+ .741	+ .01395

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm	
73	+0.261	+1.020	+0.01360
75	+ .346	+1.292	+ .01155
77	+ .392	+1.523	+ .00835
79	+ .375	+1.690	+ .00470
81	+ .284	+1.784	+ .00420
83	+ .183	+1.868	.00000
85	.000	+1.868	
85	.000	+1.868	− .00520
87	+ .084	+1.764	− .01185
89	+ .031	+1.525	− .01355
91	− .054	+1.254	− .01210
93	− .110	+1.012	− .01170
95	− .157	+ .778	− .01060
97	− .183	+ .566	− .00555
99	− .106	+ .455	− .00435
101	− .006	+ .368	− .00665
103	+ .049	+ .235	− .01175
105	.000	.000	− .01180
107	− .050	− .236	− .01050
109	− .073	− .446	
0	.000	+ .146	− .00568
2.5	− .207	+ .004	− .00016
5	− .276	.000	− .00136
7.5	− .375	− .034	+ .00040
10	− .430	− .024	+ .00352
12.5	− .408	+ .064	+ .00272
15	− .406	+ .132	+ .00488
17.5	− .350	+ .254	+ .00912
20	− .187	+ .482	+ .00748
22.5	− .066	+ .669	+ .00524
25	.000	+ .800	
85	.000	+1.868	− .00632
87.5	+ .082	+1.710	− .01320

TABLE 3—Continued

Tube No. 2—Continued

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm	
90	−0.009	+1.380	−0.01248
92.5	− .082	+1.068	− .01140
95	− .128	+ .783	− .01000
97.5	− .138	+ .533	− .00512
100	− .024	+ .405	− .00492
102.5	+0.094	+0.282	−0.01128
105	+ .051	.000	− .01168
107.5	− .002	− .292	− .00952
110	.000	− .530	

TABLE 4

Tube No. 3

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm	
1	−0.320	−0.651	+0.01865
3	− .113	− .278	+ .01390
5	.000	.000	+ .00530
7	− .058	+ .106	+ .00245
9	− .172	+ .155	+ .00595
11	− .217	+ .274	+ .00675
13	− .246	+ .409	+ .00940
15	− .223	+ .597	+ .01245
17	− .139	+ .846	+ .01400
19	− .025	+1.126	+ .01215
21	+ .053	+1.369	+ .00800
23	+ .049	+1.529	+ .00575
25	.000	+1.644	
25	.000	+1.644	+ .00580
27	+ .074	+1.760	+ .00255
29	+ .083	+1.811	+ .00030
31	+ .047	+1.817	− .00110
33	− .017	+1.795	+ .00085
35	− .042	+1.812	+ .00430
37	+ .001	+1.898	+ .00575
39	+ .074	+2.013	+ .00385
41	+0.109	+2.090	+0.00015
43	+ .070	+2.093	− .00140
45	.000	+2.065	
45	.000	+2.065	+ .00085
47	+ .012	+2.082	+ .00080
49	+ .022	+2.098	+ .00365
51	+ .090	+2.171	+ .00490
53	+ .182	+2.269	+ .00630
55	+ .302	+2.395	+ .00400
57	+ .377	+2.475	− .00350
59	+ .302	+2.405	− .00470
61	+ .202	+2.311	− .00460
63	+ .106	+2.219	− .00505
65	.000	+2.118	
65	.000	+2.118	− .00115
67	− 1.06	+2.095	+ .00100
69	+ .032	+2.115	− .00090
71	+ .031	+2.097	+ .00560
73	+ .161	+2.209	+ .00170
75	+ .212	+2.243	− .00440
77	+ .141	+2.155	− .00690

TABLE 4—Continued

Tube No. 3—Continued

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
79	+0.019	+2.017		5	+0.347	0.000	
			−0.00325				+0.00512
81	− .029	+1.952		7.5	+ .227	+ .128	
			− .00005				+ .00368
83	− .012	+1.951		10	+ .071	+ .220	
			− .00025				+ .00648
85	.000	+1.946		12.5	− .016	+ .382	
							+ .00856
				15	− .050	+ .596	
85	.000	+1.946					+ .01288
			− .00450	17.5	+ .022	+ .918	
87	+ .105	+1.856					+ .01328
			− .01480	20	+ .104	+1.250	
89	+ .003	+1.560					+ .00992
			− .01145	22.5	+ .102	+1.498	
91	− .031	+1.331					+ .00584
			− .01010	25	.000	+1.644	
93	− .039	+1.129					
			− 0.1275				
95	− .100	+ .874		85	.000	+1.946	
			− .01365				− .00640
97	− .179	+ .601		87.5	+ .043	+1.786	
			− .01320				− .01436
99	− .249	+ .337		90	− .114	+1.427	
			− .00670				− .00976
101	− .188	+ .203		92.5	− .156	+1.183	
			− .00575				− .01232
103	− .108	+ .088		95	− .263	+ .875	
			− .00440				− .01424
105	.000	.000		97.5	− .417	+ .519	
			− .00200				− .01044
107	+ .156	− .040		100	− .476	+ .258	
			− .00135				− .00540
109	+ .325	− .067		102.5	− .407	+ .123	
							− .00492
				105	− .327	.000	
0	.000	− .849					− .00180
			+ .01992	107.5	− .168	− .045	
2.5	+ .246	− .351					− .00144
			+ .01404	110	.000	− .081	

TABLE 5

Tube No. 4

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
1	+0.183	+0.268		17	+0.205	−0.054	
			−0.00830				−0.00795
3	+ .059	+ .102		19	+ .089	− .213	
			− .00510				− .00405
5	.000	.000		21	+ .050	− .294	
			− .00085				− .00315
7	+ .026	− .017		23	+ .032	− .357	
			− .00335				− .00365
9	+ .002	− .084		25	.000	− .430	
			+ .00105				
11	+ .066	− .063		25	.000	− .430	
			+ .00290				− .00700
13	+ .167	− .005		27	+ .038	− .570	
			+ .00340				− .01865
15	+ .278	+ .063		29	− .161	− .943	
			− .00585				− .01205

TABLE 5—Continued

Tube No. 4—Continued

Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$	Graduations in centimeters	x	y	$k=\frac{s-s_m}{s_m}$
	mm	mm			mm	mm	
31	−0.225	−1.184	−0.00520	89	−0.039	−2.153	+0.01585
33	− .152	−1.288	− .00805	91	+ .013	−1.836	+ .01680
35	− .135	−1.449	− .01425	93	+ .083	−1.500	+ .01790
37	− .245	−1.734	− .01145	95	+ .176	−1.142	+ .01765
39	− .297	−1.963	− .00335	97	+ .264	− .789	+ .01960
41	− .185	−2.030	− .00220	99	+ .390	− .397	+ .01135
43	− .052	−2.074	− .00630	101	+ .354	− .170	+ .00715
45	.000	−2.200		103	+ .234	− .027	+ .00135
				105	.000	.000	+ .00310
45	.000	−2.200	− .01390	107	− .200	+ .062	+ .00715
47	− .123	−2.478	− .01185	109	− .319	+ .205	
49	− .205	−2.715	− .00880				
51	− .224	−2.891	− .00875	0	.000	+ .412	− .01256
53	− .244	−3.066	− .00465	2.5	− .230	+ .098	− .00392
55	− .180	−3.159	− .00970	5	− .244	.000	− .00152
57	− .219	−3.353	− .01350	7.5	− .198	− .038	− .00260
59	− .333	−3.623	− .00650	10	− .178	− .103	+ .00236
61	− .307	−3.753	− .00060	12.5	− .035	− .044	+ .00396
63	− .162	−3.765	+ .00025	15	+ .149	+ .055	− .00568
65	.000	−3.760		17.5	+ .091	− .087	− .00828
				20	− .032	− .294	− .00200
65	.000	−3.760	− .00700	22.5	+ .002	− .344	− .00344
67	− .250	−3.900	− .00280	25	.000	− .430	
69	− .418	−3.956	+ .00460				
71	− .437	−3.864	+ .00850	85	.000	−2.641	+ .01116
73	− .380	−3.694	+ .01065	87.5	− .010	−2.362	+ .01424
75	− .280	−3.481	+ .00140	90	+ .056	−2.006	+ .01676
77	− .363	−3.453	+ .00675	92.5	+ .184	−1.587	+ .01760
79	− .340	−3.318	+ .01230	95	+ .333	−1.147	+ .01848
81	− .207	−3.072	+ .01085	97.5	+ .504	− .685	+ .01616
83	− .102	−2.855	+ .01075	100	+ .617	− .281	+ .00952
85	.000	−2.640		102.5	+ .566	− .043	+ .00172
				105	+ .323	.000	+ .00320
85	.000	−2.640	+ .01170	107.5	+ .117	+ .080	+ .00680
87	− .029	−2.406	+ .01265	110	.000	+ .250	

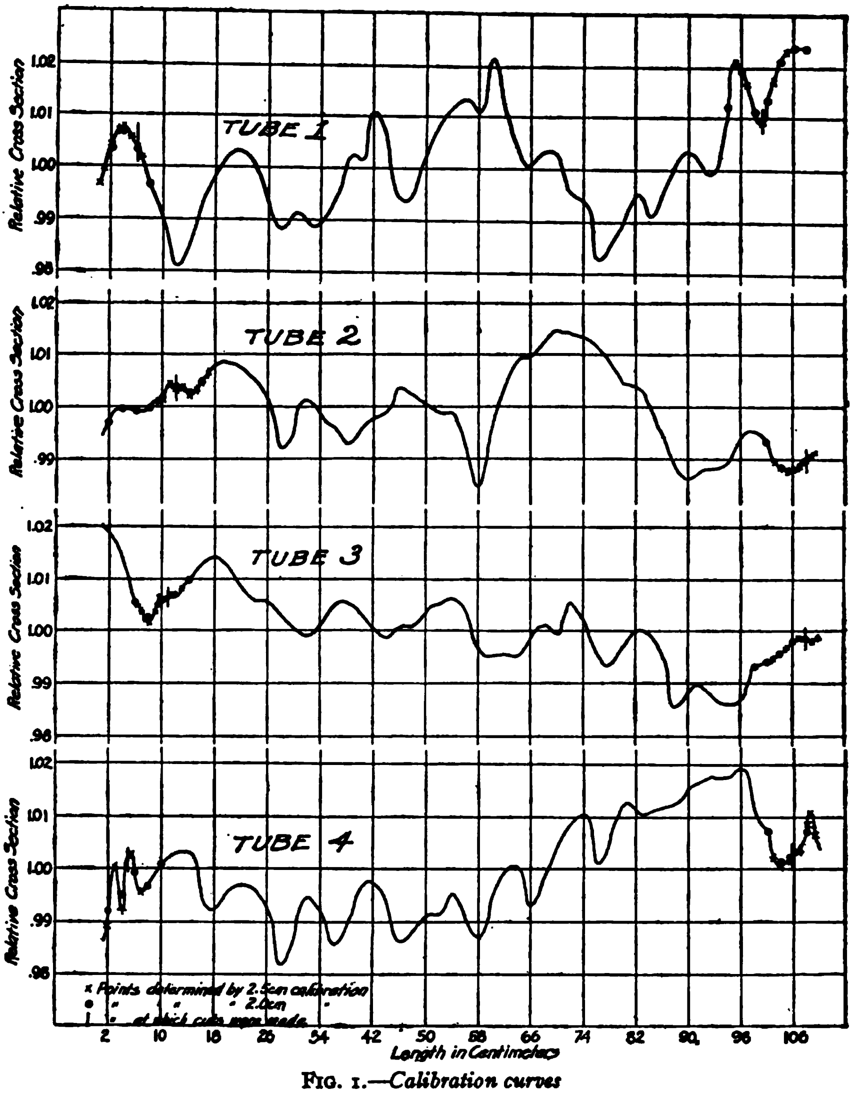


FIG. 1.—*Calibration curves*

TABLE 6

Corrections for Lack of Uniformity (In Parts per Million).

Tube	Interval	20-mm calibration		25-mm calibration	
		First approximation	Second approximation	First approximation	Second approximation
No. 1	5—25	15.27	15.54	15.06	15.21
	25—45	14.54	14.26		
	45—65	22.49	22.73		
	65—85	16.15	16.63		
	85—105	24.78	25.95	24.65	25.12
	5—105	93.23	95.11		
No. 2	5—25	5.16	5.10	5.28	5.27
	25—45	3.74	3.67		
	45—65	8.72	8.71		
	65—85	21.31	21.25		
	85—105	20.38	19.85	19.37	19.72
	5—105	59.31	58.58		
No. 3	5—25	15.84	15.81	15.72	15.58
	25—45	2.20	2.20		
	45—65	3.46	3.50		
	65—85	2.35	2.31		
	85—105	22.02	22.13	21.59	21.75
	5—105	45.87	45.95		
No. 4	5—25	3.40	3.42	3.53	3.69
	25—45	20.98	20.75		
	45—65	16.96	16.76		
	65—85	13.96	13.79		
	85—105	40.21	39.91	39.82	39.71
	5—105	95.51	94.63		

A rapid method of calibration employed at the National Physical Laboratory[4] consisted in making five separate series of measurements of 5-cm threads. Each tube was provided with a reference mark, and the measurements were usually made over an interval 10 to 15 cm beyond the limits selected for use.

The first series of readings consisted in the linear measurements (by the aid of a simple comparator) of a thread of mercury, approximately 5 cm in length, in successive 5-cm intervals. The intervals were usually designated 0–5, 5–10, 10–15, etc. The second series consisted in determining the thread lengths for the positions 1–6, 6–11, 11–16, etc., and similarly for the third, fourth, and fifth series, approximately six hours being required for the five series of measurements.

---

[4] F. E. Smith, "On the Construction of some mercury standards of resistance," etc. Phil. Trans., A. 204, pp. 66–67; 1904.

The caliber factor was computed from each series of measurements and also from the combined data of the entire set of measurements. Separate corrections were applied for conicality. There is no data given as to the order of agreement of the caliber factor as separately determined.

A preliminary simple calibration of each of the four Bureau of Standards tubes was made in order to determine the approximate magnitude of their caliber factors. The following readings on 5-cm threads (made with the aid of a hand reading glass only) together with the computation of the caliber factors of each tube are given in Table 7.

While the caliber factors of tubes No. 2 and No. 3, thus determined agree very closely with those given in Table 6, we feel that the larger discrepancies shown in the case of tubes No. 1 and No. 4 have justified the more accurate calibration made. Table 8 is a summary of four separate calculations of the caliber factor of each tube and is self-explanatory.

TABLE 7

Observational Data from a Simple Calibration of the 4 B. S. Tubes together with the Computation of Their Caliber Factors

Interval	Tube No. 1				Tube No. 2			
	Thread length in centimeters			$\frac{1}{l}$	Thread length in centimeters			$\frac{1}{l}$
	Down	Up	Mean		Down	Up	Mean	
5- 10	5.01	5.00	5.005	0.1998002	5.04	5.03	5.035	0.1986097
10- 15	5.09	5.08	5.085	.1966568	5.01	5.015	5.012	.1995211
15- 20	5.03	5.02	5.025	.1990050	5.00	5.00	5.000	.2000000
20- 25	5.00	5.00	5.000	.2000000	5.00	5.00	5.000	.2000000
25- 30	5.04	5.04	5.040	.1984127	5.03	5.04	5.035	.1986097
30- 35	5.06	5.05	5.055	.1978239	5.03	5.04	5.035	.1986097
35- 40	5.01	5.02	5.015	.1994018	5.055	5.06	5.058	.1977066
40- 45	4.96	4.965	4.962	.2015316	5.06	5.05	5.055	.1978239
45- 50	5.03	5.03	5.030	.1988072	5.015	5.02	5.018	.1992826
50- 55	4.97	4.97	4.970	.2012072	5.045	5.04	5.042	.1983340
55- 60	4.95	4.93	4.940	.2024291	5.09	5.08	5.085	.1966568
60- 65	4.96	4.96	4.960	.2016129	5.02	5.02	5.020	.1992032
65- 70	4.99	4.99	4.990	.2004008	4.975	4.98	4.978	.2008839
70- 75	5.00	5.01	5.005	.1998002	4.96	4.95	4.965	.2018163
75- 80	5.08	5.07	5.075	.1970443	4.98	4.985	4.982	.2007226
80- 85	5.04	5.03	5.035	.1986097	5.02	5.02	5.020	.1992032
85- 90	5.00	5.01	5.005	.1998002	5.09	5.09	5.090	.1964637
90- 95	4.98	4.99	4.985	.2006018	5.10	5.10	5.100	.1960784
95-100	4.91	4.91	4.910	.2036660	5.075	5.075	5.075	.1970443
100-105	4.92	4.91	4.915	.2034588	5.08	5.075	5.078	.1969279
			100.007	4.0000702			100.679	3.9734976
	$\frac{100.007\times4.0000702}{400}=1.0000876$				$\frac{100.679\times3.9734976}{400}=1.0000598$			

TABLE 7—Continued

Interval	Tube No. 3				Tube No. 4			
	Thread length in centimeters			$\frac{1}{l}$	Thread length in centimeters			$\frac{1}{l}$
	Down	Up	Mean		Down	Up	Mean	
5- 10	4.93	4.91	4.920	0.2032520	4.99	5.00	4.995	0.2002002
10- 15	4.89	4.88	4.885	.2047083	4.97	4.97	4.970	.2012072
15- 20	4.87	4.86	4.865	.2055498	5.02	5.01	5.015	.1994018
20- 25	4.89	4.89	4.890	.2044990	4.98	4.99	4.985	.2006018
25- 30	4.90	4.90	4.900	.2040816	5.04	5.04	5.040	.1984127
30- 35	4.915	4.93	4.922	.2031694	5.00	5.01	5.005	.1998002
35- 40	4.91	4.91	4.910	.2036660	5.03	5.04	5.035	.1986097
40- 45	4.94	4.94	4.940	.2024291	5.00	5.00	5.000	.2000000
45- 50	4.93	4.93	4.930	.2028398	5.04	5.03	5.035	.1986097
50- 55	4.90	4.90	4.900	.2040816	5.02	5.02	5.020	.1992032
55- 60	4.945	4.93	4.938	.2025111	5.03	5.03	5.030	.1988072
60- 65	5.95	4.95	4.950	.2020202	4.98	4.98	4.980	.2008032
65- 70	4.94	4.94	4.940	.2024291	4.99	5.00	4.995	.2002002
70- 75	4.905	4.91	4.908	.2037490	4.95	4.94	4.945	.2022245
75- 80	4.95	4.95	4.950	.2020202	4.95	4.955	4.952	.2019386
80- 85	4.94	4.95	4.945	.2022245	4.92	4.92	4.920	.2032520
85- 90	4.99	4.985	4.988	.2004812	4.91	4.92	4.915	.2034588
90- 95	4.98	4.99	4.985	.2006018	4.89	4.89	4.890	.2044990
95-100	4.985	4.99	4.988	.2004812	4.89	4.89	4.890	.2044990
100-105	4.95	4.95	4.950	.2020202	4.94	4.94	4.940	.2024291
			98.604	4.0568151			99.557	4.0181581
	$\frac{98.604\times4.0568151}{400}=1.0000455$				$\frac{99.557\times4.0181581}{400}=1.0000894$			

TABLE 8

Calculated Caliber Factors, 5 to 105 cm Sections

How calculated	Tube No. 1	Tube No. 2	Tube No. 3	Tube No. 4
From eye observations on 5-cm threads	1.0000876	1.0000598	1.0000455	1.0000894
From abbreviated calibration as made, first approximation	1.0000931	1.0000598	1.0000458	1.0000955
From abbreviated calibration, second approximation	1.0000951	1.0000586	1.0000459	1.0000946
From relative cross section of 4-cm sections, second approximation curves	1.0000844	1.0000539	1.0000428	1.0000871

C. DETERMINATION OF THE APPROXIMATE OHM LENGTHS

The approximate ohm length of each tube was derived from the mean of three determinations, made as follows:

The tube under observation was first cleaned and dried in the usual manner and the micrometer microscopes set very nearly 100 cm apart. A thread of mercury of approximately this length was then introduced and observations made on both the base and tip of the meniscus at each end of the column in the 2.5 to 102.5, 5 to 105, and 7.5 to 107.5 cm intervals and on the adjacent graduations. These observations were then repeated in the reverse order. The thread lengths were thus referred to the graduations and the micrometer settings. The temperature at each end of the tube was recorded for each position of the thread and all thread lengths reduced to a common temperature. The mercury was then drawn out and carefully weighed and the ohm length at 0° C. derived from the following formula, after applying suitable temperature corrections:

$$L_0 = \frac{(106.3)^2}{14.4521} \frac{M}{L}$$

The resulting lengths in centimeters of the graduations, after correcting for the terminal bulbs and applying the caliber corrections, are given in Table 9.

TABLE 9

Approximate Ohm Lengths

Tube	Ohm length
	cm
1	95.15
2	96.32
3	97.02
4	100.66

D. LOCATION OF CUTS

Knowing the ohm lengths of the tubes, the locations of the cuts were so chosen that the terminal planes would lie where the cross section of the bore could be most accurately interpolated. This was done in each case by selecting two points on the cross-section curve, the proper distance apart, where the curvature was most free from irregularities.

E. CUTTING, GRINDING, AND POLISHING

The tubes were cut at points 1 mm outside of the selected points, to allow for a possible irregular fracture. After grinding down to approximately the right points they were tapered for the end bulbs and cleaning and drying connections, the tapers being the same so as to make all fittings interchangeable. The ends were then given an optical polish. The details of the grinding and polishing are as follows:

To prevent the chipping of the bore, which must be very carefully guarded against on account of its possible influence on the correction for the terminal bulbs, the ends of the tube were plugged with a slightly tapering rod of the same kind of glass set in with sealing wax, the slight heating required being local.

The tube was inserted in a holder provided with a number of supports to prevent flexure of the tube. This holder, consisting of a brass tube, was clamped vertically between two sets of rollers, one of each set being carried by springs pressing the holder against the companion rollers mounted directly from the massive frame of the apparatus. The lower end of the holder was supported at one end of a lever, to the other end of which weights could be applied for counterbalancing. The holder was, therefore, free to move in the direction of the axis of the tube, the end of which was pressed lightly upward against the grinding or polishing surface. This had a bearing surface on a steel ring, mounted from the frame and capable of adjustment so as to be at right angles to the axis of the holder. The grinding plate was of brass turned plane in a lathe. The motion in grinding was irregular, the grinding plate being rotated, and any lateral motion being limited by a ring (screwed to the plate) of slightly smaller diameter than the opening of the steel ring. The polishing plate was constructed as shown in Fig. 2. The pitch surface, *a*, about 0.5 mm thick, was applied by covering the central metal surface with the melted material, the ring limiting the motion having been removed, so that by pressing the plate on a plane glass surface a plane polishing surface was secured in the same plane as the bearing surface of the tool.

The polishing was continued until the surface was plane to within one-half wave length (0.3 micron), as shown by the interference fringes between the surface and an optically polished glass plate resting on the same, light from a helium tube being employed for testing. Tests were also made which showed that the end surfaces were in each case approximately at right angles to the axis of the tube.

The actual location of the cuts with respect to the graduations was determined as follows: The fixed distance between the two microscopes, set at approximately 12 cm apart, was determined by taking a double set of observations on the graduations, using the micrometers. The end of the tube was then brought under one microscope and the position of the other microscope with respect to the graduations noted. Repeated observations were made in each case; the mean values obtained and referred to the graduations are given in Table 10.

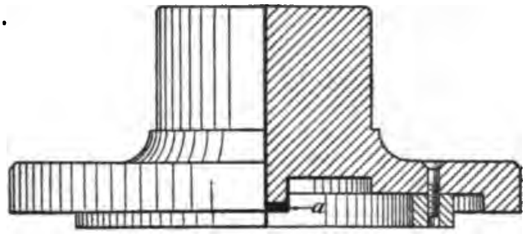


FIG. 2.—*Polishing plate used in grinding and polishing ends of tubes*

TABLE 10

Location of Cuts

Tube	0 end	100 end	Length
	cm	cm	cm
No. 1.....	5.976	101.055	95.079
No. 2.....	12.037	108.297	96.260
No. 3.....	10.996	107.997	97.001
No. 4.....	4.993	105.695	100.702

F. CALCULATION OF THE CALIBER FACTORS FOR THE FINISHED TUBES

Having determined the location of the terminal planes, the caliber factors for the finished tubes were determined from the formula

$$1+C=\left\{1+\frac{\Sigma_1^n k+\alpha k_\alpha+\beta k_\beta}{n+\alpha+\beta}\right\}$$

$$\left\{1-\frac{\Sigma_1^n k+\alpha k_\alpha+\beta k_\beta}{n+\alpha+\beta}+\frac{\Sigma_1^n k^2+\alpha k^2_\alpha+\beta k^2_\beta}{n+\alpha+\beta}-\frac{\Sigma_1^n k^3+\alpha k^3_\alpha+\beta k^3_\beta}{n+\alpha+\beta}\right\} \quad (16)$$

The values of α and β are given below:

TABLE 11

Tube	α	β
1.........	0.5120	0.0275
2.........	.5185	.6485
3.........	.0020	.4485
4.........	.0035	.3475

In order to make the effect of the term $\frac{\Sigma_1^n k+\alpha k_\alpha+\beta k_\beta}{n+\alpha+\beta}$ negligible, corrections were applied to the k's in Tables 2 to 5 which are expressed in terms of the mean cross sections in the interval 5–105, to refer them to the mean cross sections of the finished tubes. The simplified formula (17) becomes as shown on page 381,

$$C=\frac{\Sigma_1^n k^2+\alpha k^2_\alpha+\beta k^2_\beta}{n+\alpha+\beta}-\frac{\Sigma_1^n k^3+\alpha k^3_\alpha+\beta k^3_\beta}{n+\alpha+\beta}.$$

The caliber factor computed by the above formula is based on the assumption that the resistance of the tube is equal to the sum of the resistances of the separate elements, each regarded as cylinders. As a further approximation they may be taken as uniformly conical, having terminal areas given by the cross-section curve. From these areas a correction factor for conicality[6] expressing the mean ratio of the resistance of the conical to the corresponding cylindrical elements was computed. The part contributed by each element is given by the very approximate formula

---

[6] Smith, Trans. Roy. Soc., **204**, p. 40; 1904.

$$\frac{R_0'}{R_0} = 1 + \frac{1}{12} d^2$$

in which *d* denotes the ratio of the terminal cross sections. This correction is not included in Tables 2 to 5 and has only been computed for the finished tubes.

A sample computation of the caliber factor of Tube No. 1 is given in Table 12. The k^2 and k^3 terms are taken directly from tables, so that, as will be seen, the computation has been very much simplified.

TABLE 12

Caliber Factor, Tube No. 1

Interval	$k_0 = \frac{s - s_0}{s_0}$	$k = \frac{s - s_m}{s_m}$	k^2	k^3
			m	m
5.976-7	+0.00230	+0.00305	(4.8)	(0.0)
7-9	− .00355	− 0280	7.8	.0
9-11	− 0995	− 0920	84.6	.8
11-13	− 1885	− 1810	327.6	− 5.9
13-15	− 1505	− 1430	204.5	− 2.9
15-17	− 0665	− 0590	34.8	− .2
17-19	− 0155	− 0080	.6	.0
19-21	+ 0220	+ 0295	8.7	.0
21-23	+ 0340	+ 0415	17.2	+ .1
23-25	+ 0010	+ 0085	.7	.0
25-27	− 0740	− 0665	44.2	− .3
27-29	− 1280	− 1205	145.2	− 1.7
29-31	− 0850	− 0775	60.1	− .5
31-33	− 1095	− 1020	104.0	− 1.1
33-35	− 1065	− 0990	98.0	− .9
35-37	− 0675	− 0600	36.0	− .2
37-39	+ 0130	+ 0205	4.2	.0
39-41	+ 0125	+ 0200	4.0	.0
41-43	+ 1095	+ 1170	136.9	+ 1.6
43-45	+ 0395	+ 0470	22.1	+ .1
45-47	− 0570	− 0495	24.5	− .1
47-49	− 0530	− 0455	20.7	− .1
49-51	+ 0220	+ 0295	8.7	.0
51-53	+ 0820	+ 0895	80.1	+ .7
53-55	+ 1145	+ 1220	148.8	+ 1.8
55-57	+ 1355	+ 1430	204.5	+ 2.9
57-59	+ 1080	+ 1155	133.4	+ 1.5
59-61	+ 2145	+ 2220	492.8	+10.9
61-63	+ 1080	+ 1155	133.4	+ 1.5

TABLE 12—Continued

Interval.	$k_o=\frac{s-s_o}{s_o}$	$k=\frac{s-s_m}{s_m}$	k2	k3
			m	m
63–65	+0.00355	+0.00430	18.5	+ .1
65–67	+ .00060	+ 0135	1.8	.0
67–69	+ 0360	+ 0435	18.9	+ .1
69–71	+ 0285	+ 0360	13.0	.0
71–73	− 0435	− 0360	13.0	.0
73–75	− 0555	− 0480	23.0	− .1
75–77	− 1750	− 1675	280.6	− 4.7
77–79	− 1535	− 1460	213.2	− 3.1
79–81	− 1015	− 0940	88.4	− .8
81–83	− 0445	− 0370	13.7	− .1
83–85	− 0920	− 0845	71.4	− .6
85–87	− 0645	− 0570	32.5	− .1
87–89	+ 0025	+ 0100	1.0	.0
89–91	+ 0360	+ 0435	18.9	+ .1
91–93	+ 0045	+ 0120	1.4	.0
93–95	− 0030	+ 0045	.2	.0
95–97	+ 1200	+ 1275	162.6	+ 2.1
97–99	+ 1905	+ 1980	392.0	+ 6.6
99–101	+ 1230	+ 1305	170.3	+ 2.2
101–101.055	+ 0965	+ 1040	(.3)	(.0)

$$\Sigma k^2=4127.6^m \quad \Sigma k^3=+9.0^m$$

Length of tube $=95.079$ cm
Number of 2 cm sections $=47.54$

$$C.=\frac{4127.6}{47.54}-\frac{9.0}{47.54}=86.82-.19=86.63 \text{ millionths}$$

Table 13 gives a summary of the data obtained for the four tubes. The part contributed by the k^3 term is very small and it was therefore unnecessary to compute the next term. The probable errors of the caliber factors for the finished tubes were not computed. They will be given, for the interval 5–105 cm, in a separate paper on "The probable error in the determination of the caliber factors of ohm tubes," by F. A. Wolff, to be published in this Bulletin. The computed probable errors found, which of course do not take into account the irregularities not disclosed by the calibrations, are in every case less than one part in two million. These values are sufficiently close, since the lengths of all the tubes differed but a few centimeters from a meter.

TABLE 13

Caliber Factor for Finished Tubes

[Corrections in parts per million.]

Tube	"k^2" term	−"k^4" term	Correction for conicality of elements	Caliber factor 1+C
1	86.82	−.19	+2.62	1.00008925
2	64.63	−.02	+1.32	1.00006593
3	45.82	+.05	+0.86	1.00004673
4	94.31	−.30	+1.86	1.00009587

G. DETERMINATION OF M_0

The methods available for the determination of the mass of mercury required to fill the tube between selected points may be briefly described as follows:

First. A mass of mercury approximately equal to that corresponding to the ohm length is introduced into the tube, preferably before cutting, in the form of a continuous column, the length of which is determined usually in a number of positions near the points selected from preliminary measurements. The correction for the menisci, computed on the assumption of spehrical bounding surfaces, more or less uncertain, must be applied. If measurements are not made at 0°, errors may be introduced through errors in the coefficients of thermal expansion employed for reduction as well as in the temperature measurements.

From the mass of the column, the corrected length at 0°, together with the relation between the mean cross section of the particular interval to that of the finished tube as given by the calibration curve, the mass of mercury required to fill the tube between the selected points may be calculated.

Attempts have been made to eliminate the meniscus correction by flattening the menisci[6] by means of ebonite plugs introduced into the capillary, but this can not be accomplished to a sufficient degree of accuracy for the precision now attainable in ohm work.

---

[6] Huchinson and Wilkes, Phil. Mag., 28, p. 17; 1889.

Second. This method consists in carefully filling the tube, closing one end by a plane surface and stroking off the excess mercury at the other end. As first carried out, the tube was filled in a trough of mercury and then the mercury contained in the tube was separated from the rest by covering the ends still under the mercury with pieces of leather fastened to the fingers.

This method was perfected at the Reichsanstalt, and at the same time very perfect exhaustion preliminary to filling was provided for. The desirability of such exhaustion would seem apparent in view of what is known of the condensation of moisture and gases on glass, for even though the adsorbed material is not perfectly removed by exhaustion without heating, which is not permissible on account of thermal hysteresis, variations in the thickness of the film should be reduced as the thickness itself is reduced.

Third. This method, devised by Smith[7] at the National Physical Laboratory, is a modification of the first method, and consists in rendering the meniscus correction negligible and at the same time reducing the influence of errors in the measurement of thread length by means of capillary extensions coupled to the ends of the tube. The mass of mercury required to fill the tube is equal to that required to fill the tube plus extensions diminished by that required to fill the extensions alone, the differences in the readings of the capillaries being allowed for.

The method is, therefore, similar in principle to the method here employed for the determination of the lengths of the finished tubes.

Details of the Reichsanstalt Method.—The mass determinations employed in the calculation of the resistance of the four tubes were all made by the Reichsanstalt method.

The tube, after being cleaned and dried in the usual way, was placed in an inclined rack, the lower end being closed by a plane glass plate, *b*, Fig. 3, clamped to it by means of a hexagonal-headed steel screw with an intermediate ball-and-socket joint, the screw being threaded through the lower part of a steel yoke, which was in turn screwed to a steel collar cemented to the tube about 2½

---

[7] Smith, Nat. Phys. Lab. Collected Researches, Vol. V, pp. 149–157; 1909.

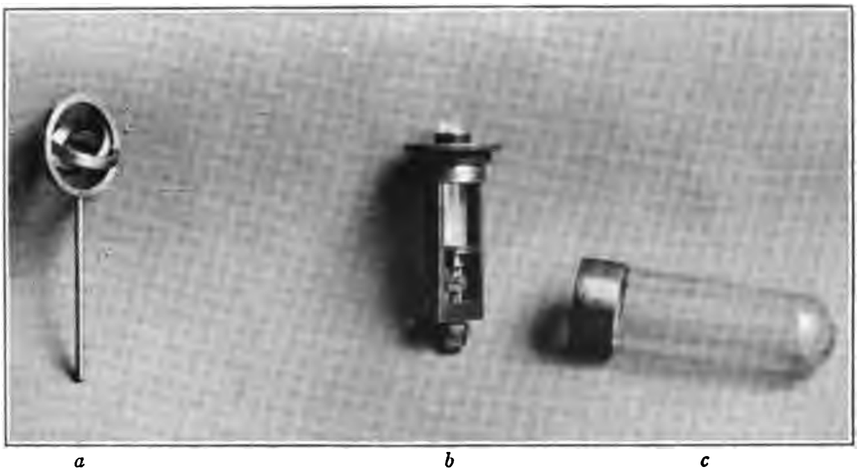


FIG. 3.—*Parts used in the determination of M_0 by Reichsanstalt method. a, Stroking-off plate in Gimbal mounting. b, Plane-polished plate mounted by ball and socket in steel yoke, used to seal lower end of tube. c, Tube used to protect b from water of ice bath*

centimeters from the end. The whole was made secure by suitable lock nuts and submerged in double-distilled mercury, contained in a cylindrical glass reservoir free to rotate about the axis of the ohm tube. At the bottom of this reservoir was a fixed steel plate with a hexagonal hole to engage the head of the screw carrying the plane polished plate. The upper end of the tube was connected to a Geryk pump by means of a ground-glass extension and a mercury seal. After exhausting the tube for 15 minutes sufficient mercury to fill it was admitted by rotating the mercury reservoir. The plate was then reclamped to the end of the tube in a similar manner.

After disconnecting the tube from the pump and removing it from the rack, the lower end was inclosed in a glass vessel, *c*, Fig. 3, screwed to the upper part of the yoke. To exclude moisture, this was then dipped in liquid rubber cement. The upper end of the tube was protected by a small ground-glass cap. The filled tube was then placed vertically within a specially constructed ice bath. This bath, Fig. 4, consisted of glass tubes 110 cm long and 5 cm in diameter, provided with constrictions near the top which served as supports for split rubber stoppers fastened to the ohm tubes to hold them in position. A tube was filled with distilled water to within 1 or 2 cm of the top, and after introducing an ohm tube a layer of finely shaved ice was added and the tube closed with a ground-glass cap. Around the glass tubes were two cylindrical copper baths, each of which was provided with a separate cover, so that the inclosed ohm tube could be completely surrounded with a double layer of ice and water. The outer copper wall was protected by a layer of cork placed between it and the heavy wooden box and cover inclosing the whole.

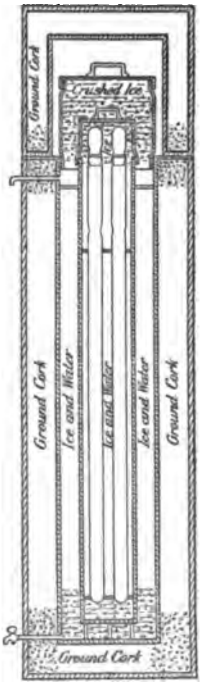


FIG. 4.—*Ice bath used in the determination of M_0 by the Reichsanstalt method*

After allowing the tube to remain in the ice bath sufficiently long to come to temperature equilibrium, the top was uncovered and the excess of mercury removed by stroking off with a plane polished glass plate in a gimbal mounting, *a*, Fig. 3. Condensation of moisture was eliminated by directing currents of dry, cooled air at both the end of the tube and the plane plate for a few seconds immediately before stroking off. Suitable illumination was employed, and in nearly every case interference fringes could be seen during the stroking-off process. After replacing the cap the tube was removed from the bath and allowed to come to room temperature before transferring the mercury to a porcelain crucible.

The Weighings.—The weighings for the determination of M_0 were all made in a basement room without windows, especially fitted up for use in silver voltameter work, in which connection it has been more fully described.[8] In one end an inner room was constructed with walls of hollow terra-cotta tiling and a double door and small double windows for the observing telescopes, and with openings for the rods and levers operating the balances. Temperature changes within the room were therefore found to be very small, even from day to day.

The weighings were made on a 200-gram Stückrath balance, so designed that the transposition of load and weights, as well as the manipulation of multiple riders and the other operations, are accomplished from a distance. There was, therefore, no necessity of the observer entering the inner room after the load and weights were put upon the balance pans.

Two sets of weights were used, one a set of one-piece gold-plated brass weights ranging from 1 to 50 grams, made by Gurley, and the other a set of platinum weights ranging from 0.0001 to 1 g, made by Christian Becker. Both sets were very carefully calibrated in the weights and measures division of the Bureau before and after making the series of weighings, and only one of the brass weights used was found to have changed by as much as 0.05 mg, the remaining ones have remained constant to 0.02 mg. None of the platinum weights changed by more than 0.002 mg.

The method of double weighings was used, the equilibrium points being determined by observing on the image of a scale reflected by a mirror on the balance beam and a fixed right-angle prism. The sensibility of the balance was determined for every weighing and was approximately 37–38 scale divisions per milligram, so that in view of the constancy of the temperature the weighings may be considered correct to 0.01 mg, as was also shown by check weighings occasionally made.

---

[8] The Silver Voltameter, this bulletin, 9, p. 174; 1913.

The mercury to be weighed was collected in porcelain crucibles and for the purpose of weighing was transferred to a particular crucible, the weight of which was determined before and after each set.

The temperature, relative humidity, and barometric pressure were always recorded and corresponding buoyancy corrections applied.

Six fillings for each of the four ohm tubes were first made. In Table 14 the separate values of M_0, the deviations from the mean value, and the mean probable error of the result are given.

TABLE 14

Results of M_0 Determinations, Reichsanstalt Method

Tube	Filling	M_0 in grams	Deviation from the mean in milligrams
No. 1	1	11.574772	+0.028
	2	825	+ .081
	3	621	− .123
	4	799	+ .055
	5	738	− .006
	6	712	− .032
Mean		11.574744	± .054
No. 2	1	11.864017	+ .056
	2	3953	− .008
	3	3997	+ .036
	4	3920	− .041
	5	4015	+ .054
	6	3862	− .099
Mean		11.863961	± .049
No. 3	1	12.042690	− .025
	2	818	+ .103
	3	767	+ .052
	4	670	− .045
	5	680	− .035
	6	664	− .051
Mean		12.042715	± .052

TABLE 14—Continued.

Tube	Filling	M_0 in grams	Deviation from the mean in milligrams
No. 4	1	12.974942	− .008
	2	4976	+ .026
	3	4838	− .112
	4	4922	− .028
	5	5027	+ .077
	6	4994	+ .044
Mean		12.974950	± .049

Average deviation from mean:	
Tube No. 1	±4.7 parts per million.
Tube No. 2	±4.1 parts per million.
Tube No. 3	±4.3 parts per million.
Tube No. 4	±3.9 parts per million.
Probable error of result:	
Tube No. 1	±1.73 millionths.
Tube No. 2	±1.43 millionths.
Tube No. 3	±1.44 millionths.
Tube No. 4	±1.40 millionths.

Supplementary Fillings.—In making the above fillings for the determination of M_0 only slightly more mercury than required to fill it was admitted into the tube. In filling a tube for electrical comparison it was, however, necessary to pass enough additional mercury through it to fill one of the end bulbs. The fillings for M_0 and for the electrical comparisons were thus made under slightly different conditions, and to test the effect, if any, of the sweeping action upon the surface film produced by a relatively large volume of mercury passing through the tube, six subsequent fillings for mass determination were made on tube No. 3. The upper end of the tube, placed in the inclined rack, was connected to a bulb approximately 4 cm in diameter, which was in turn connected to the Geryk pump. When filling the tube enough mercury was drawn through it to fill this bulb also. The results of six determinations by this method are given in Table 15. As will be seen, the difference found was negligible.

TABLE 15

Supplementary Fillings, Tube No. 3, P. T. R. Method

Filling	M_0 in grams	Deviation from mean in milligrams
1	12.042585	−0.136
2	803	+ .082
3	711	− .010
4	725	+ .004
5	766	+ .045
6	737	+ .016
Mean	12.042721	± .049

Average deviation of the mean = ±4.1 parts per million.
Probable error of result = ±1.70 millionths.

Check Determinations of M_0 by Smith's Method.—The Smith method of determining M_0, as previsously stated, involves the two operations of filling the extensions alone and of filling the tube with the extensions attached, the difference in the masses of mercury required being the mass contained in the tube.

Some difficulty was experienced with this method, as the fillings frequently showed irregular effects when submitted to reduced pressure, even when made after very perfect exhaustion. Most of the difficulty was eliminated by modifications of the form of coupling first employed. The agreement of individual results in the fillings of the extensions alone is, however, not as good as that obtained by the Reichsanstalt method, so that only a relatively few check fillings were made.

The first form of coupling used was of the ball-and-socket pattern employed by Smith.[9] In the present case, however, they were made of steel with the sleeves threaded to screw onto the steel collars cemented to the tubes, instead of being secured by small set screws seated in dimples ground near the ends of the tube.

The coupling was provided with a mercury seal made as shown in Fig. 5; *a* is a glass cylinder with opening *d*, provided with a ground-glass cap; *b* and *b* are annular disks of rubber, while

---

[9] Smith, Nat. Phy. Lab. Collected Researches, Vol. V, p. 153; 1909.

c and *c* are steel cup-shaped pieces threaded to engage the collars of the tube and hold the rubber disks firmly against the ends of the glass seal.

The extensions (Fig. 6) were of Jena 59^{III} glass of 0.158 mm bore for the greater part of their length, but terminating in a bore of very approximately the same size as that of the ohm tubes. Each extension was graduated in millimeters over 3 cm of the vertical portion. The lower ends were ground to the same taper as that of the ohm tubes to permit using the same cleaning and drying fittings, and the end sections were ground and polished to a plane surface. The upper ends were tapered and ground to fit the connecting bulbs *a* and *b* as shown, while *c* and *d* are further ground glass extensions used in making connections to the exhaust pump. Both sides of *a* and *b* are provided with mercury seals.

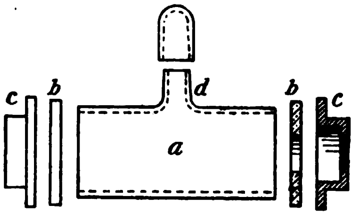


Fig. 5.—*Mercury seal for steel coupling; used in determination of M_0 by Smith's method*

The coupled extensions were mounted within a suitable Dewar vessel which in turn rested upon a small turntable. This arrangement made it possible to read the mercury columns from opposite sides thus eliminating the effects of parallax.

In filling the coupled extensions, the bulbs *a* and *b* are brought into place and the extensions *c* and *d* connected to the pump, bulb *a* being completely filled with mercury beforehand. After exhausting 15 minutes, the pressure at *c* is momentarily increased enough to start the mercury through the capillary. After filling, the bulb *a* is removed and extension *d* connected to the house vacuum and enough mercury withdrawn from the tube to bring the level of the two ends of the thread within the graduated portion of the capillaries. A cathetometer was used in making the observations, simply as a matter of convenience, the observing telescope turning about a vertical axis so as to bring into view either end of the mercury column as required.

Readings were taken with the mercury column alternately under atmospheric pressure and then under reduced pressure until a series of several observations were made, the relative positions of the ends of the column being shifted for each set of readings. The last two observations were made at normal pressure. In many cases, when under reduced pressure, the mercury rose in the the extensions. When this rise was appreciable the filling was rejected and a new one made. The true mean height of the ends of the mercury thread was taken as the mean of the heights at normal pressure in the two positions. After completing a set of observations the mercury was drawn off into a clean bulb by aid of the house vacuum. Weighings were made in the same manner and on the same balance as were the fillings made by the Reichsanstalt method. Thirteen fillings were made all of which were reduced to correspond to a definite height of mercury in the capillaries. The correction factors were determined from data obtained by a careful calibration of the capillaries (approximate cross section 0.158 sq mm) before sealing them onto the larger portions of the extensions. The results of the fillings are given in Table 16.

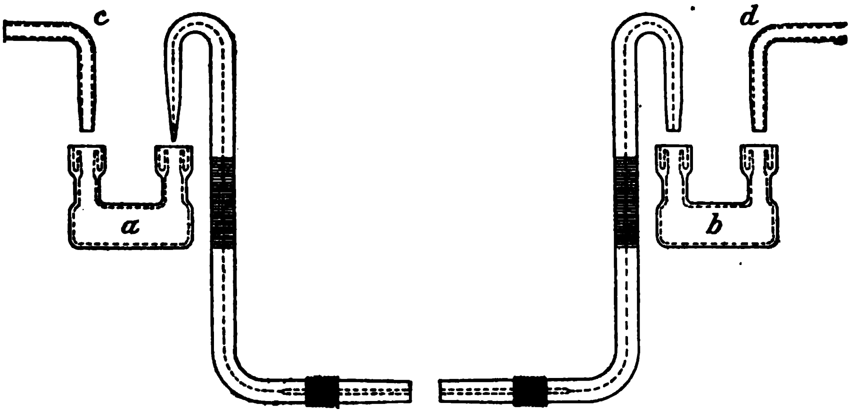


FIG. 6.—*Ohm tube extensions and fittings; used in determination of M_0 by Smith's method*

TABLE 16

Fillings of Extensions

Filling	Mass in grams	Deviation from mean in milligrams
1	1.125238	−0.26
2	467	− .03
3	341	− .16
4	624	+ .13
5	515	+ .02
6	553	+ .06
7	521	+ .02
8	394	− .10
9	585	+ .09
10	732	+ .24
11	377	− .12
12	620	+ .12
13	483	− .01
Mean	1.125496	± .10

Average deviation from mean = 88.8 parts per million.
Probable error of result = 22.5 millionths.

Fillings of a whole tube by Smith's method were first made when using two couplings similar to the one used in filling the extensions. These proved unsatisfactory, owing to fact that the introduction of additional joints multiplied the effect of reduced pressure on the mercury column. In many cases each end of the thread rose several centimeters, the original mean height not being recovered when again under normal pressure. A universal coupling was therefore devised. This is shown in Fig. 7; *a* is a ring to which are fastened the two cylindrical threaded parts *b* and *c*, riveted so they are free to move in planes at right angles to each other; *c* and the thin

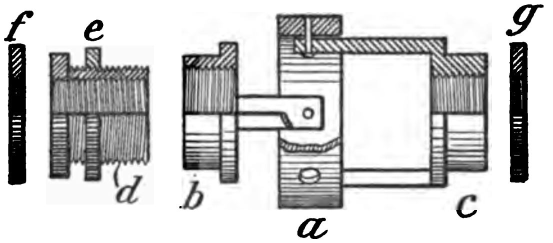


FIG. 7.—*Universal steel coupling for tube and extension or for extensions alone; used in the determination of M_0 by Smith's method*

double-threaded cylinder *d* are designed to engage the collars of the tubes and extensions; *d* also engages the part *b* of the coupling and is provided with a milled head and a lock nut *e*, while *f* and *g* are additional lock nuts. In making a coupling, the lock nuts *f* and *g*, together with the several parts of the seal, are first placed in their proper order beyond the collars of the tube and extension; *c* is then screwed onto the collar of the ohm tube until the end of the tube is about at the center of the ring *a*. The lock nut *g* is then made fast; *d* is made to engage *b* and the collar of the extension at the same time. By turning *d* the end of the extension is brought to the center of *a* and against the end of the tube. The lock nuts *e* and *f* are then brought into place, after which the seal is assembled.

In trying out these new couplings the same difficulty was experienced as when using the old ones. A relatively large quantity of air appeared to be trapped about the ends of the tube, which caused a rise of the ends of the mercury column when under reduced pressure. As the seating of the face of the tube against that of an extension appeared to be all that could be desired, it was thought that the difficulty lay in the fact that the metal of the coupling was too close to the joint of the tubes to allow of the intimate contact of the mercury with the glass at that point. Couplings of the same design but of larger diameter were therefore made and were used with greater success. Any small rise of the mercury in the capillaries, under reduced pressure, was succeeded by the columns recovering their mean height when again under normal pressure. Fillings were made in the same ice bath used when making electrical comparisons, the operations being the same as when making the fillings of the extensions. Six satisfactory fillings were made of tube No. 3, the results of which are shown in Table 17.

TABLE 17

Check Fillings, Tube No. 3, Smith's Method

Filling	M_0 in grams, tube and extensions	Deviation from mean in milligrams
1	13.168216	+0.089
2	8293	+ .166
3	8312	+ .185
4	7827	− .300
5	7965	− .162
6	8147	+ .020
Mean	13.168127	± .154
M_0 of end sections	1.125496	
M_0 of tube	12.042631	

Average deviations from the mean—11.7 parts per million, tube and extensions.
Probable error of result—4.0 millionths.
Probable error of result, tube alone—4.6 millionths.

A comparison of the determinations of M_0 for tube No. 3, as given in Tables 14 and 17, shows that the average deviation from the mean of six fillings and the probable errors of the results are greater in the Smith method than in the Reichsanstalt method of determination.

Effect of Pressure on Filling.—A calculation was made of the effect of internal pressure on the mass of mercury contained in an ohm tube of Jena 16^{III} glass when in a vertical position. In such a case the observed M_0 would be too large by approximately 4.7 parts per million, due to the internal pressure alone. This is reduced by 0.3 parts per million, due to the external pressure exerted by the surrounding water of the ice bath, making the resultant M_0 too large by 4.4 parts per million. As there was no complete data available on the constants of Jena 59^{III} tubing, such as that used in the present investigation, no correction for the effect of pressure was applied to the M_0 as determined by the Reichsanstalt method, although the correction is probably approximately the same. In the investigations at the Reichsanstalt no correction of this kind was applied.

The Effect of Capillary Pressure in Stroking Off.—At the Reichsanstalt, when making the M_0 determinations, both a plane plate and one with a small groove along half its diameter were used in stroking off. A practically uniform difference of 0.15 mg in the fillings was noted, the lower values being found with the use of the grooved plate. The above difference corresponded to 17.8, 20.3, and 17.8 parts per million, respectively, for the three tubes XI, XIV, and 114. No allowance was made for capillary pressure at the Bureau, although an ungrooved plate was used throughout. However, the plate received some accidental scratches upon it through usage. An error in stroking off of 0.15 mg would make the calculated resistance of the four tubes too small by approximately 12.5 parts per million.

The resultant effect of possible slight heating in stroking off is uncertain, and it requires excess internal pressure to fill the space around the periphery of the capillary with mercury. These effects should be further investigated.

Accuracy of Some of the Fillings Made at the Reichsanstalt and at the National Physical Laboratory.—At the Reichsanstalt 10 fillings were made of each of the 1-ohm tubes XI and XIV, and six fillings of the 1-ohm tube 114. The average deviations of the individual fillings from their means, as calculated from available data,[10] were ±6.1, ±11.6, and ±6.7 parts per million, respectively, the mean average deviation for the 3 tubes, 26 fillings, being ±8.1 parts per million.

At the National Physical Laboratory 3 to 5 fillings were made on each of 11 tubes, 46 fillings being made in all. The measurements were made before the tubes were cut, the method employed consisting of comparing a column of mercury (the length and position of which very nearly coincided with the x and y limits as derived from the calibration) directly with a standard meter. The mean average deviation of all tubes, as calculated from data in published tables,[11] was ±8.1 parts per million. Omitting two tubes, the ohm length of which exceeded 1 meter and for which the length measurements were less accurate, the corresponding mean average deviation for the 9 tubes was ±6.4 parts per million.

---

[10] Wiss. Abh. P. T. R., Vol. 2, pp. 458 and 465; 1895; Vol. 3, pp. 130, 1900.

[11] Collected Researches, N. P. L., Vol. 1, pp. 170 and 171.

H. DETERMINATION OF L_0

The problem of determining the length of the ohm tubes reduces practically to that of comparing the two types of length standards, the usual type in which the standard length is given by the distance between two lines, and the now less common type where it is given as the distance between end surfaces. These two types of standards can be compared by various methods of which only three were considered.

Direct Sighting.—In this method the micrometer settings are made on some point on the outer edges of the plane end faces of the standard and then compared with settings made on a line standard. The length of the bore in the case of the ohm tube would be given by the average of values obtained with the tube orientated in various positions. This method may be modified by sighting on the edge of the bore instead of the outer edges of the end faces, being more direct but also more troublesome. In some ohm tubes previously constructed by other investigators the end faces were slightly curved to facilitate making observations on the bore.

In the first method a difficulty arises in that the micrometer settings are made on points identified by sharply contrasting fields of illumination, thus introducing the possibility of a personal error.

In sighting directly on the edge of the bore[12] an error due to a displacement of the image would in general result from a dissymmetry of the optical system of the observing microscope, since only part of the objective is used in forming the image, the other half being "stopped" by the tube, while in making settings on the line standard the whole of the objective is employed. This error is not in any sense eliminated by giving the end surfaces a small curvature.

The Optical Method.—This method makes use of the property of a reflecting plane that an object and its image are equidistant from the plane. The principle is utilized by bringing a fine hair or set of points near the face of the end standard and setting the cross wires symmetrically with respect to the object and image, thus locating the plane. Possibilities of error due to dissymmetry of the optical system are also introduced in this method as above described.

---

[12] This may be greatly facilitated by inserting a slightly tapering plug into the capillary, the settings being made on the line of demarcation between the tube and plug.

The Contact Method.—A modification of the contact method, devised by L. A. Fischer of this Bureau and applied by him with much success in comparing line and end standards,[18] when adapted for use with the ohm tubes, was found so satisfactory that it was used throughout.

The method consists in applying end pieces or extensions to the end faces of the standard. These are provided with ruled lines upon which the micrometer settings may be directly made. This reduces the end standard or ohm tube to a line standard the length of which is equal to the length to be determined plus the added length due to the extensions. The amount added can be accurately determined by directly abutting the contact faces of the extensions and determining the distance between the lines.

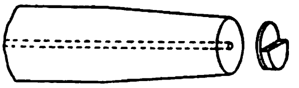


FIG. 8.—*End of tube and platinum-iridium end piece; used in the determination of L_0*

The end pieces used were those employed by Mr. Fischer in the work above referred to and were constructed as shown in Fig. 8. Each consisted of a lug of platinum-iridium of semicircular cross section, fastened to one face of a circular plate of the same material 6.5 mm in diameter, the other face of the plate serving as the contact area. The surface of the lug bearing the rulings, upon which the settings were made, was at right angles to the plane of the disk and intersecting it along a diameter. The average distance of the ruling from the surface of the end standard was approximately 0.8 mm, thus reducing to a minimum errors caused by any lack of parallelism of the end surfaces.

The extensions were held in place, as shown in Fig. 9, by the pressure of a flat spiral spring exerted through a small cupped brass lug mounted in the center of the spring. The spring was carried on an arm projecting from a circular nut screwing on to the steel collar of the tube. This nut was secured by a jam nut which formed part of a casing to protect the ends from the ice and water of the bath in which the comparisons were made. This consisted of a cylinder of brass tubing closed at one end and threaded at the other. The threaded portion screwed into a flange of the jam nut above mentioned, thus inclosing the end of the tube. The lines on the extensions were viewed through a tube of smaller diameter fastened at right angles to the side of the brass cylinder, long enough to project well above the water level of the comparing bath.

---

[18] Bull. Phil. Soc. (Wash.), 13, 241; 1898.

Method of Mounting the Tubes.—In order to correct for the very slight residual curvature of the tubes, they were mounted in accurately aligned V-shaped supports, against which they were firmly held by the pressure of springs. The supports were clamped to one of the two adjustable T-shaped bars of the comparator box, provided for the support of standards under comparison, and were about 11 cm apart except in case of the end supports, which were adjusted to accommodate the tube under measurement.

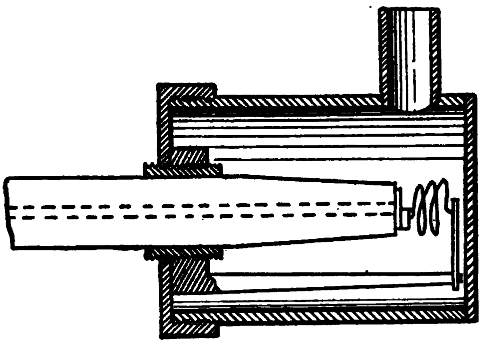


FIG. 9.—*Mounting for end piece and casing; used to protect the ends from ice and water of the comparing bath*

The Comparison Standard.—The comparison standard employed was an Invar bar of H section, known as meter 39. This, together with a companion bar belonging to the University of Wisconsin, was purchased from the Société Genevoise in January, 1903. The ruled surface has a high polish, and the lines are very fine and sharp. The bar was ruled throughout its length to millimeters, the terminating millimeters, which lay outside of the 0 and 100 cm marks, being further subdivided to tenth millimeters. In order to determine the length of tube No. 4, which slightly exceeded the length of the graduated portion, an additional line was ruled on the bar by R. Y. Ferner, of this Bureau.

The bar was calibrated and its length carefully determined when received. As changes with time have been observed in other nickel-steel standards, including the one purchased by the University of Wisconsin, the length was redetermined by comparison with platinum-iridium national prototype meter No. 21, after the completion of the measurements on the ohm tubes. The old calibration was used in determining the value of the subintervals used in the ohm tube comparisons and corrected for the new value of length found. The previous calibration did not include the terminating millimeters, necessitating the determination of the values of the interval 100.0–100.1 cm and its subdivisions. This, together with the determinations of the additional interval required for tube No. 4, will be described below. In order to protect the lines upon which observations were made from the ice and water of the comparing bath, a small brass tube, Fig. 10, was held vertically over the rulings by a brass plate through which it passed. This plate fitted over the top of the bar and held the lower end of the tube close to the surface of the graduations, the top of the tube extending above the water line. Before putting this device in place the lower end of the tube was supplied with an extension of soft wax, which, when the tube was in place, filled the gap between the tube end and the ruled surface. By perfecting this wax seal by means of a hot wire, the water was effectively prevented from entering. During observations, as is customary, the bar rested on two supports placed at the 23 and 77 cm points.

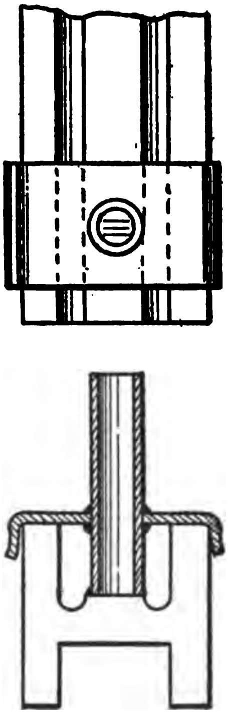


FIG. 10. — *Brass tube mounted on meter bar to protect lines under observation from ice and water of the comparing bath*

The comparator employed was the precision comparator, Fig. 11, of the Bureau, designed by L. A. Fischer. As this is to be described in detail at some future date, a brief outline only will be given here.

The micrometers of the comparator are mounted on heavy plates which are clamped to a beam supported at either end by concrete piers, having separate foundations. One end only of the beam is attached to its supporting pier, the other end resting freely on a plate set in the top of the other pier. The metal parts, including the micrometer screws, are composed of nickel steel.

The standards under comparison were mounted on two independent parallel supports provided with means for horizontal and vertical adjustments at either end. These supports were carried in the heavy metal inner compartment of a double brass box which rested on an open frame on the floor of the outer box, the latter being provided with leveling screws resting on the comparator carriage. The carriage ran on two rails mounted on a pier having an independent foundation. Its movement was perpendicular to the length of the standards, so that they could be brought successively beneath the micrometers by a slight shifting back and forth of the carriage, course and fine adjustment being provided. The inner and outer boxes were provided with lids through which apertures were cut for observations and to admit the keys which operated the various adjustments of the mountings. These lids, however, were used only in the comparisons of meter 21 and meter 39, made in air and at room temperature.

The Illumination.—The illumination for each microscope was secured by the light reflected from an incandescent lamp by a small totally reflecting prism. The latter was immediately beneath the observing microscope, covering one-half the field, and was carried on an arm extending from a sleeve on the microscope tube.

The sleeve could be rotated, and adjustments were provided for placing the prism in any desired position.

Preparation of the Ohm Tubes and Their Comparison with Length Standards.—The extensions were applied to the carefully cleaned ends of the ohm tube in the following manner: With the holders screwed on the collars of the tube, the distance of the spring from the end faces was adjusted so that a small pressure would be applied to the end pieces when in position. The tube was then placed in the mounting employed in its calibration and rotated into the

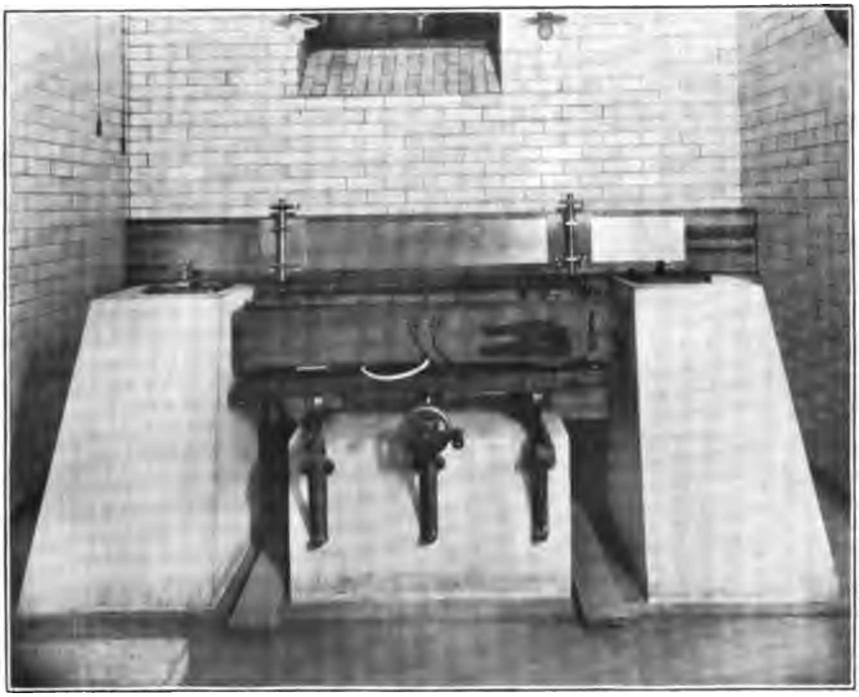


FIG. 11.—*Precision comparator used in the determination of the lengths of the tubes*

position in which it was to be compared. After a careful inspection of the tube ends and extensions for lint and dust particles, the extensions, which were handled throughout by small forceps, were applied to the tube. They were then centered by eye, the operation being greatly facilitated by illumination from the opposite end of the tube leaving a bright annulus around the extension. The ruled surfaces of the extensions were then made parallel by means of two paper targets placed on a line parallel to the axis of the tube, the proper orientation being determined by the targets, and their images being in line with the centers of the respective extensions. The adjustments were repeated when necessary until satisfactory in all particulars. The protecting boxes were next attached, wax being applied to the threads to insure waterproofing and the tube then placed in the mounting previously described.

The surface of the standard was prepared for observation by cleaning with benzene to remove traces of wax and oil that might have been left from a previous comparison and in order to give the ruled surface uniform treatment.

The approximate lengths of the tubes and extensions were separately obtained. The combined length was compared with an interval on the meter bar determined by the distance between one of the millimeter divisions toward the zero end and one of the lines of the subdivided millimeter at the 100-cm end. In this way the difference read on the micrometers was reduced to a minimum. The protecting tubes were then placed over the lines selected and the bar was placed on its supports in the comparator box.

After spacing the microscopes the surface of the bar was adjusted to be perpendicular to the axes by means of the leveling screws on the outer comparator box, the illumination in this position being a maximum. With the bar properly orientated it was approximately adjusted for observations. The tubes and meter bar were compared at 0° C, maintained by immersing them in a mixture of ice and water. Fine ice was placed directly about the tube and bar, being heaped to a height of several centimeters, except where it would have interfered with the observations and adjustments, while coarser ice was carefully packed in the space between the sides and bottoms of the inner and outer boxes. The water level was maintained at a height of several centimeters above the standard. The ice was replenished as needed, and the excess of water was drained off from time to time.

In order to insure certainty in the temperature, an interval of at least 15 minutes was allowed to elapse after completing the icing before the observations were begun.

There was a tendency for atmospheric moisture to condense upon the surfaces under observation, thus obscuring the lines. This was effectively disposed of by blowing a current of dried air against the surfaces, after which they would remain clear for a considerable time.

The final adjustments were made after the icing was completed. The microscopes were spaced so that the distance between them was approximately the average of the two intervals compared. The standard and tube were then put in alignment so that the center of the lines came in the center of the microscope field.

In order to use the micrometers to the best advantage the bars were adjusted endwise until in passing from an observation on the tube to an observation on the bar the readings would shift from one side to about an equal distance on the other side of the center of the microscope field.

The focusing was done by raising or lowering the support of the standard under observation by means of the adjusting screws provided. The proper focus was obtained when the parallax, produced by oscillating the sleeve upon which the illuminating prism was mounted, was a minimum. Though this had a natural tendency to disturb the microscopes the results derived indicated that this effect was negligible.

The microscopes were focused preceding each group of four simultaneous observations, the two observers interchanging positions for the last two readings. The comparator carriage was then shifted, bringing the other standard in position, the microscopes were focused, and readings taken as before, the observers coming back to their original position for the last two readings. A comparison consisted of the average of three groups of observations on the ohm tube alternated with two groups on meter 39.

The difference between the two standards was obtained from the average of a set of six comparisons. Alternate sets were made with the initial positions of the observers interchanged, in order to eliminate as far as possible personal errors in focusing and setting.

The results indicated that the personal error was quite small, the mean difference of the means of alternate comparisons not exceeding one-tenth micron. Comparisons were made in four positions of the tube about its axis 90° apart, necessitating dismounting the tube, recleaning the contact surfaces, resetting the extensions, and remounting the tube.

The final length was taken as the mean of values obtained for the four positions.

The length of meter 39 was determined following the work on the ohm tubes. This was done by comparisons made with National Prototype meter No. 21. The comparisons were made at temperatures of 20° C and 0° C in the same general manner as above described. The measurements at the higher temperature were made in order to check the temperature coefficient upon which the calibration depended. The comparisons at 0° C were made in two positions, meter 21 being turned end for end in the last half of the observations, and included 16 sets of observations. In the observations on the bar at the higher temperature the general routine in such work was followed, the bars being compared in eight relative positions, one being duplicated. Four observations were taken for each position and, in order to minimize temperature errors, several hours were allowed to elapse after placing the bars in position before observations were made. The comparisons therefore extended over several days.

The results showed that an appreciable change, amounting to approximately 5 microns, had taken place in the total length of the bar since its original calibration in January, 1903. There was also found a slightly different temperature coefficient, corresponding to a relative change of approximately 1½ microns for a range of 20° C. Changes of a similar character had been previously observed at the Bureau in case of the nickel-steel meter submitted for verification by the University of Wisconsin. The corrections of the graduations employed were recomputed on the assumption that the observed change in length was uniformly distributed throughout the bar.

The comparisons for the determinations of the values of the intervals −0.9 to 0.0 cm and 100.0 to 100.1 cm, the subdivisions of the latter, together with the check comparisons on the extensions, were made at room temperature on a Geneva Society dividing engine. The objects to be compared were aligned on the carriage of the dividing engine, the microscopes being adjusted so that the left-hand lines of the intervals compared came near the center of the field. Two micrometer readings were taken and the carriage was then shifted, bringing the right-hand lines into the center of the field. Four readings were then taken on each of the lines and the carriage again shifted to the left-hand lines, two readings being taken in this last position. In this manner the differences read on the micrometers were small, and errors due to progressive changes were also reduced. The standard intervals used were selected intervals on nickel-steel decimeter No. 24.

The interval −0.9 to 0.0 cm was determined by comparisons with intervals 0.0 to 0.9 cm and 0.1 to 1.0 cm of decimeter No. 24 and the results reduced to 0° C.

The value of the interval 100.0 to 100.1 cm was obtained by comparisons with the first 9-mm intervals of decimeter No. 24, the result also being reduced to 0° C.

The abbreviated method used in the calibration of the ohm tubes was employed in the subcalibration of this interval. In this the 0.4 mm, the 0.5 mm, and the 0.6 mm intervals were each intercompared by the aid of the corresponding fixed intervals selected on the subdivided millimeter of decimeter No. 24. From the data thus obtained and the length at 0° C of the whole interval the corrections of the subdivisions were computed.

The correction for the extensions was frequently checked by measurements made on the dividing engine at room temperature. The results agreed within the limits set by the errors of observation. It was found that the interval was unaffected by a slight relative displacement of the two abutting faces.

Correction for the Extensions.—To eliminate errors due to the variation in the appearance of the lines as viewed through different microscopes and under different illumination, the final correction for the extensions was determined on the precision comparator in order to secure the same conditions which obtained in the measurement of the ohm tubes. The same general plan employed in the comparisons above described was adopted. The extensions and standard scale were mounted on either end of the same support in the comparator box, the shifting being accomplished by the aid of the adjustment provided for moving the support longitudinally. The comparison intervals selected were the five 2-mm intervals constituting the first centimeter of decimeter No. 24.

In the determination of the correction for the extensions it was highly important to include an optical constant, due to the proximity of the lines to the end plates in the same manner as it occurred in use with the tubes. This was realized by shifting the support so that the settings on the extensions always came in the central region of the field, the difference being taken up on the other micrometer. The final value was obtained from observations made in the manner previously described with the position of extensions and standard interchanged for half the readings.

The micrometer constants were redetermined, following the work on the tubes, by comparisons made with the half millimeters of the subdivided millimeters of decimeters No. 24 and No. 43, one scale being placed beneath each microscope. Observations were made simultaneously on both microscopes, observers changing positions in the same order as in the length comparisons. After a series of observations the decimeters were interchanged and readings taken as before. The constants were thus determined under conditions obtaining in the comparisons of the ohm tubes and extensions.

Summary of Results, Length Measurements.—The results obtained in the length measurements are shown in Tables 18 to 23, which also give the component and resultant probable errors, calculated from the observational data and from the probable errors of the comparison standards.

The probable error of the National Prototype meter No. 21 was assumed to be 0.20 μ, which is regarded as a liberal allowance. The probable error of the recomparison of meters 39 and 21 at

0° C was 0.15 μ. The probable error in the meter interval of meter 39 was therefore taken as 0.25 μ, and this was prorated according to the length of the subinterval employed.

As no probable errors had been assigned to the corrections of decimeters 24 and 43, calibrated at the International Bureau, liberal values were assumed.

No recalibration of the subdivisions of meter 39 was made, the corrections employed being derived from the calibration made in 1903, from which the probable errors of the graduations were also derived. On tubes 1, 2, and 3 the probable error in the determination and subcalibration of the interval 100.0 to 100.1 cm is also involved. Tube No. 4 includes the probable error in the determination of the interval −0.9 to 0.0 cm. Owing to the small differences between the measured lengths of the tubes and extensions and the comparison intervals the error due to the uncertainty in the micrometer values is sensible only in the constant for the extensions and in the measurements on tube No. 4. In the latter case the difference happened to be very approximately equal to that dealt with in the determination of the constant for the extensions, and consequently no error due to the micrometer value is introduced in the length of the tube itself.

In the following table are given the computed probable errors contributed by the comparison standard for each of the four tubes:

TABLE 18

Probable Errors of Comparison Intervals

	Tube 1	Tube 2	Tube 3	Tube 4
	μ	μ	μ	μ
Graduation at 0 cm end of meter 39	0.17	0.33	0.30	0.15
Graduation at 100 cm end of meter 39	.04	.08	.09	.00
Error of meter 39	.24	.24	.24	.25
Resultant error of comparison interval	.30	.42	.39	.29

In Tables 19 to 23 are given the set means and final values obtained in the determination of the correction for the extensions and of the corrected lengths of the tubes. In these tables the component and resultant probable errors are also given.

TABLE 19

Summary of Length Measurements

Value of Correction for Extensions

Set:	Value at 0°
1	1629.01 μ
2	1629.09 μ
3	1629.31 μ
4	1629.49 μ
Mean	1629.22 μ

Average deviation from mean	±0.18 μ
Probable error of observation of mean	±0.07 μ
Probable error of correction, including probable error of comparison scale and micrometer value (tubes 1, 2, and 3)	±0.15 μ
Probable error of correction, including probable error of comparison scale (tube No. 4)	±0.14 μ

TABLE 20

Tube No. 1, Differences Measured by Micrometer

Tube+Extensions—Comparison Interval

Position:	Observed differences
1	−2.82 μ
2	−2.68 μ
3	−3.43 μ
4	−3.17 μ
Mean	−3.02 μ

Average deviation from mean	±0.28 μ
Probable error of mean	±0.12 μ
Comparison interval, meter 39	4.8—100.05 cm
Correction at 4.8 cm	− 4.98 μ
Correction at 100.05 cm	−49.64 μ
Correction to interval 4.8 cm to 100.05 cm	−44.66 μ
Corrected comparison interval	95.245534 cm ±0.30 μ
Corrections for extensions	− .162922 cm ±0.15 μ
Micrometrically measured difference	− .000302 cm ±0.12 μ
Corrected length at 0° C	95.082310 cm ±0.36 μ

TABLE 21

Tube No. 2, Differences Measured by Micrometer

Tube+Extensions—Comparison Interval

Position:	Observed differences
1	−18.92 μ
2	−18.40 μ
3	−18.70 μ
4	−18.45 μ
Mean	−18.62 μ

Average deviation from the mean	± 0.19 μ
Probable error of mean	± 0.08 μ
Comparison interval, meter 39	3.6 cm—100.03 cm
Correction at 3.6 cm	− 3.95 μ
Correction at 100.03 cm	−50.24 μ
Correction to interval 3.6 cm to 100.03 cm	−46.29 μ
Corrected comparison interval	96.425371 cm±0.42 μ
Corrections for extensions	− .162922 cm±0.15 μ
Micrometrically measured difference	− .001862 cm±0.08 μ
Corrected length at 0°C	96.260587 cm±0.45 μ

TABLE 22

Tube No. 3, Differences Measured by Micrometer

Tube+Extension—Comparison Interval

Position:	Observed differences
1	−62.03 μ
2	−62.53 μ
3	−62.07 μ
4	−63.25 μ
Mean	−62.47 μ

Average deviation from the mean	± 0.42 μ
Probable error of mean	± 0.19 μ
Comparison interval, meter 39	2.9—100.07 cm
Correction at 2.9 cm	− 4.30 μ
Correction at 100.07 cm	−50.32 μ
Correction to interval 2.9 cm to 100.07 cm	−46.02 μ
Corrected comparison interval	97.165398 cm±0.39 μ
Corrections for extensions	− .162922 cm±0.15 μ
Micrometrically measured difference	− .006247 cm±0.19 μ
Corrected length at 0°C	96.996229 cm±0.46 μ

TABLE 23

Tube No. 4, Differences measured by Micrometer

Tube+Extensions—Comparison Interval

Position:	Observed differences
1	−369.63 μ
2	−371.70 μ
3	−371.73 μ
4	−371.25 μ
Mean	−371.08 μ
Average deviation from the mean	± 0.72 μ
Probable error of mean	± 0.33 μ
Comparison interval, meter 39	−0.9−100.0 cm
Correction at −0.9 cm	+0011.81 μ
Correction at 100.0 cm	−50.19 μ
Correction to interval −0.9 cm to 100.0 cm	−0062.00 μ
Corrected comparison interval	100.893800 cm±0.29 μ
Correction for extensions	− .162922 cm±0.10 μ
Micrometrically measured difference	− .037108 cm±0.33 μ
Corrected length at 0°	100.693770 cm±0.45 μ

Discussion of Results, Length Measurements.—From the above tables it will appear that the probable error of the final length in no case exceeds 0.50μ. Moreover, except in the case of tube No. 4, the magnitude of the probable error is largely determined by the probable error of the comparison interval. It must be pointed out that the errors of comparison include errors due to variation in mounting the extensions, to lack of parallelism of the end faces of the tubes, and to variations in curvature of the tube between the supports when mounted in the four different positions. The error due to the curvature of a tube into a circular arc is given to a very high degree of approximation for any practical case by $\frac{8}{3}\frac{h^2}{l^2}$, h denoting the height of the arc above the chord and l the length of the cord. With 10 supports, 11 cm apart, the error due to an elevation of 0.1 mm of the middle point would correspond to an error of 2.2 parts per million in the length of a section. Since there were nine sections, the variation in the average curvature would determine the error of the result, but close agreement of the values obtained for the four positions in which the tubes were measured indicates that this source of error must have been quite small, particularly since the tubes had been straightened as previously mentioned and the slight residual curvature further reduced by the mounting in which they were measured.

I. TERMINAL BULBS

The terminal bulbs were constructed as shown in Fig. 12. They were made spherical, and very approximately 4 cm internal diameter. At one end of and on a line with a horizontal diameter of the bulb, a ground-glass extension was provided for the seating of the tapered end of the ohm tube. This extension was made by sealing on a short piece of thick-walled tubing with the outer half of its length enlarged so as to form part of a mercury seal. The grinding was then very carefully done, so that when completed the taper and diameter permitted the end of the seated tube to be flush with and to practically form part of the inner surface of the bulb. To the upper side of the seal portion of this extension was fused a short tube, provided with a ground-glass cap, serving for the introduction of mercury.

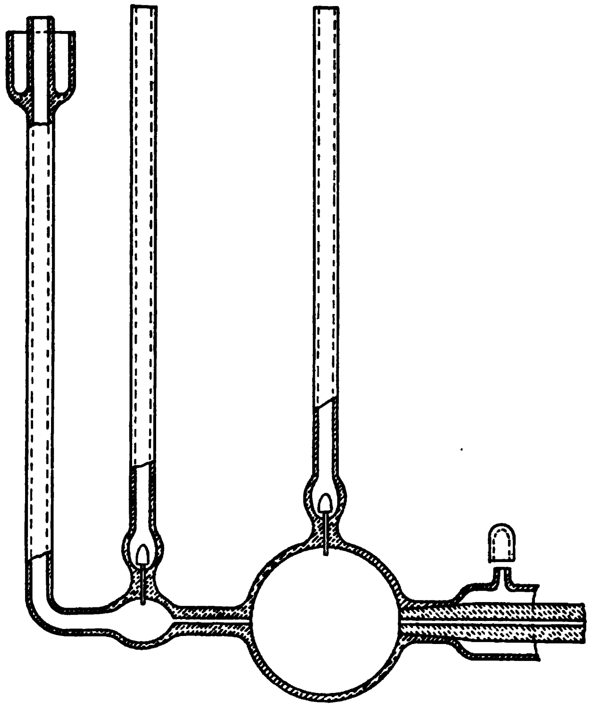


FIG. 12.—*Type of terminal bulb (exhaust end), forming part of a completed mercurial resistance standard*

Diametrically opposite the above opening to the bulb, a piece of capillary tubing with a bore somewhat larger than that of the ohm tube was fused on. The central portion of this capillary was enlarged to provide for the current terminal. Beyond the enlargement, the capillary terminated in a tube of larger bore bent vertically upward to serve as an exhausting or filling tube. The potential terminal was attached to the bulb at the upper end of a vertical diameter.

The current and potential terminals were made by sealing in short platinum wires pointed at the inner ends and carrying conical copper lugs at the outer ends. Care was taken in sealing in these terminals to preserve the spherical contour of the bulbs. They were protected from the water of the ice bath by glass tubing of about 4-mm internal diameter, and of sufficient length to extend well above the surface of the bath. These protecting tubes were sealed onto the bulbs and were slightly enlarged about the copper lugs.

The above figure shows a bulb provided with an exhaust tube terminating in a ground seat and mercury seal for the connection to the exhaust pump. The companion bulb to the above is provided with a filling tube consisting of a thick-walled capillary, or a tube with a number of constrictions, bent downward for a few centimeters at the top and drawn out to a fine point before exhausting.

A distinct advantage of the type of bulb described lies in the fact that there is but one glass joint and this is protected by a mercury seal.

The resistance contributed by the terminal bulbs was calculated on the assumption that the length of the mercury column is increased at each end by eight-tenths of the radius.

The corresponding resistance is, therefore, given by the formula:

$$\begin{aligned} R_{T_1}+R_{T_2} &= \frac{1}{1063}\left(\frac{0.8r_1}{\pi r_1^2}+\frac{0.8r_2}{\pi r_2^2}\right) \\ &= \frac{0.8}{\pi 1063}\left(\frac{1}{r_1}+\frac{1}{r_2}\right) \end{aligned}$$

r_1 and r_2 being expressed in millimeters.

The mean cross sections of the finished tubes are given by the L_0 and M_0 tabulations above, together with the density of mercury, which is taken as 13.5956 at 0° C.

The calibration data and the caliber and cross section curves were, however, all referred to the interval 5 cm to 105 cm, so that it was necessary to first determine the mean cross section in the above interval and from this the mean cross section at the ends.

The data employed in these calculations are given in Table 24.

TABLE 24

Data for the Calculation of the Correction for the Terminal Bulbs

	Tube No. 1	Tube No. 2	Tube No. 3	Tube No. 4
Caliber correction at lower end (mm)	+0.042	+0.050	+0.274	±0.000
Caliber correction at upper end (mm)	− .675	− .363	− .048	+ .010
Ratio of mean cross section of finished tube to mean cross section 5 cm–105 cm	.999246	.999571	.999669	1.000010
Ratio of cross section at lower end to mean cross section 5 cm–105 cm	1.00302	1.00334	1.00648	1.00196
Ratio of cross section at upper end to mean cross section 5 cm–105 cm	1.00968	.98978	.99875	1.00297
Mean cross section of finished tube (sq mm)	.895392	.906531	.913211	.947774
Mean cross section in interval 5 cm–105 cm (sq mm)	.896067	.906920	.913514	.947765
Cross section at lower end (sq mm)	.898773	.909949	.919434	.949623
Cross section at upper end (sq mm)	.904741	.897652	.912372	.950580
Correction for terminal bulbs in international ohms	.0008943	.0008933	.0008873	.0008712

The above values of the corrections for the terminal bulbs are correct to 0.000001 ohm as estimated by the accuracy of the calibration data, which, however, do not include irregularities within the subsections into which the tube is divided. The uncertainty arising therefrom, however, is believed to be small.

J. THEORETICAL RESISTANCES OF THE FINISHED TUBES

The theoretical resistances of the finished tubes can now be computed from the measured lengths, the mass of mercury required to fill the tubes and the corrections for the observed variations in cross section and for the terminal bulbs.

These fundamental constants together with the calculated resistances are summarized in Table 25.

TABLE 25

Theoretical Resistances of Finished Tubes

	Tube No. 1	Tube No. 2	Tube No. 3	Tube No. 4
L_0 in centimeters	95.082310	96.260589	96.996229	100.693770
M_0 in grams	11.574744	11.863961	12.042715	12.974950
Caliber factor	1.0000892	1.0000659	1.0000467	1.0000959
Resistance at 0°C of tube proper	.9990593	.9989882	.9992404	.9995525
Correction for terminal bulbs in international ohms	.0008943	.0008933	.0008873	.0008712
Total resistance in international ohms	.9999536	.9998815	1.0001277	1.0004237

To the above values certain corrections are still to be applied, but owing to the absence of reliable data for their calculation, they have merely been discussed (p. 456).

K. THE ELECTRICAL MEASUREMENTS

The electrical comparisons of the ohm tubes with auxiliary wire standards were all made by a modification of the Thomson bridge method.

The wire standards were five in number and were of the small type previously described,[14] the cases being filled with oil and hermetically sealed to eliminate changes in resistance due to variations in atmospheric humidity. Two of them, Nos. 3939 and 3940, were constructed by Otto Wolff in accordance with specifications furnished, while the others were constructed at the Bureau.

The comparisons were made by substitution (see Fig. 13), each standard being in turn substituted in the same bridge arm, S, the auxiliary standard, R, in the opposite arm being an open 1-ohm Wolff coil, readjusted to have a resistance approximately 0.12 per cent higher than its nominal value, so that the shunt required for balancing the bridge would always be on the auxiliary standard, and so that it would be unnecessary to employ shunts of very high value.

The wire standards, together with a 100:100 ohm main ratio coil and a 10:10 ohm auxiliary ratio coil, were contained in a well-stirred, thermostatically controlled, and electrically heated oil bath. In the comparisons they were connected in the desired manner by copper blocks mounted on suitable stands and provided with mercury cups, with the exception that the connection bridged over by the auxiliary ratio arms consisted of a short No. 10 copper wire, *L*, the ends dipping into the mercury cups, serving as potential terminals of the standards. When not in use the five reference standards were mounted on a metal stand, so as to be always completely surrounded by oil. The ratio coils were selected so as to obtain a very exact equality between the main and auxiliary ratios, thus eliminating any necessity of balancing the auxiliary arms. The main and auxiliary ratios were within one

---

[14] A New Form of Standard Resistance, this bulletin, **5**, pp. 413–434; 1908–9.

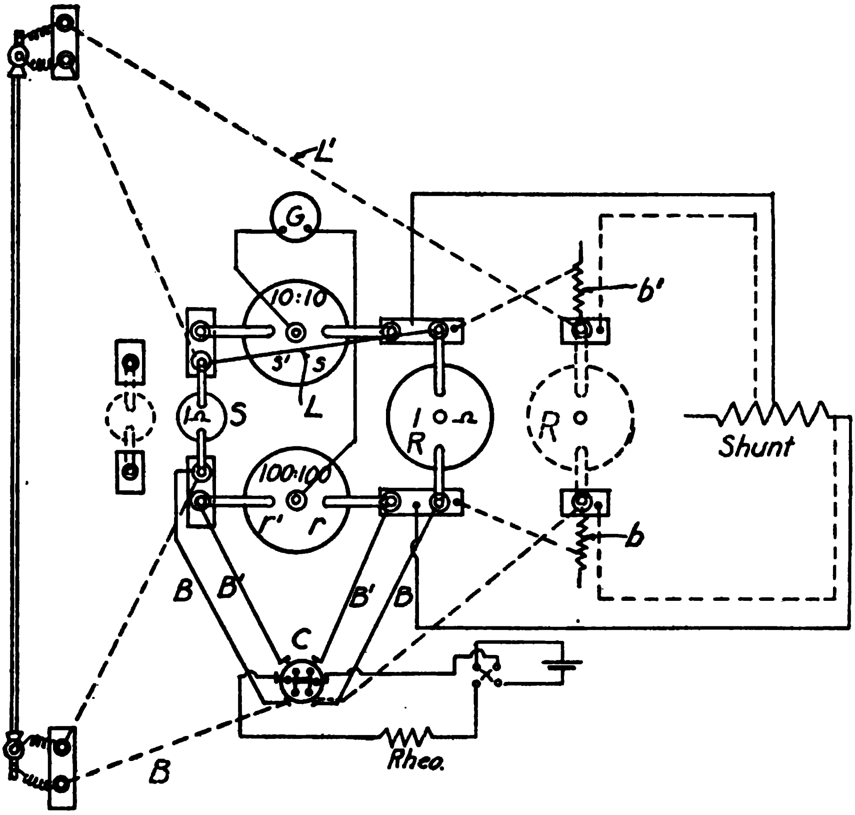


FIG. 13.—*Diagram of connections employed in making intercomparisons of mercurial and wire standards*

millionth of each other and differed from unity by but 13 parts in 1 000 000.

As before stated, the bridge was balanced by adjusting the shunt on the auxiliary comparison standard, R, until the galvanometer, G, permanently connected to the bridge, gave no deflection on reversal of the test current. G is a special high sensitivity moving coil galvanometer and was employed throughout. Its external critical resistance is approximately 50 ohms, and the deflection obtained for a difference in balance of one hundred parts per million amounted to 50 or 60 mm, with the test current of 0.035 ampere employed, so that comparisons could be easily made to within a few parts in 10 million.

The battery connections were normally applied to the potential terminals, adjacent to the main ratio arms, of the auxiliary and reference standards under comparison. By the aid of a mercury commutator, C, the connections could be shifted to the terminals of the main ratio coil, and the correction for the inequality of the connecting resistances from the latter to the standards could be interpolated from the deflection resulting on reversing the commutator. Since the resistances in the main ratio arms were approximately 100 ohms and the connecting resistances were themselves small, the correction never exceeded a few parts in 10 million.

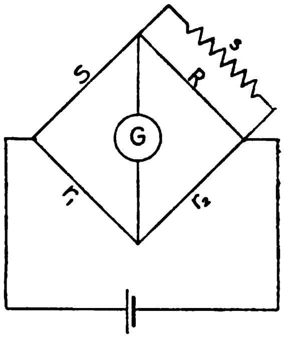


FIG. 14.—*Diagram of simple bridge with shunt*

Computation of the Results.—In a simple bridge, Fig. 14, let r_1 and r_2 denote the resistances of the main ratio arms; R that of the auxiliary standard; S_1, S_2, etc., that of the standards to be compared; and s_1, s_2, etc., that of the respective shunts applied to R to produce a balance.

Then

$$\frac{S_m}{\left(\dfrac{s_m R}{R+s_m}\right)}=\frac{r_1}{r_2}$$

or

$$S_m=\frac{s_m R}{R+s_m}\frac{r_1}{r_2}$$

As the direct calculation by the above formula is quite laborious, and as only the relative values of the standards under comparison are required, the computation was simplified by expressing the results in terms of the decrease in the resistance of the auxiliary standard produced by the balancing shunt.

The above formula may be written:

$$S=\frac{r_1}{r_2}\left(R-\frac{R^2}{R+s}\right)$$

or

$$S=\frac{r_1}{r_2}R-\frac{r_1}{r_2}\,\frac{R^2}{R+s}$$

Let $R=(1+\alpha)\,R_0$ and $s=m\,R_0$

Then

$$\frac{R^2}{R+s}=\frac{1+2\alpha+\alpha^2}{1+m+\alpha}R_0$$

$$=\frac{1+2\alpha+\alpha^2}{1+\dfrac{\alpha}{1+m}}\,\frac{R_0}{1+m}$$

The error due to neglecting α^2 in the numerator and $\dfrac{\alpha}{1+m}$ in the denominator is represented by

$$\left(\alpha^2-\frac{\alpha}{1+m}\right)\frac{R_0}{1+m}$$

and is readily shown to be negligible in all practical cases.

Hence

$$S=\frac{r_1}{r_2}R_0(1+\alpha)-\left(\frac{R_0}{1+m}+\frac{2\alpha}{1+m}\,R_0\right)\frac{r_1}{r_2}\equiv K-D$$

The first term of the expression which is denoted by K is constant, and its exact value is not required to obtain the relative values of standards substituted in the arm S. Moreover, since the second term, denoted by D, is a correction term, $\dfrac{r_1}{r_2}$ may be put equal to the nominal ratio in estimating its balance. In the special case here dealt with R_0 is equal to 1 ohm and $\dfrac{r_1}{r_2}$ differs only very slightly from unity, so that $D=\left(\dfrac{1}{1+m}+\dfrac{2\alpha}{1+m}\right)$. The first part is given directly by reciprocal tables, and the second part is so small that it can be assumed constant for a particular standard, the slight differences in *m* found in different comparisons producing negligible effects. Disregarding the second term entirely would, however, have produced a maximum error only slightly exceeding 1 part in 1 million even in the comparison of the mercury standards.

The results of a series of comparisons made in the manner above described may be written

$$\begin{aligned} S_1 &= K - D_1 \\ S_2 &= K - D_2 \\ &\cdot \quad \cdot \quad \cdot \\ S_n &= K - D_n \end{aligned}$$

Hence, assuming the value of one of the coils, or the mean value of a number of them, *K* is obtained. This value of *K*, substituted in the observational equations, gives the value of all standards substituted in the given bridge arm on the particular basis to which the assumed values are referred (in this case the mean of the 5 coils employed). The differences between the values thus computed and the assumed values of the separate coils, found from day to day and tabulated below, gives a measure both of the accuracy of the observations and of the relative constancy of the coils.

TABLE 26

Relative Constancy of Wire Standards During their Intercomparison with the Mercury Standards

Date	No. 3	No. 7	No. 8	No. 3939	No. 3940
1911	m	m	m	m	m
February 10	+0.8	−0.2	−0.6	+1.3	+0.4
February 13	+1.0	− .3	− .8	.0	+ .4
February 14	+ .9	− .6	− .4	.0	.0
February 15	+ .7	− .9	− .4	.0	+ .7
February 16	+ .6	− .6	− .6	+ .5	+ .4
February 18	+ .8	− .4	− .4	− .1	+ .5
February 21	+ .9	− .8	− .7	+ .1	+ .5
February 23	+1.0	− .6	− .5	+ .1	+ .3
February 24	+1.0	− .2	− .6	− .2	.0
February 25	+1.0	− .2	− .6	− .3	+ .1
February 27	+ .8	.0	− .9	− .4	+ .4
February 28	+1.0	− .2	− .6	− .4	− .1
March 1	+1.0	− .3	− .8	+ .4	.0
March 2	+1.0	− .1	− .8	− .1	+ .1
March 3	+1.0	− .4	− .8	− .1	+ .2
March 4	+ .8	− .2	− .8	− .2	+ .5
March 6	+ .8	− .2	− .8	− .2	+ .5
March 8	+1.0	− .2	− .8	− .1	+ .3
March 9	+1.0	− .1	− .7	− .1	− .1
March 16	+1.1	.0	− .7	− .1	− .1
March 17	+1.2	− .1	− .7	− .2	− .3
March 18	+1.1	− .1	− .7	− .1	− .1
March 20	+1.0	− .3	− .4	− .1	+ .1

Table 27 gives the temperature coefficients (in the general formula $R_t = R_{25}\{1 + \alpha\,(t-25) - \beta\,(t-25)^2\}$) as well as the values of the wire standards used in the intercomparisons, and covers the period from December 1, 1910, to June 14, 1912. The first five standards were used in the 1911 measurements. Two supplementary series of measurements were made in 1912, in which coils Nos. 3939 and 3940 were replaced by Nos. 4 and 15.

TABLE 27

One Ohm Wire Standards

[Excess in millionths over nominal value in international ohms]

Cell	α	β	Dec. 1, 1910	Feb. 17, 1911	Mar. 11, 1911	Jan. 11, 1912	Mar. 1, 1912	May 4, 1912	June 14, 1912
3	+10.8	−.48	+ 63	+ 63	+ 63.5	+ 66.5	+ 66.5	+ 66.5	+ 68
7	+ 9.2	−.49	+ 65	+ 64	+ 65	+ 65.5	+ 64.5	+ 64	+ 65.5
8	+ 9.2	−.50	+ 70	+ 70	+ 69.5	+ 69	+ 68	+ 68.5	+ 68
3939	+13.0	−.46	+ 99	+ 99	+ 99	+ 97.5	+ 99	+ 99	+100
3940	+13.1	−.45	+ 98	+ 97.5	+ 99	+ 98.5	+100	+101	+101
Mean			+ 79.0	78.7	+ 79.2	+ 79.4	+ 79.6	+ 79.8	+ 80.5
4	+ 8.2	−.46	+ 31	+ 31	+ 31.5	+ 29.5	+ 28	+ 28	+ 28.5
15	+ 6.0	−.50	+153	+155.5	+158	+167.5	+166.5	+168.5	+168

As stated above, the close equality of the ratios of the main and auxiliary arms of the bridge and the relatively low resistance of the connecting link between the two standards eliminated any error due to the link. The correction arising from the inequality of the connecting resistances in the main ratio arms was calculated from the deflection produced by shifting the battery connections as described above, a double deflection of 5 mm corresponding to a correction of one part in ten million. The observed deflection rarely exceeded 3 mm.

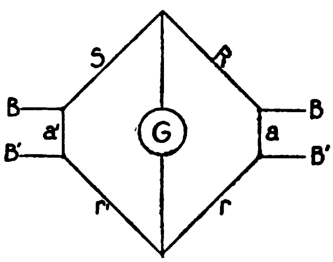


FIG. 15.—*Diagram of simple bridge with connecting resistances*

The considerably larger inequalities met with in the measurements of the mercury standards made it desirable to introduce compensating resistances in the opposite arms of the bridge.

The following discussion relates to the principles governing the method of adjusting the compensating resistances, so as to eliminate the correction involved, or to reduce it to such a small value that it can be computed with adequate accuracy:

Consider a simple Wheatstone bridge as shown in Fig. 15, in which S is the standard to be measured, R the standard in terms of which it is to be expressed, r and r' ratio arms, and a and a' connecting resistances, one or both adjustable, between the ratio arms and the standards under comparison.

$$\begin{aligned} \text{Let } S &= m\,R\,(1+x) \\ r &= n\,R \\ r' &= mn\,R\,(1+\alpha) \\ a &= c\,R \\ a' &= mc'\,R \end{aligned}$$

Then if the bridge be balanced (by preliminary adjustment of one of the connecting resistances) with the battery applied at B′ B′

$$\frac{S+a'}{R+a}=\frac{r'}{r}$$

Substituting

$$\frac{m\,R\,(1+x)+mc'\,R}{R+cR}=\frac{mn\,R\,(1+\alpha)}{nR}$$

From this it follows that

$$x=\alpha+(1+\alpha)\,c-c'$$

Next shift the battery connections from B' B' to BB and rebalance the bridge by adjustment of R to R′, keeping the connecting resistances a and a' unchanged and

$$\text{let } R'=R\,(1+\beta)$$

Then

$$\frac{S}{R'}=\frac{r'+a'}{r+a}$$

or,

$$\frac{mR(1+x)}{R(1+\beta)}=\frac{mnR(1+\alpha)+mc'R}{nR+cR}$$

On simplification we obtain,

$$1+x=(1+\beta)(1+\alpha)-(1+\beta)\frac{(1+\alpha)c-c'}{n+c}$$

In the first position of the battery connections, the error due to ignoring the connecting resistances would have made x too small by $(1+\alpha)c-c'$; in the second position the value found for x would have been too large by $(1+\beta)\frac{(1+\alpha)c-c'}{n+c}$. The ratio of the second to the first of these quantities is $\frac{1+\beta}{n+c}$.

From the above relations the following conclusions can be drawn:

(1) If the connecting resistances are entirely compensated,

$$x=\alpha,\ (1+\alpha)c=c' \text{ and } \beta=0.$$

That is,

$$\frac{S}{R}=\frac{r'}{r}=\frac{a'}{a}=m(1+\alpha)$$

In this case the bridge is in balance for both positions of the battery connections without adjusting *R*. Moreover, it is the only condition under which the connecting resistances are completely eliminated, as could have been seen directly from Fig. 15, by supposing the battery and galvanometer connections interchanged, there being in the above case no differences in potential between the pairs of terminals *B'B'* and *BB*.

(2) The ratio $\frac{r}{R}$, designated by n, must be greater than unity in order that the error due to the connecting resistance may be reduced by rebalancing in the manner described. That is, the battery connections, when adjusting the bridge balance by varying one of the connecting resistances, must be applied so that the resistances a and a' are included in the arms of lower resistance. In other words, the connecting resistances are adjusted in such a manner as to make the ratio of the connecting resistances approximate the ratio of the bridge arms having the higher resistance. They can not, by the above process, be adjusted to the ratio of the arms of lower resistance.

(3) The original error, due to the presence of the connecting resistances remaining after the adjustment of one of them to balance the bridge with the battery applied at *B'B'*, is determined solely by the difference $x-\alpha$, the relative difference in balance if there be no connecting resistances. Rebalancing by adjustment of *R* (or *S*) with the battery applied at *BB*, the above error is reduced in the ratio $\frac{1+\beta}{n+c}$, or since β and c are generally small in comparison, respectively, with unity and n, the error is virtually reduced in the ratio $\frac{1}{n}$.

(4) The repetition of the above process, first rebalancing with the battery at $B'B'$, by adjustment of a or a', and then again rebalancing with the battery at BB by adjustment of R, will each time reduce the error in the approximate ratio $\frac{1}{n}$.

(5) In the special case in which $n=1$, the error in x in the two positions of the battery will be $(x-\alpha)$ and $-\frac{1+\beta}{1+c}(x-\alpha)$. That is, they will be approximately of the same magnitude but of opposite signs if β and c are small in comparisons with unity. For $n=1$, the method of eliminating the connecting resistances therefore fails.

(6) When $n \geqq 10$, the steps need only be repeated a very few times to eliminate the connecting resistances entirely, since $(x-\alpha)$ rarely exceeds 0.001. Moreover, in the preliminary adjustments the balance need not be exact, so that the final adjustment is quickly attained.

To apply these conclusions to the case of the Thomson bridge, it need only be remembered that the Thomson bridge is equivalent to a simple bridge having arms $R+K\frac{s}{s+s'}$, and $S+K\frac{s'}{s+s'}$, Fig. 16, where K is the resistance between the points bridged by the auxiliary ratio arms, and s and s' are the total resistances in the auxiliary ratio arms.

The connecting resistances a and a' must in the first instance be adjusted, so as to have the same ratio as r and r', no choice being left, as the ratio arms are larger than the resistances under comparison in practically all cases where the Thomson bridge is employed.

The procedure employed in the comparison of the mercury and wire standards was briefly as follows:

The wire standard, S (Fig. 13), was replaced by the mercury standard, connections to the bridge being made through the leads as shown. The auxiliary 1-ohm standard R was shifted to the position as indicated by the dotted lines and connected to the ratio arms by the adjustable low resistances[15] b and b' consisting of movable amalgamated copper rods in porcelain tubes containing mercury.

---

[15] Phys. Rev., **32**, p. 614, 1911; **33**, p. 215, 1911.

A rough balance of the bridge, with the link L' in, was first made by adjusting the shunt on R, the battery being applied at BB. The battery connections were next shifted[16] to B' B' by means of the commutator C, and the bridge rebalanced by adjustment of the mercury resistance b. With the battery again in the normal position and the link L' out, a balance was obtained by adjusting b'. . The above operations were repeated and the final balance obtained by adjusting the shunt. After the approximate value of the resistance of a tube and its corresponding approximate balancing shunt had once been determined subsequent measurements could very quickly be made.

Preparation of the Tubes for Electrical Measurements.—Two tubes are made ready for electrical measurements at the same time. After cleaning and drying them in the usual way (this is always done before making a new filling) they are mounted on either side of a nickel-plated steel beam in carefully aligned V-grooved supports and clamped with suitable spring clips. The terminal bulbs are then seated, being adjusted so that the extensions are in a vertical plane. The bulbs are prevented from rotating by means of hard-rubber strips made to slip over the protecting tubes of the potential terminals and secured to uprights attached to the supporting beam. Under the center of each bulb is a vertical screw carrying horizontally a flat metal disk about 2 cm in diameter which is adjusted to support the bulb. A further support is provided between the bulb proper and the smaller bulb of the current terminal. This support consists of a rectangular piece of copper with a V-notch at the upper end

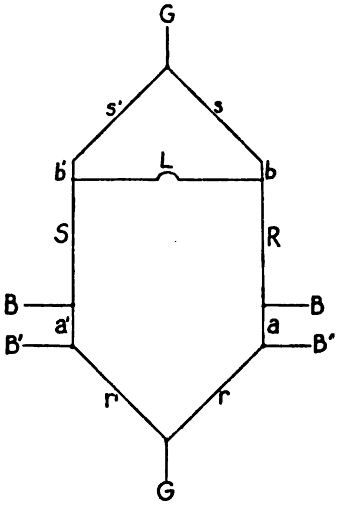


FIG. 16.—*Diagram of the Thomson bridge, with connecting resistances*

and with its lower end slotted so that the support may be adjusted to its proper height and held in place by a screw.

---

[16] This bulletin, 8, p. 585; 1912.

The mercury seals are next made secure by means of soft-rubber washers snugly fitting the ohm tubes and clamped against the face of the glass portion of the seal by screws through annular hard-rubber disks placed on the tubes and engaging slotted disks, also of hard rubber, slipped over the outlet of the bulbs just behind the enlargement forming part of the seal. The seals are filled with mercury and the proper seating of the tubes tested by partial evacuation of the tubes and bulbs. If satisfactory, the seals are capped and coated with celloidin to make them waterproof.

Preparatory to filling, a tube is connected to a Geryk pump and exhausted 15 minutes, the resulting observed pressure in most cases being less than 0.01 mm of mercury. It is filled by breaking the sealed and drawn-out end of the filling tube under pure mercury twice distilled in a Hulett still. The greater portion of the mercury is next removed from the upright portions of the filling and exhaust tubes and the final height of the mercury in each above the level of the ohm tubes recorded. Mercury is also introduced into the protecting tubes of the current and potential terminals. The filled tubes (see Fig. 17) are next removed to the ice bath and "iced up" preparatory to the electrical measurements.

The ice bath consists of a rectangular copper tank (Fig. 18) 128 cm long, 22 cm wide, and 16 cm deep, supported within a wooden tank with a clearance of about 6 cm all around. The inner tank is of double-walled, nickel-plated copper with intermediate air space of 0.6 cm. The outer tank is lined with nickel-plated copper sheeting and has the corners at the bottom rounded to facilitate its proper filling with ice. The outer tank is also provided with a drain pipe and a water gage.

Finely chipped, pure plate ice, made from distilled water, was used, care being taken to keep it free from contamination at all times. The ice was carefully packed about the tubes and terminal bulbs and extended to within 1 or 2 centimeters of the top of the terminal tubes. The outer bath was well packed with

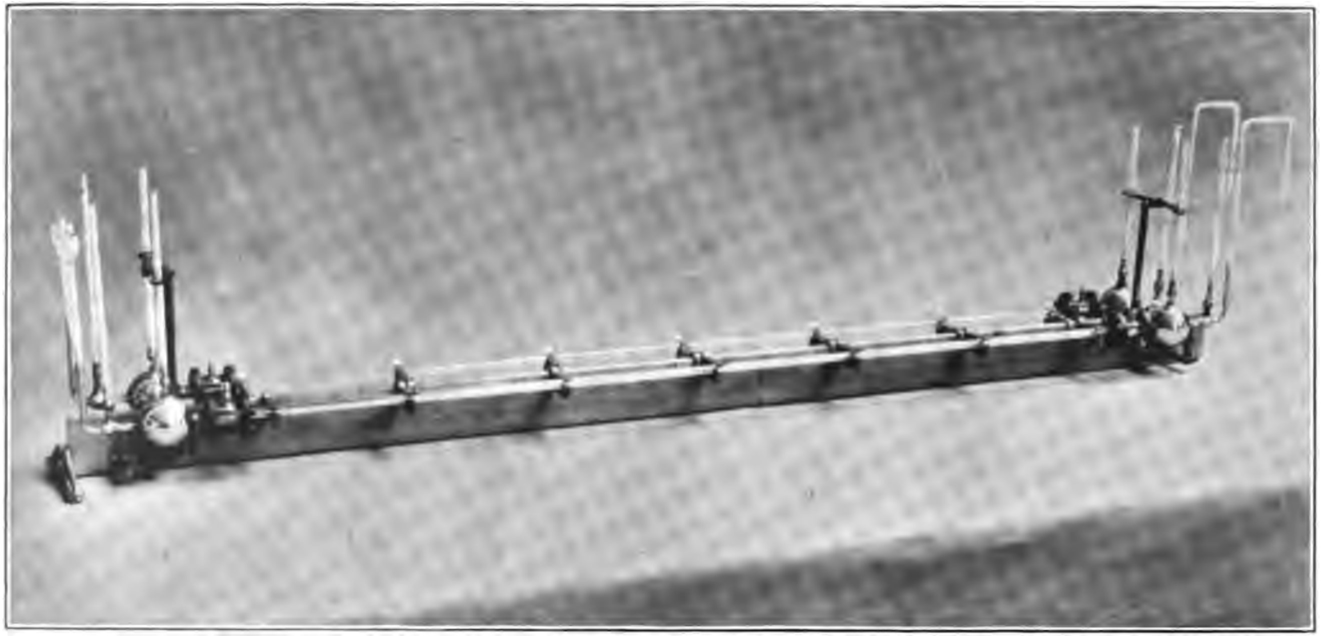


FIG. 17.—*View of two mercury units mounted and filled preparatory to placing in ice bath for electrical comparisons with wire standards*

coarser ice, and both tanks were filled to the top with distilled water. The resistance of the water in the inner bath was determined in nearly every case after the completion of the electrical measurements and averaged 3300 ohms. That of distilled water averaged 5500 ohms in the same apparatus. In one instance the measured resistance was abnormally low, 300 ohms, in which case the bath was cleaned and reiced and the electrical measurements repeated. With this one exception there was no cause for a rejection of the ice, the conductivity of the water being considerably less than that corresponding to a concentration of NaCl, which would affect the melting point by 0.001° C.

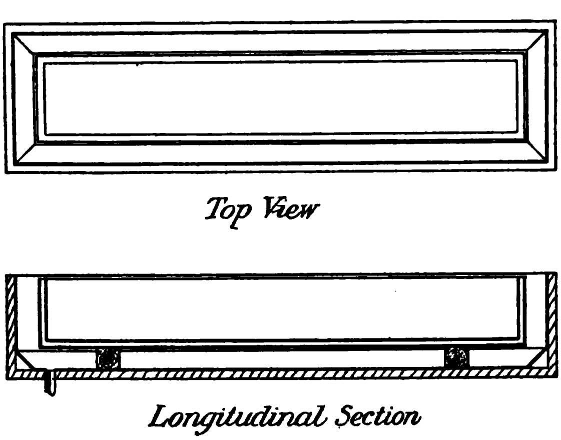


FIG. 18.—*Ice bath for mercury units under electrical comparison; also used in the determination of M_0 by Smith's method*

Insulation Resistance.—The insulation resistance of the inclosed column of mercury was measured in practically every case, and was found in most instances to exceed 100 megohms. The few exceptions were among the first fillings, where no attempt was made to protect the mercury seals from the water by coating them with celloidin.

The Connecting Leads.—Flexible stranded copper wire leads, cotton covered and 3 mm in diameter, were used for making electrical connections to the bridge. These leads terminate in well amalgamated No. 8 single copper wires, dipping into the mercury of the current and potential tubes. The height of the mercury in these tubes was so adjusted that with the leads in place the mercury was but slightly below the level of the ice. Heat conduction through the terminals into the end bulbs was thereby eliminated even when using the heavy conductors.

During the electrical measurements one of these terminals was several times heated in mercury at 90° C. and quickly placed in the potential terminal tube. The effect was principally thermoelectric, and the normal resistance was recovered within two or three minutes. No measurements were ever made after inserting the terminals until the galvanometer zero was steady, the usual time of waiting being from 5 to 10 minutes.

Effect of Test Current.—The effect of the test current of 0.035 ampere employed was found to be inappreciable. A current of 0.88 ampere steadily applied gave a resultant change in resistance of 238 millionths, corresponding to a change of approximately 300 parts per million per watt. The effect produced became constant within five minutes and the resistance returned to its original value within two minutes after opening the circuit. The current employed in the measurements corresponded to a load of 0.0012 watts, and therefore would correspond to a heating effect of but four parts in ten million. The experiments made also give a measure of the rapidity with which thermal equilibrium is established when the tube is placed in the ice bath. However, no electrical measurements were made without waiting at least half an hour after icing. Measurements occasionally repeated for various means confirm the above conclusions.

Effect of Pressure on Resistance.—The effect of the pressure on the conductivity of the mercury within the tube, due to the head in the filling and exhaust tubes, was necessarily very small, since the head in all cases was less than 6 cm. The change in resistance per unit resistance per atmosphere (internal and external pressure) of a column of mercury under pressure in ordinary glass as determined by A. De F. Palmer,[17] is approximately 33 parts per million. In tube No. 1 a decrease in resistance of but 3.7 parts per million was found for an increase in head of 15.6 cm of mercury, corresponding to a change of 1.4 parts per million for a head of 6 cm.

Effect of Curvature on Electrical Resistance.—A calculation was made to ascertain the influence of impressed curvature on the electrical resistance of an ohm tube on the assumption that the length of the axis remains unchanged and that the shape of the cross section remains the same. The calculated resistance of the flexed tube was found to be

---

[17] American Jr. Sci. (4), 4; p. 1; 1897.

$$R_0\left\{1+\frac{1}{4}\left(\frac{a}{h}\right)^2+\frac{1}{8}\left(\frac{a}{h}\right)^4\right\}$$

in which R_0 represents the resistance of the unflexed tube, a the radius of the bore, and h the radius of curvature. It is apparent that there can be no appreciable effect due to this cause.

L. RESULTS OF ELECTRICAL COMPARISONS

In the electrical comparisons, readings were made, first on each of two tubes, then on the five wire coils with which they were compared, after which the readings on the tubes were repeated. The average difference between the first and last readings on the tubes (62 pairs in all) was less than 0.8 part per million.

The results of electrical comparisons of the ohm tubes with the wire standards in 1911 are shown in Table 28. Seven fillings were

TABLE 28

Results of Electrical Measurements, 1911

[The observed minus the calculated value in international ohms]

Filling	Tube No. 1		Tube No. 2		Tube No. 3		Tube No. 4	
	Difference in millionths	Δ	Difference in millionths	Δ	Difference in millionths	Δ	Difference in millionths	Δ
1	−27.8	+0.6	−27.0	+5.5	−17.7	+1.8	−14.4	+7.1
2	−28.0	+ .4	−34.8	−2.3	−19.2	+ .3	−16.4	+5.1
3	−28.2	+ .2	−34.0	−1.5	−18.1	+1.4	−16.6	+4.9
4	−31.4	−3.0	−28.7	+3.8	−20.0	− .5	−25.6	−4.1
5	−28.4	.0	−35.8	−3.3	−21.4	−1.9	−24.1	−2.6
6	−33.4	−5.0	−38.6	−6.1	−22.0	−2.5	−26.8	−5.3
7	−21.4	+7.0	−28.5	+4.0	−18.4	+1.1	−26.3	−4.8
Mean	−28.4	±2.3	−32.5	±3.8	−19.5	±1.4	−21.5	±4.8

Probable error of mean:

Tube No. 1	±0.95 millionths
Tube No. 2	±1.12 millionths
Tube No. 3	±0.42 millionths
Tube No. 4	±1.38 millionths

made of each tube, and the values tabulated are differences in international ohms of the observed and calculated values. The Δs represent the deviation of the individual measurements from their mean. The average deviation from the mean and the probable error of the mean are also given. The calculated or theoretical resistances of the tubes have been given in Table 25, page 443.

The following table shows the mean results for the four tubes. The average deviation was 5 millionths, a very satisfactory agreement.

TABLE 29

Summary of Electrical Measurements, 1911

[Observed minus calculated resistance in international ohms]

Tube	Difference in millionths. Mean of seven fillings	Δ
1	−28.4	−2.9
2	−32.5	−7.0
3	−19.5	+6.0
4	−21.5	+4.0
Mean	−25.5	±5.0

Discussion of Results.—Table 29 shows that the international ohm, as defined by the four tubes, was found in 1911 to be 25.5 millionths smaller than the international ohm as represented by the manganin coils of the Bureau. These coils maintain the unit agreed upon in the 1910 international investigation, by intercomparison of the coils of England, France, Germany, and the United States.[18] The "1910 ohm" thus agreed upon represented the international ohm as closely as it was known at that time. The "1910 ohm" was so selected as to represent the mean of the English and German mercury ohm tubes, as no other countries had suitable ohm tubes at that time.

According to the comparisons made in the 1910 investigation, the international ohm, as represented by the German ohm tubes, was 10 millionths larger than that represented by the English ohm tubes. The first, therefore, differed by + 5 millionths and the second by − 5 millionths from the "1910 ohm." It follows that the international ohm as represented by the Bureau of Standards ohm tubes in 1911 was 30.5 millionths smaller than the unit defined by the German tubes and 20.5 millionths smaller than the unit defined by the English tubes.

---

[18] Report to the International Committee on Electrical Units and Standards (issued by the Bureau of Standards, Jan. 1, 1912), pp. 82, 203.

PART III. SUPPLEMENTARY MEASUREMENTS

In July, 1912, two additional electrical comparisons of each of the four Bureau of Standards tubes were made. As the observed resistance of each tube minus its calculated resistance was greater than found in 1911 (see Tables 29 and 33), a third series of comparisons was made in November and December, 1912, which, in general, confirmed the results of the July series.

Redeterminations of the M_0 and L_0 of each tube were therefore made early in 1913 and the theoretical resistances recalculated. All measurements were made in the same way as those of 1910 to 1911, with the exception that in the redetermination of L_0 readings were made on but two positions of each tube with a smaller number of readings for each position, and hence with a correspondingly larger probable error. The work involved a recalibration of the weights employed and a recomparison of meters 39 and 22 at 0° C. The results are summarized in the following tables.

Tables 30, 31, and 32 show that the average change in L_0, M_0, and the calculated R_0 for all four tubes was 1.1, 9.4, and 8.2 parts per million, respectively.

Table 34 shows that the international ohm as defined by the four tubes in December, 1912, was 12.5 millionths smaller than the international ohm as represented by the manganin coils of the Bureau at that time. The ohm tubes and the manganin coils of the Bureau have thus come into better agreement since 1911, when the difference was 25.5 millionths. It would appear that the manganin coils changed in such a way that the unit which they define decreased by 13 millionths. That the change may actually be attributed to the manganin coils rather than to the ohm tubes was indicated by comparisons made with the standards in other national laboratories. Coils were sent by the Bureau to England and Germany in 1911 and 1912 and measured in terms of the manganin standards of the National Physical Laboratory and the Reichsanstalt, respectively. According to the former the unit defined by the Bureau manganin coils decreased 9 millionths, and according to the latter 5 millionths during the interval.

TABLE 30

Redetermined L_0, 1913

Tube	L_0 in centimeters	L_0, 1913, minus L_0, 1911, in parts per million
1	95.082500	+2.00
2	96.260476	−1.17
3	96.996315	+0.89
4	100.693824	+0.53
Average change	....	1.14

TABLE 31

Redetermined M_0, 1913

Tube	Fillings	Mean M_0 in grams	Average deviation from mean in parts per million	Probable error of results in parts per million	M_0, 1913, minus M_0, 1911, in parts per million
1	7	11.574870	±2.19	±0.91	+10.89
2	7	11.864101	±3.17	±0.95	+11.80
3	7	12.042745	±3.57	±1.07	+ 2.49
4	7	12.975108	±2.77	±0.90	+12.28
Mean	....	....	±2.92	±0.96	+ 9.36

TABLE 32

Recalculated R_0, 1913

Tube	R_0 in international ohms	Calculated R_0, 1913, minus R_0, 1911, in parts per million
1	0.9999467	− 6.88
2	0.9998674	−14.13
3	1.0001270	− 0.71
4	1.0004127	−11.20
Average change	....	8.23

TABLE 33

Observed R_0 − Calculated R_0 in International Ohms, Series 2, July, 1912

[The calculated R_0 was prorated on the assumption that the M_0 and the L_0 changed uniformally between the two determinations]

Tube	Fillings	Observed minus calculated R_0 in parts per million	Δ
1	2	−13.7	− 4.9
2	2	−11.8	− 3.0
3	2	−12.7	− 3.9
4	2	+ 3.0	+11.8
Mean	....	− 8.8	± 5.9

TABLE 34

Observed R_0 − Calculated R_0 in International Ohms, Series 3, November and December, 1912

[Calculated R_0 from 1913 determination of M_0 and L_0]

Tube	Fillings	Observed minus calculated R_0 in parts per million	Δ
1	4	−19.5	− 7.0
2	5	− 8.6	+ 3.9
3	4	−19.7	− 7.2
4	4	− 2.1	+10.4
Mean	....	−12.5	± 7.1

The unit defined by the manganin coils of the Bureau, therefore, decreased by 7 millionths in terms of the mean of the English and German manganin standards, and decreased by 13 millionths in terms of the mercury tubes of the Bureau. This rather good agreement is partly accidental, since the mean change of 13 millionths is an average for the four tubes of changes ranging from 0 to 24 millionths, as may be seen from the following table:

TABLE 35

Observed R_0 — Calculated R_0, in Parts per Million

Tube	1911	December, 1912	Change	Δ
1	−28.4	−19.5	− 8.9	+ 4.1
2	−32.5	− 8.6	−23.9	−10.9
3	−19.5	−19.7	+ 0.2	+13.2
4	−21.5	− 2.1	−19.4	− 6.4
Mean	−25.5	−12.5	−13.0	± 8.6

The mean deviation of 8.6 millionths is a rough measure of the accuracy of reproduction of the mercury unit at different times, using the same tubes.

COMPARISON OF OHM TUBES OF SEVERAL COUNTRIES

A comparison of the mercury ohm tubes of the various national laboratories was made in 1913 by means of measurements upon a number of manganin coils. The English National Physical Laboratory sent coils to the Laboratoire Central d'Electricité of France, the Reichsanstalt of Germany, the Bureau of Measures and Weights of Russia, and to the Bureau of Standards. Each laboratory made measurements upon the coils and gave data by which the results could be expressed in terms of the mercury ohm tubes of the laboratory. The values given at that time by this Bureau were in terms of the mercury ohm measurements of July, 1912. In the following table these have been changed to correspond with the latest mercury ohm measurements, those of December, 1912. Mercury ohm tubes have also been set up in the Electrotechnical Laboratory at Tokyo, Japan,[19] and compared with tubes supplied by the National Physical Laboratory of England. The unit defined by the Japanese tubes was found to be 8 millionths smaller than the unit defined by these English tubes. Assuming (1) that these English tubes give the same mean unit as the tubes kept in the National Physical Laboratory, and (2) that the Japanese laboratory takes the mean of both sets of tubes to be the mercury resistance unit of Japan, it follows that the Japanese mercury unit is 4 millionths smaller than the English mercury unit. The first of these assumptions is probably not exact, but the result can not be far wrong.

---

[19] Proc. Math.-Phys. Soc. Tokyo, 7, p. 346; 1914. Science Abstracts, 17, No. 1890; 1914.

TABLE 36

Comparison of Units Defined by Ohm Tubes in 1913

Country	Mercury unit of country minus mercury unit of United States
	Millionths
England	+10
Germany	+28
Japan	+6
Russia	+5
United States	0

New tubes have been set up in France, but the results have not yet become available.

As has been stated above, the unit defined by the ohm tubes of this Bureau in 1911 was 20.5 millionths smaller than the unit defined by the ohm tubes set up in England prior to 1910. The English mercury unit used in Table 36 is that defined by a set of new tubes. It would appear that the new English tubes gave a unit 10.5 millionths smaller than the old. This is corroborated by an estimate given by the National Physical Laboratory, viz, the new tubes define a unit 11 millionths smaller than the old.

Table 36 shows that the mercury unit of the Bureau of Standards was 28 millionths smaller than the German mercury unit in 1913. This is a good agreement with the difference of 30.5 millionths found in 1911.

In conclusion, the mercury ohm tubes of the various countries have been found to exhibit a very satisfactory agreement.

The authors wish to acknowledge the valuable assistance given them by Dr. J. H. Dellinger, Dr. F. Wenner, and other members of the staff.

WASHINGTON, January 1, 1915.

NOTE ON THE RESISTANCE OF RADIOTELEGRAPHIC ANTENNAS

By L. W. Austin.

According to the theory the portion of the resistance of an earthed antenna due to its radiation is[1]

$$R_r = 160\pi^2 \frac{h^2}{\lambda^2}$$

which is approximately $1600 \frac{h^2}{\lambda^2}$

Where h is the height from the earth to the center of capacity of the antenna and λ the wave length, assuming that the earth below the antenna is a good conductor like salt water. As a matter of fact, in most land stations the effective height is less than the actual height and in many cases not more than one-half of it. In all antennas, however, radiation resistance must, according to the theory, decrease as the wave length increases and in accordance with the square law.

It was first noted by C. Fischer[2] that in certain cases the resistance[3] of antennas increased as the period was increased by introducing inductance. Fischer believed that this indicated an increase of radiation with wave length.

The whole course of the phenomenon has been studied by the United States Naval Radio Laboratory and the results published.[4]

Fig. 1 shows the resistance curve of the antenna used by the naval laboratory at the Bureau of Standards and that of the U. S. S. *Maine*. The first is typical of a land station with poor ground conditions, the curve falling rapidly as the true radiation resistance drops with increasing wave length and then rising again in a straight line as the wave length is further increased. The curve for the *Maine* shows the same drop at the shorter wave lengths followed by a rise which is very slight. Fig. 2 shows the antenna resistance of the high power station at Arlington, Va., with the towers grounded. Here there is almost no rise as the wave length is increased, showing how perfect ground conditions can be made by the use of an extensive network of ground wires.

---

[1] Zenneck Lehrbuch d. drahtlosen Telegraphie, 1913, p. 202.

[2] Phys. Zeit. 1911, p. 295.

[3] Leaving out of account the resistance of the inductance coils.

[4] Proc. Wash. Acad. I, p. 9, 1911; and this Bulletin, 9, p. 65, 1912; Scientific Paper No. 189.

Up to the present there has been no satisfactory explanation of the rise of the antenna resistance curve, although it seems certain that it has something to do with the ground. Experiments by H. True [5] show, however, that the ground resistance, as indicated by fall of potential methods, is greater at short than at long wave lengths. If, however, we consider the ground as a dielectric rather than as a conductor and consider it as a portion of the total dielectric lying between the antenna, regarded as the upper plate of a condenser, and the ground water, regarded as the lower plate, we reach a very probable explanation of the peculiar form of many antenna resistance curves. For it is well known that the equivalent resistance of an imperfect dielectric increases as the wave length is increased.

As an example [6] of experiments showing this, the equivalent resistance of a certain glass condenser of approximately 0.002 mf capacity measured in the Naval Radio Laboratory is shown in Fig. 3. The equivalent resistance of the condenser at the various wave lengths is determined by placing it in a circuit acted upon by a buzzer-excited wave meter. The current in the circuit containing the condenser is measured by means of a thermoelement. The condenser under test is then replaced by a variable air condenser adjusted to the same capacity in series with which resistance is introduced until the thermoelement indicates the same high frequency current as in the case of the glass condenser. The series resistance then represents the equivalent resistance of the glass condenser. Fig. 3 shows that the equivalent resistance of the glass condenser increases in direct proportion to the wave length, just as does the resistance of the antenna at the Bureau of Standards beyond 1500 meters.

---

[5] Jahrbuch d. drahtlosen Telegraphie, V, p. 125, 1911.

[6] Proc. of the Inst. of Radio Engineer, I, p. 35, 1913.

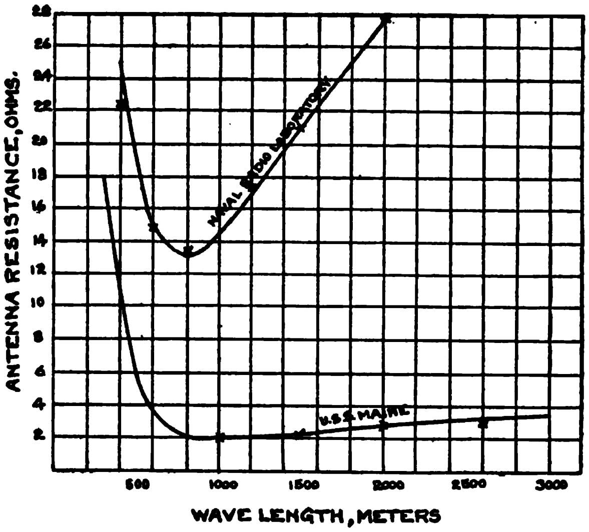


FIG. 1.—*Antenna resistance, Naval Radio Laboratory, U. S. S. "Maine"*

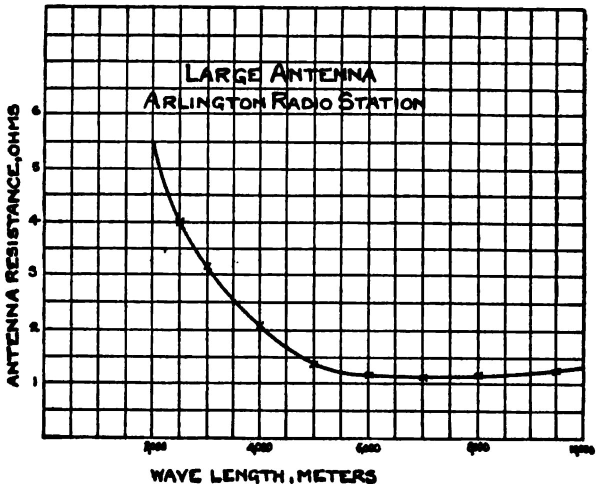


FIG. 2

The rise in the resistance curve of the *Maine*, as shown in Fig. 1, is probably due to the fact that the measurements were made with the ship in dock, as it is impossible to use sufficiently sensitive galvanometers under other circumstances. With the ship in dock, of course, a considerable portion of the field passes through poor dielectric material before reaching the water.

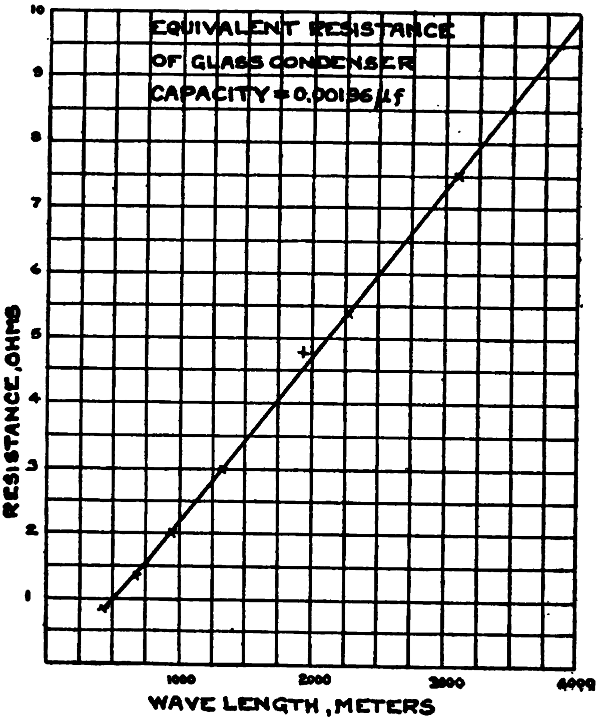


FIG. 3

U. S. NAVAL RADIOTELEGRAPHIC LABORATORY,
WASHINGTON, February 13, 1915.

A METHOD OF MEASURING EARTH RESISTIVITY

By Frank Wenner

1. INTRODUCTION

A knowledge of earth resistivity[1] (or specific resistance) may be of value in determining something of the composition of earth, such for example, as moisture content, whether it contains oil, ore of high conductivity, etc., or in the calculation or mitigation of damages to pipe systems by electrolysis caused by the return current of street-railway systems. For some of these or other reasons we may wish to determine the resistivity of limited portions of the earth.

For those cases in which we desire the resistivity of a fairly large portion of earth, extending to a considerable depth, or where there are reasons why we should not disturb the portion of earth to be measured, the following method is suggested: Four holes are made in the earth approximately uniformly spaced in a straight line. The diameter of the holes is not more than 10 per cent of the distance between them, and all extend to approximately the same depth, which is usually that at which we are most concerned with the resistivity. In each hole is placed an electrode, which makes electrical contact with the earth only near the bottom, as shown in Fig. 1.

This constitutes a four-terminal conductor[2] the resistance of which depends upon the distance between the electrodes and the resistivity, mainly in a region whose linear dimensions are of the same order as the distance between the outside electrodes, but does not depend appreciably upon the size of the electrodes nor the kind of electrical connection they make with the earth.

---

[1] Here we are concerned with the volume resistivity, which is the resistance of a portion of a conductor having unit length and unit cross section. It is usually expressed as the resistance in ohms of a centimeter cube.

[2] A four-terminal conductor is a conductor provided with two terminals to which current leads may be connected and two terminals to which potential leads may be connected. The resistance of such a conductor is the difference in potential between the potential terminals divided by the current entering and leaving through the current terminals. For a more complete discussion of the four-terminal conductor see this Bulletin, 8, p. 360, 1912, Reprint No. 181.

Therefore, if the depth of the holes, the distance between them, and the resistance (using 1 and 4 as current and 2 and 3 as potential terminals, or 2 and 3 as current and 1 and 4 as potential terminals) are measured, we have data from which the effective resistivity in the vicinity can be calculated.

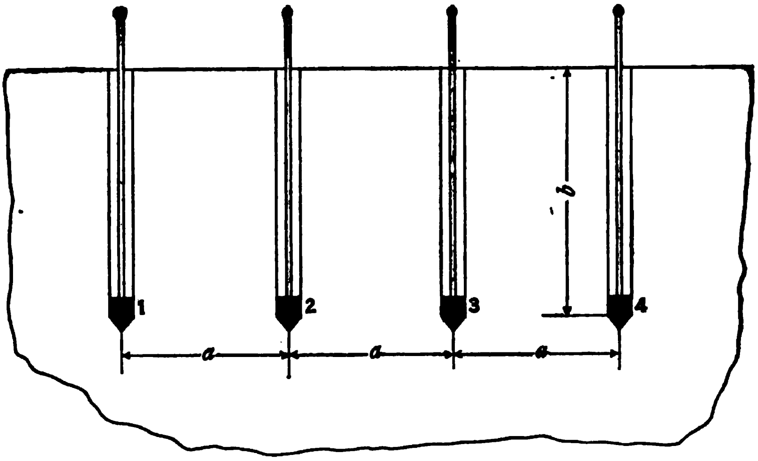


FIG. 1.—*Diagram showing four electrodes in earth constituting four-terminal conductor as used in measuring earth resistivity*

In case a is the distance between the holes (1 to 2, 2 to 3, and 3 to 4), b the depth of the holes, ρ the resistivity, and R the measured resistance, then

$$\rho=\frac{4\pi aR}{1+\frac{2a}{\sqrt{a^2+4b^2}}-\frac{2a}{\sqrt{4a^2+4b^2}}}=\frac{4\pi aR}{n} \qquad (1)$$

where n has a value between 1 and 2 depending upon the ratio of (b to a) the depth of the electrodes to the distance between them. Where $b=a$, $n=1.187$; $b=2a$, $n=1.038$; and $b=4a$, $n=1.003$.

In case b is large in comparison with a,

$$\rho=4\pi a\,R \qquad (2)$$

and in case b is small in comparison with a,

$$\rho=2\pi a\,R \qquad (3)$$

If the holes are not in a straight line, or are not of a uniform depth or spacing, the resistivity is easily calculated when the depth of each of the holes and the distances of each from each of the other three are known.

2. DERIVATION OF EQUATIONS

To derive equation (1) and the more general relation we may proceed as follows:

Referring to Fig. 2, which is intended to represent a part of an infinite conductor of uniform resistivity, let a unit current enter at the point designated 1. This current flows radially away from 1 and at a distance r its density is $1/4\pi r^2$. This follows from the fact that at any distance r from 1 the current density is uniform over the surface of a sphere whose center is at 1 and whose area is $4\pi r^2$. Since the potential gradient is the current-density times the resistivity,

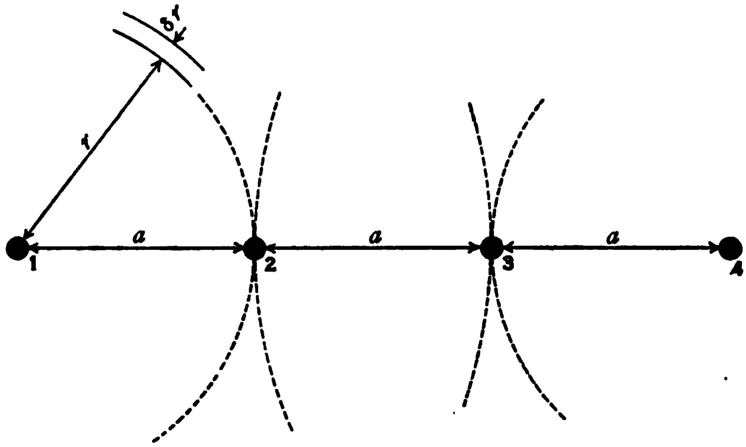


FIG. 2.—*Diagram used in showing the relation between the resistivity, resistance, and distances between terminals in an infinite conductor*

$$-\frac{\delta e}{\delta r} = \frac{\rho}{4\pi r^2} \tag{4}$$

where e is the potential at the distance r from 1.

To get the drop in potential, $e' - e''$ between two points distant r' and r'' from 1 we must integrate the potential gradient from $r = r'$ to $r = r''$ or

$$e' - e'' = \frac{\rho}{4\pi}\int_{r'}^{r''} r^{-2}\delta r = \frac{\rho}{4\pi}\left[\frac{1}{r'} - \frac{1}{r''}\right] \tag{5}$$

If e_x is the drop in potential between the points 2 and 3, distant a and $2a$ from 1, caused by unit current flowing radially from 1, equation (5) gives

$$e_x = \frac{\rho}{8\pi a} \tag{6}$$

Also if e_y represents the drop in potential between 2 and 3 caused by unit current flowing radially toward 4

$$e_y = \frac{\rho}{8\pi a} \tag{7}$$

If unit current enters the conductor at 1 and leaves at 4, the current density at any point[8] is the vector sum of the current density resulting from unit current flowing radially from 1 and unit current flowing radially toward 4. Likewise, the potential difference between any two points is that which results from unit current flowing radially from 1 plus that which results from unit current flowing radially toward 4.

Therefore the difference in potential between 2 and 3 caused by unit current entering the conductor at 1 and leaving at 4 is

$$e_x + e_y = \frac{\rho}{4\pi a} \tag{8}$$

Since the difference in potential for unit current, using 1 and 4 as current terminals and 2 and 3 as potential terminals, is the resistance R,

$$R = \frac{\rho}{4\pi a} \tag{9}$$

R is the resistance of the earth between the equipotential surfaces on which the potential electrodes are placed. It is equal to the resistance of a cylinder of the earth of radius $2a$ and length a.

However, in the measurement of earth resistivity, unless the distance between the electrodes is small in comparison with their distance below the surface, we can not assume that we have an infinite conductor, so equation (9) does not apply.

Referring to Fig. 3, which again represents a part of an infinite conductor, if we let e_1 be the drop in potential between the points

---

[8] The current distribution and equipotential surfaces are the same as the "lines of force" and equipotential surfaces in the field about two spheres with equal opposite charges.

2 and 3, caused by unit current entering the conductor at 1, from equation (5) we have

$$e_1 = \frac{\rho}{4\pi}\left[\frac{1}{r_{12}} - \frac{1}{r_{13}}\right] \tag{10}$$

Likewise, if we designate the drop in potential between 2 and 3, caused by unit current leaving at 4, by e_4; that caused by unit current entering at 5, by e_5; and that caused by unit current leaving at 6, by e_6; we have

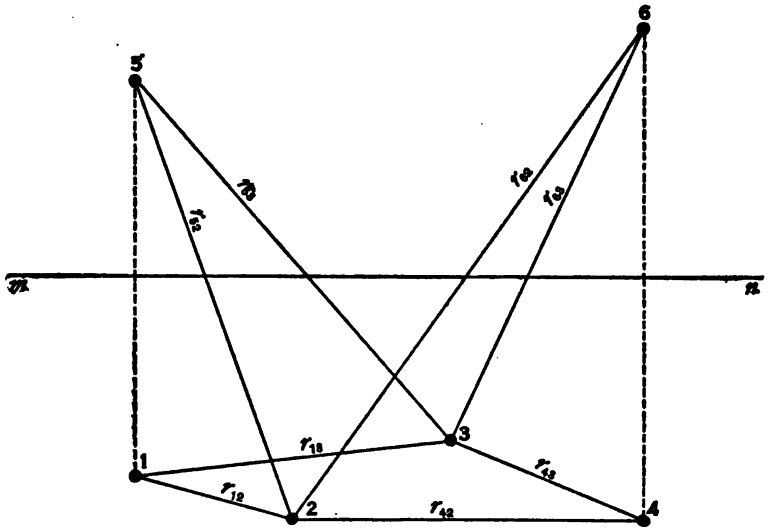


Fig. 3.—*Diagram used in showing relation between the resistivity, resistance, and distances between terminals or electrodes in a semiinfinite conductor*

$$\begin{aligned} e_4 &= \frac{\rho}{4\pi}\left[\frac{1}{r_{43}} - \frac{1}{r_{42}}\right] \\ e_5 &= \frac{\rho}{4\pi}\left[\frac{1}{r_{52}} - \frac{1}{r_{53}}\right] \\ e_6 &= \frac{\rho}{4\pi}\left[\frac{1}{r_{63}} - \frac{1}{r_{62}}\right] \end{aligned} \tag{11}$$

Now, if a current I enters at 1 and leaves at 4 and at the same time an equal current enters at 5 and leaves at 6, the drop in potential, E, between 2 and 3 is $I\ (e_1+e_4+e_5+e_6)$ or

$$E = \frac{I\rho}{4\pi}\left[\frac{1}{r_{12}} - \frac{1}{r_{13}} + \frac{1}{r_{43}} - \frac{1}{r_{42}} + \frac{1}{r_{52}} - \frac{1}{r_{53}} + \frac{1}{r_{63}} - \frac{1}{r_{62}}\right] \tag{12}$$

In the derivation of this equation it has been assumed that 1, 2, 3, 4, 5, and 6 represent points. However, the equation can be used without appreciable error where 1, 2, 3, 4, 5, and 6 represent metallic electrodes or terminals in a conductor of relatively high resistivity, providing the dimensions of the electrodes are small in comparison with the distances between them.

If 5 and 6 are so located that we can choose a plane (represented by the line *m n* of Fig. 3) through the conductor, such that the lines connecting 1 and 5 and 4 and 6 are normal to and bisected by it, it will be evident, on account of the symmetrical arrangement, that no current passes through this plane. Therefore we may remove that part of the conductor on one side of the plane without disturbing conditions on the other side. That is, the equation applies to a semi-infinite conductor having four terminals, providing we consider that the current terminals have images and we take into consideration the distances of the potential terminals both from the current terminals and their images. This, however, does not require that the potential terminals be in the same plane as the current terminals and their images, as shown in the Fig. 3.

Since the drop in potential E between 2 and 3 divided by the current I entering at 1 and leaving at 4 is the resistance R, it follows from equation (12) that

$$R=\frac{\rho}{4\pi}\left[\frac{1}{r_{12}}-\frac{1}{r_{13}}+\frac{1}{r_{43}}-\frac{1}{r_{42}}+\frac{1}{r_{52}}-\frac{1}{r_{53}}+\frac{1}{r_{63}}-\frac{1}{r_{62}}\right] \tag{13}$$

It will, therefore, be evident that the equation gives the relation between the resistivity, resistance, depth, and distance between small electrodes in the earth, as shown in Fig. 1, or in the more general case where the electrodes are not in a straight line.

If the electrodes are all at a uniform depth b and at a uniform distance apart a in a straight line, then $r_{12}=a$, $r_{13}=2a$, $r_{43}=a$, $r_{42}=2a$, $r_{52}=\sqrt{4b^2+a^2}$, $r_{53}=\sqrt{4b^2+4a^2}$, $r_{63}=\sqrt{4b^2+a^2}$, and $r_{62}=\sqrt{4b^2+4a^2}$. Therefore,

$$R=\frac{\rho}{4\pi}\left[\frac{2}{a}-\frac{1}{a}+\frac{2}{\sqrt{a^2+4b^2}}-\frac{2}{\sqrt{4a^2+4b^2}}\right] \tag{14}$$

which, when solved for ρ, gives equation (1).

In the derivation of these equations we have assumed a uniform resistivity. If the resistivity is not uniform, a solution (except in special cases) is not possible. However, the measured resistance, and consequently the value found for the resistivity, using the equations, depends mainly on the resistivity of the portion of earth between the inner electrodes having a cross section equal to the square of half the distance between the outer electrodes.

3. MEASUREMENT OF THE RESISTANCE

Since the electrodes are small, the resistance between the current electrodes and earth, or between any of the electrodes and earth, is large in comparison with the resistance R of the four-terminal conductor, and usually not very constant. Also, since earth acts as an electrolyte there is, in general, an electromotive force in any circuit containing two of the electrodes and polarization at any electrode through which appreciable current passes, even though the current may be alternating. For these reasons none of the usual methods for measuring the resistance of a four-terminal conductor, such as the Thomson bridge and Matthiessen and Hockin bridge methods, seem to meet the conditions very well.

Fortunately there is, in general, no need to measure the resistance to a high accuracy. There is, therefore, no reason why we may not use an ammeter for measuring the current and a voltmeter for measuring the resulting difference in potential between the potential terminals, providing the voltmeter is so used that its readings are not affected by the high resistance between each of these terminals and earth.

The potentiometer arrangement shown in Fig. 4 (using alternating current to obviate the more serious difficulties which might arise on account of polarization with direct current) seems to meet conditions fairly well. The current terminals or electrodes are connected to a source of alternating voltage of suitable value (50 to 150 volts for electrodes about 3 or 4 cm in diameter and 30 to 50 cm apart). Across the line is connected a transformer stepping down, usually to one-tenth or one-twentieth of the voltage across the current terminals. The low-voltage side of the transformer is connected to the ends of a slide wire, one end

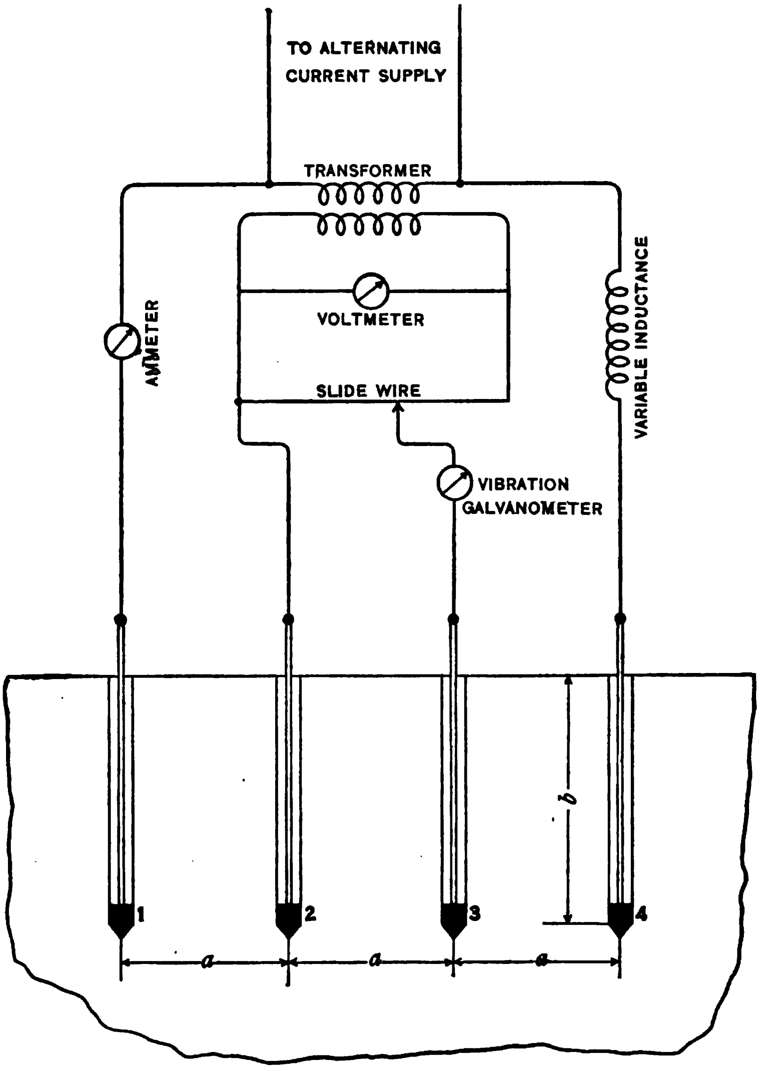


FIG. 4.—*Diagram showing arrangement for measuring four-terminal earth resistance by potentiometer-voltmeter-ammeter method*

of which is also connected to one of the potential terminals. The other potential terminal is connected through a vibration galvanometer[4] (tuned to the frequency of the test current) to the adjustable contact on the slide wire. An ammeter is connected into a lead to one of the current terminals and a voltmeter across the ends of the slide wire, or, if the ratio of the transformer is known when connected to the terminals of the slide wire, the voltmeter may be connected across the line. If the resistance per division of the slide wire is known, we may use an ammeter in series with it instead of the voltmeter.

On account of phase displacement resulting from polarization at the current electrodes, a variable inductance (or inductance with variable resistance in parallel) is connected into one of the leads so that the test current may be brought in phase with the voltage of the low side of the transformer. This is one of the conditions necessary for a zero current through the galvanometer. The other condition is that the magnitude of the ri drop in that part of the slide wire between the adjustable contact and the end connected to one of the potential terminals be equal to the voltage drop across the potential terminals.

If, then, adjustments[5] are made so that no current flows through the galvanometer, and the position of the sliding contact, the value of the test current, and the voltage across (or current in) the slide wire are read, we have data from which the resistance R is readily calculated.

From the measured resistance and the depth and distance between electrodes the effective resistivity may be calculated as explained above. The result obtained depends mainly upon the resistivity near and between the inner or potential electrodes, and very little upon the resistivity at distances from them equal to or more than the distance between the outer or current electrodes, providing the four electrodes are approximately uniformly spaced.

---

[4] If alternating current of from 300 to 1000 cycles is available, a telephone receiver could be used to advantage in place of the vibration galvanometer.

[5] As the accuracy required is not high it is not necessary to take special precautions in regard to induced currents, distributed capacity, etc.

4. USE OF THE METHOD

The method has been tried out by the author using approximately the arrangement shown in Fig. 4. It has also been used by McCollum and Logan in their investigation of earth resistivity, the results of which, it is expected, will be published soon in one of the Bureau's Technological Papers.

In a particular measurement the electrodes were placed at a depth of approximately 125 cm and so spaced that (referring to Fig. 3) $r_{12}=25$, $r_{13}=58$, $r_{43}=31$, and $r_{42}=64$ cm. This gives 251 cm for r_{52} and r_{63}, and 255 cm for r_{53} and r_{62}. These values substituted in equation (14) give

$$R=\frac{\rho}{4\pi}\ .0397 \text{ or } \rho=316\ R$$

The measured value of R, the mean of several ratios of potential difference to current, was found to be 25.3, which give 8000 for ρ, the resistivity expressed as the mean resistance in ohms of a centimeter cube.

So far the method has been used only for determining resistivities in a region a few meters or less in radius and very close to the surface. To measure the effective resistance of a much larger portion of earth, extending to a considerable depth, the electrodes would be placed much farther apart. By using large electrodes a considerable distance apart, a large test current and a very sensitive detector (either of the vibration galvanometer or the separately excited dynamometer type) it should be possible to get some idea of the effective resistivity from the surface to a considerable depth, even though the electrodes are placed practically on the surface. The result obtained would depend to a considerable extent upon the resistivity of a surface layer should it be low, as is usually the case. The effect of the surface layer could be estimated roughly from the results obtained from measurements with the electrodes closer together, and thus some idea obtained as to the resistivity at different depths. Such a measurement might be of assistance in locating deposits of ore of high conductivity.

WASHINGTON, JULY 15, 1915.

A NEW RELATION DERIVED FROM PLANCK'S LAW

By Paul D. Foote, Assistant Physicist

Of the many possible relations which may be derived from the radiation law of Planck only a few have received any attention. The most important of these are the two displacement laws of Wien; $\lambda_{max}\theta$ = constant and $J_{max}\theta^{-5}$ = constant. Salpeter[1] has defined a function of λ and θ as follows:

$$\eta(\lambda, \theta) = \frac{J_\lambda}{\int_0^\infty J_\lambda d\lambda}$$

where J_λ is the intensity of radiation at absolute temperature θ and wave length λ as determined by Planck's law. The functions $\frac{\partial\eta}{\partial\lambda} = o$ and $\frac{\partial\eta}{\partial\theta} = o$ have roots, the former expression giving $\lambda_m\theta$ = constant = 2910 micron degrees for $C_2 = 14\,450$ micron degrees, i. e., identical with Wien's displacement law, and the latter expression $\lambda\theta_{max} = 3686$ micron degrees for the same value of C_2.

It is evident that any number of displacement laws may be derived from the Planck equation.[2] Thus, $\lambda'\theta$ = constant where λ' may refer to any kind of corresponding points on the energy curves at various temperatures, for example, the point of inflection on each side of the maximum ordinate, etc. The present paper concerns the relation that the product of the absolute temperature and the λ-component of the center of gravity of the spectral energy curve is a constant. The utility of this relation is also briefly considered.

Let

$$(1) \qquad J_\lambda = c_1\lambda^{-5}\left(e^{\frac{C_2}{\lambda\theta}} - 1\right)^{-1} = \text{Planck's Law}$$

λ_c = the λ-component of the center of gravity of the curve J_λ versus λ.

---

[1] Salpeter, Phys. Z., 15, pp. 764–765; 1914.

[2] See Buckingham and Dellinger, Bureau of Standards Scientific Paper No. 162, p. 405.

Then by the definition of center of gravity

$$(2) \qquad \lambda_c = \frac{\int_0^\infty J_\lambda \lambda d\lambda}{\int_0^\infty J_\lambda d\lambda}$$

Put $x = \frac{C_2}{\lambda\theta}$. Equation (2) becomes:

$$(3) \qquad \lambda_c = \frac{C_2}{\theta} \frac{\int_0^\infty x^2 (e^x - 1)^{-1} dx}{\int_0^\infty x^3 (e^x - 1)^{-1} dx}$$

Expanding $(e^x - 1)^{-1}$ by division each term in both numerator and denominator takes the gamma function form, and by the relation:

$$\int_0^\infty e^{-kx} x^{n-1} = \frac{\Gamma(n)}{k^n}$$

equation (3) may be expressed as follows:

$$(4) \qquad \lambda_c = \frac{C_2}{\theta} \frac{\left(1 + \frac{1}{2^3} + \frac{1}{3^3} + \cdot \cdot \cdot\right)}{\left(1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdot \cdot \cdot\right)} \cdot \frac{\Gamma(3)}{\Gamma(4)}$$

whence

$$(5) \qquad \lambda_c = 0.37021 \frac{C_2}{\theta}$$

Equation (5) is the new relation derived from Planck's law. Thus, for $C_2 = 14450$, $\lambda_c\theta = 5350$ micron deg.

The displacement law for the center of gravity holds, of course, for the entire family of curves (1) λ versus J; (2) λ versus J/λ; (3) λ versus J/λ^2, etc. This is illustrated as follows:

	Center of gravity λ_c of curve from 0 to ∞	Displacement law
(6)	J vs. λ	$\lambda_c^{(0)} = .37021 C_2/\theta$
(7)	J/λ vs. λ	$\lambda_c^{(1)} = .26094 C_2/\theta$
(8)	J/λ^2 vs. λ	$\lambda_c^{(2)} = .20385 C_2/\theta$
(9)	J/λ^3 vs. λ	$\lambda_c^{(3)} = .16815 C_2/\theta$
(10)	J/λ^n vs. λ	$\lambda_c^{(n)} = \frac{\Gamma(n+3)\left[1 + \frac{1}{2^{n+3}} + \frac{1}{3^{n+3}} + \cdot \cdot \cdot\right]}{\Gamma(n+4)\left[1 + \frac{1}{2^{n+4}} + \frac{1}{3^{n+4}} + \cdot \cdot \cdot\right]} \cdot \frac{C_2}{\theta}$

Utility of the Above Relations.—The obvious use to which any of the above equations might be put is the determination of the important constant C_2 from experimental observation on J and λ. The observations in general cover a range from $\lambda = .5\mu$ to $\lambda = 6$ to 20μ while the curves extend to infinity. The center of gravity of a finite portion of the curve (equation (6)) say from $\lambda = 0$ to $\lambda = \lambda'$ can be determined with a mechanical integrator such as the Amsler integrator No. 1. But the center of gravity λ'_c of this portion of the curve will not in general be near enough to the center of gravity λ_c of the complete energy curve, to permit its use in equations (5) or (6) for the determination of C_2. The following relation, however, holds for the center of gravity of a finite portion of the curve from $\lambda = 0$ to $\lambda = \lambda'$ where $p = \frac{C_2}{\lambda'\theta}$

$$(11) \qquad \lambda'_o = \frac{C_2}{\theta} \frac{\int_p^\infty x^2\ (e^x - 1)^{-1} dx}{\int_p^\infty x^3\ (e^x - 1)^{-1} dx} = \frac{C_2}{\theta} f\ (p)$$

Equation (11) is readily integrable and a curve of $f\ (p)$ versus p was constructed. By use of this plot the expression

$$\lambda'_c = \frac{C_2}{\theta} f\left(\frac{C_2}{\lambda'\theta}\right)$$

can be solved for C_2 by successive approximation. The solution can be computed in less than five minutes. The actual value of this method of obtaining C_2 depends upon the accuracy with which λ'_c can be determined. Consider a curve extending from 0 to 11μ for a temperature of 1800° abs. The center of gravity of this portion of the curve will lie around 2.8μ. The reading of the integrator is good to about .2 per cent of the total range of wave lengths, 11μ, i. e., to $\pm .02\mu$. Thus λ'_c can be determined to about ± 1 per cent and hence, C_2 may be computed to this accuracy, neglecting any consideration of errors in θ, etc., since we are considering only mathematical precision. The method accordingly, although probably accurate enough for most radiometric data, and possessing the advantage of being very simple and rapid, does not afford as satisfactory a means of computing C_2 as the laborious Paschen method or similar methods based on the determination of λ_{max}.

The present brief note was written, not with the primary object of suggesting a new method for the determination of the constant C_2, but for the purpose of calling attention to a new and interesting displacement law following directly from the Planck spectral distribution equation.

WASHINGTON, August 17, 1915.

PUBLICATIONS ISSUED BY THE BUREAU OF STANDARDS

SCIENTIFIC PAPERS

1. Recomparison of the United States Prototype Meter . . . . *L. A. Fischer*
2. A Study of the Silver Voltameter . . . . *K. E. Guthe*
3. The So-called International Electrical Units . . . . . . *Frank A. Wolff*
4. The Spectra of Mixed Gases. . . . . *P. G. Nutting*
5. On Secondary Spectra and the Conditions under which They May Be Produced . . . . *P. G. Nutting*
6. Some New Rectifying Effects in Conducting Gases . . . . . *P. G. Nutting*
7. On Fibers Resembling Quartz in Their Elastic Properties . . . *K. E. Guthe*
8. On the Temperature of the Arc . . . . *C. W. Waidner and G. K. Burgess*
9. The Absolute Measurement of Inductance . . *E. B. Rosa and F. W. Grover*
10. The Absolute Measurement of Capacity . . . *E. B. Rosa and F. W. Grover*
11. Optical Pyrometry . . . . *C. W. Waidner and G. K. Burgess*
12. On the Theory of the Matthews and the Russell-Leonard Photometers for the Measurement of Mean Spherical and Mean Hemispherical Intensities . . . . *E. P. Hyde*
13. The Testing of Clinical Thermometers . . . *C. W. Waidner and L. A. Fischer*
14. Measurement of Inductance by Anderson's Method, Using Alternating Currents and a Vibration Galvanometer . *E. B. Rosa and F. W. Grover*
15. Use of Serpentine in Standards of Inductance . . *E. B. Rosa and F. W. Grover*
16. The Silver Coulometer . . . . *K. E. Guthe*
17. History of Standard Weights and Measures of the United States . . *L. A. Fischer*
18. Wattmeter Methods of Measuring Power Expended upon Condensers and Circuits of Low Power Factor . . . . *E. B. Rosa*
19. The Relative Intensities of Metal and Gas Spectra from Electrically Conducting Gases . . . . *P. G. Nutting*
20. The Use of White Walls in a Photometric Laboratory . . . . *E. P. Hyde*
21. Influence of Wave Form on the Rate of Integrating Induction Wattmeters . . . . *E. B. Rosa, M. G. Lloyd, and C. E. Reid*
22. Detector for Small Alternating Currents and Electrical Waves . . *L. W. Austin*
23. The Positive Charges Carried by the Canal Rays . . . . . *L. W. Austin*
24. Radiation from Platinum at High Temperatures. . . . . . *G. K. Burgess*
25. A Five-Thousand-Volt Generator Set . . . . *P. G. Nutting*
26. Talbot's Law as Applied to the Rotating Sectored Disk . . . *E. P. Hyde*
27. A New Determination of the Electromotive Force of Weston and Clark Standard Cells by an Absolute Electrodynamometer . . *K. E. Guthe*
28. The Gray Absolute Electrodynamometer . . . . *Edward B. Rosa*
29. Construction and Calculation of Absolute Standards of Inductance . *J. G. Coffin*
30. An Efficiency Meter for Electric Incandescent Lamps . . . . *E. P. Hyde and H. B. Brooks*
31. Calculation of the Self-Inductance of Single-Layer Coils . . . *Edward B. Rosa*
32. Heat Treatment of High-Temperature Mercurial Thermometers . . . . *Hobert C. Dickinson*
33. A New Potentiometer for the Measurement of Electromotive Force and Current . . . . *H. B. Brooks*
34. Spectrum Lines as Light Sources in Polariscopic Measurements . *Frederick Bates*
35. Polarimetric Sensibility and Accuracy . . . . *P. G. Nutting*
36. On the Platinum-Point Electrolytic Detector for Electrical Waves . *L. W. Austin*
37. The Influence of Frequency upon the Self-Inductance of Coils . . *J. G. Coffin*
38. Experiments on the Heusler Magnetic Alloys . . *K. E. Guthe and L. W. Austin*
39. A Pocket Spectrophotometer . . . . *P. G. Nutting*

40. Preliminary Measurements on Temperature and Selective Radiation of Incandescent Lamps . . . . . . *C. W. Waidner and G. K. Burgess*
41. Revision of the Formulæ of Weinstein and Stefan for the Mutual Inductance of Coaxial Coils . . . . *Edward B. Rosa*
42. The Mutual Inductance of Two Circular Coaxial Coils of Rectangular Section . . . . *Edward B. Rosa and L. Cohen*
43. On the Determination of the Mean Horizontal Intensity of Incandescent Lamps by the Rotating Lamp Method . . . *E. P. Hyde and F. E. Cady*
44. Purity and Intensity of Monochromatic Light Sources . . . . *P. G. Nutting*
45. Radiometric Investigations of Infra-Red Absorption and Reflection Spectra . . . . *W. W. Coblentz*
46. A Vacuum Radiomicrometer . . . . *W. W. Coblentz*
47. On the Geometrical Mean Distances of Rectangular Areas and the Calculation of Self-Inductance . . . . *E. B. Rosa*
48. The Compensated Two-Circuit Electrodynamometer . . . . *E. B. Rosa*
49. Complete Form of Fechner's Law . . . . *P. G. Nutting*
50. A Comparison of the Unit of Luminous Intensity of the United States with Those of Germany, England, and France . . . . . . *E. P. Hyde*
51. Geometrical Theory of Radiating Surfaces, with Discussion of Light Tubes . . . . *E. P. Hyde*
52. The Influence of Basic Lead Acetate on the Optical Rotation of Sucrose in Water Solution . . . . *F. J. Bates and J. C. Blake*
53. On the Calorimetric Determination of Iron, with Special Reference to Chemical Reagents . . . . *H. N. Stokes and J. R. Cain*
54. On Sulphocyanic Acid . . . . *H. N. Stokes and J. R. Cain*
55. Radiation from and Melting Points of Palladium and Platinum . . . . *C. W. Waidner and G. K. Burgess*
56. The Mutual Inductance of a Circle and a Coaxial Single-Layer Coil—The Lorenz Apparatus and the Ayrton-Jones Absolute Electrodynamometer . . . . *E. B. Rosa*
57. On the Establishment of the Thermodynamic Scale of Temperature by Means of the Constant-Pressure Thermometer . . . *Edgar Buckingham*
58. An Exact Formula for the Mutual Inductance of Coaxial Solenoids . . . . *Louis Cohen*
59. The Mutual Inductance of Coaxial Solenoids . . . *E. B. Rosa and L. Cohen*
60. The Production of High-Frequency Oscillations from the Electric Arc . . . . *L. W. Austin*
61. An Explanation of the Short Life of Frosted Lamps . . . . *E. P. Hyde*
62. Melting Points of the Iron Group Elements by a New Radiation Method . . . . *G. K. Burgess*
63. On the Determination of the Mean Horizontal Intensity of Incandescent Lamps . . . . *Edward P. Hyde and F. E. Cady*
64. Simultaneous Measurement of the Capacity and Power Factor of Condensers . . . . *F. W. Grover*
65. A New Determination of the Ratio of the Electromagnetic to the Electrostatic Unit of Electricity . . . . . *E. B. Rosa and N. E. Dorsey*
66. A Comparison of the Various Methods of Determining the Ratio of the Electromagnetic to the Electrostatic Unit of Electricity . . . . *E. B. Rosa and N. E. Dorsey*
67. Preliminary Specifications for Clark and Weston Standard Cells . . . . *F. A. Wolff and C. E. Waters*
68. Calorimetric Resistance Thermometers and the Transition Temperature of Sodium Sulphate . . . . . *H. C. Dickinson and E. F. Mueller*
69. On the Standard Scale of Temperature in the Interval 0° to 100° C. . . . . *C. W. Waidner and H. C. Dickinson*
70. Clark and Weston Standard Cells . . . . . *F. A. Wolff and C. E. Waters*
71. The Electrode Equilibrium of the Standard Cell. *F. A. Wolff and C. E. Waters*
72. A Comparative Study of Plain and Frosted Lamps . *E. P. Hyde and F. E. Cady*
73. The Variation of Resistances with Atmospheric Humidity . . . . *E. B. Rosa and H. D. Babcock*
74. On the Self-Inductance of a Toroidal Coil of Rectangular Section . . . . *Edward B. Rosa*

75. On the Self-Inductance of Circles . . . . *Edward B. Rosa and Louis Cohen*
76. The Influence of Frequency on the Resistance and Inductance of Solenoidal Coils . . . . *Louis Cohen*
77. The Atomic Weight of Hydrogen . . . . *W. A. Noyes*
78. The Best Method of Demagnetizing Iron in Magnetic Testing . *C. W. Burrows*
79. A Deflection Potentiometer for Voltmeter Testing . . . . . *H. B. Brooks*
80. The Self and Mutual Inductance of Linear Conductors . . . *Edward B. Rosa*
81. The Atomic Weight of Chlorine . . . . *W. A. Noyes and H. C. P. Weber*
82. The Preparation of Chloroplatinic Acid by Electrolysis of Platinum Black . . . . *H. C. P. Weber*
83. The Self-Inductance of a Coil of any Length and any Number of Layers of Wire . . . . *Edward B. Rosa*
84. Self-Inductance of a Solenoid of any Number of Layers . . . *Louis Cohen*
85. Instruments and Methods Used in Radiometry . . . . . *W. W. Coblentz*
86. A Quartz Compensating Polariscope with Adjustable Sensibility . . *F. J. Bates*
87. An Apparatus for Determining the Form of a Wave of Magnetic Flux . . . . *M. G. Lloyd and J. V. S. Fisher*
88. Effect of Wave Form upon the Iron Losses in Transformers . *Morton G. Lloyd*
89. The Luminous Properties of Electrically Conducting Helium Gas . . . . *P. G. Nutting*
90. Function of a Periodic Variable Given by the Steady Reading of an Instrument, with a Note on the Use of the Capillary Electrometer with Alternating Voltages . . . . *Morton G. Lloyd*
91. Selective Radiation from the Nernst Glower . . . . . . *W. W. Coblentz*
92. The Testing of Glass Volumetric Apparatus . . *N. S. Osborne and B. H. Veazey*
93. Formulæ and Tables for the Calculation of Mutual and Self-Inductance . . . . *Edward B. Rosa and Louis Cohen*
94. Some Contact Rectifiers of Electric Currents . . . . *L. W. Austin*
95. A Method for Producing Feebly Damped High-Frequency Electrical Oscillations for Laboratory Measurements . . . . *L. W. Austin*
96. On the Advantages of a High Spark Frequency in Radio-Telegraphy . . . . *L. W. Austin*
97. Selective Radiation from Various Solids . . . . *W. W. Coblentz*
98. Remarks on the Quartz Compensating Polariscope with Adjustable Sensibility . . . . *Frederick Bates*
99. Methods of Obtaining Cooling Curves . . . . *George K. Burgess*
100. Note on the Approximate Values of Bessel's Functions for Large Arguments . . . . *Louis Cohen*
101. The Influence of Terminal Apparatus on Telephonic Transmission . *Louis Cohen*
102. The Principles Involved in the Selection and Definition of the Fundamental Electrical Units to be Proposed for International Adoption . . . . *F. A. Wolff*
103. The Luminous Equivalent of Radiation . . . . *P. G. Nutting*
104. The Temperature Formula of the Weston Standard Cell . . . *F. A. Wolff*
105. Radiation, Constants of Metals . . . . *W. W. Coblentz*
106. Dependence of Magnetic Hysteresis upon Wave Form . . . *Morton G. Lloyd*
107. A New Form of Standard Resistance . . . . *Edward B. Rosa*
108. Errors in Magnetic Testing with Ring Specimens . . . . *Morton G. Lloyd*
109. The Testing of Transformer Steel . . . . *M. G. Lloyd and J. V. S. Fisher*
110. A New Method of Determining the Focal Length of a Converging Lens . . . . *Irwin G. Priest*
111. A New Method for the Absolute Measurement of Resistance . *Edward B. Rosa*
112. The Theory of Coupled Circuits . . . . *Louis Cohen*
113. A Volt Scale for a Watts-per-candle Meter . . . . . . *Herbert E. Ives*
114. The Coefficient of Reflection of Electrical Waves at a Transition Point . . . . *Louis Cohen*
115. A Tungsten Comparison Lamp in the Photometry of Carbon Lamps . . . . *Herbert E. Ives and L. R. Woodhull*
116. The Determination of the Ratio of Transformation and of the Phase Relations in Transformers . . . . *E. B. Rosa and M. G. Lloyd*
117. The Determination of the Magnetic Induction in Straight Bars . . . . *Charles W. Burrows*

118. A Method for Constructing the Natural Scale of Pure Color . . *P. G. Nutting*
119. An Approximate Method for the Analysis of EMF. Waves . . *P. G. Agnew*
120. The Thermoelectric Properties of Tantalum and Tungsten . *W. W. Coblentz*
121. The Estimation of the Temperature of Copper by Means of Optical Pyrometers . . . . . . *George K. Burgess, assisted by J. F. Crowe*
122. The Resolving Power of Objectives . . . . *P. G. Nutting*
123. The Theory of the Hampson Liquefier . . . . . . *Edgar Buckingham*
124. Platinum Resistance Thermometry at High Temperatures . . . . *C. W. Waidner and G. K. Burgess*
125. The Daylight Efficiency of Artificial Illuminants . . . . . *H. E. Ives*
126. Coupled Currents in which the Secondary has Distributed Inductance and Capacity . . . . *Louis Cohen*
127. Effect of Phase of Harmonics upon Acoustic Quality . . . . *M. G. Lloyd and P. G. Agnew*
128. White Light from the Mercury Arc and its Complementary . *Herbert E. Ives*
129. The Regulation of Potential Transformers and the Magnetizing Current . . . . *M. G. Lloyd and P. G. Agnew*
130. The Determination of the Constants of Instrument Transformers . . . . *P. G. Agnew and T. T. Fitch*
131. Selective Radiation from Various Solids, II . . . . *W. W. Coblentz*
132. Luminous Efficiency of the Firefly . . *Herbert E. Ives and W. W. Coblentz*
133. Luminosity and Temperature . . . . *P. G. Nutting*
134. A Theoretical and Experimental Study of the Vibration Galvanometer . . . . *F. Wenner*
135. Specific Heat of Some Calcium Chloride Solutions between −35° C. and +20° C . . . . *H. C. Dickinson, E. F. Mueller, and E. B. George*
136. On the Definition of the Ideal Gas . . . . *Edgar Buckingham*
137. Mica Condensers as Standards of Capacity . . . . . . *Harvey L. Curtis*
138. The Mutual Induction of Two Parallel Coaxial Circles in Terms of Hypergeometrical Series . . . . *Frederick W. Grover*
139. A New Method for the Absolute Measurement of Electric Quantity . . . . *Burton McCollum*
140. The Comparative Sensitiveness of Some Common Detectors of Electrical Oscillations . . . . *Louis W. Austin*
141. Photometric Units and Nomenclature . . . . *E. B. Rosa*
142. A Modified Method for the Determination of Relative Wave Lengths, Especially Adapted to the Establishment of Secondary Standards . . . . *Irwin G. Priest*
143. Note on the Temperature Scale between 100 and 500° C. . . . . *C. W. Waidner and G. K. Burgess*
144. A New Form of Direct-Reading Candlepower Scale and Recording Device for Precision Photometers . . . . . . *George W. Middlekauff*
145. A Device for Measuring the Torque of Electrical Instruments . . . . *F. G. Agnew*
146. The Intensities of Some Hydrogen, Argon, and Helium Lines in Relation to Current and Pressure . . . *P. G. Nutting and Orin Tugman*
147. The Temperature Coefficient of Resistance of Copper . . . *J. H. Dellinger*
148. The Electrical Conductivity of Commercial Copper . . . . *F. A. Wolff and J. H. Dellinger*
149. On the Constancy of the Sulphur Boiling Point . . . . *C. W. Waidner and G. K. Burgess*
150. Note on Oscillatory Interference Bands and Some of their Practical Applications . . . . *G. O. Squier and A. C. Crehore*
151. The Effect of Preliminary Heat Treatment upon the Drying of Clays . . . . *A. V. Bleininger*
152. The Reflecting Power of Various Metals . . . . *W. W. Coblentz*
153. The Action of Sunlight and Air upon Some Lubricating Oils . *C. E. Waters*
154. The Visibility of Radiation. A Recalculation of Konig's Data . *P. G. Nutting*
155. A Photometric Attachment for Spectroscopes . . . . . . *P. G. Nutting*
156. Selective Radiation from Various Substances, III . . . . *W. W. Coblentz*
157. The Measurement of Electrical Oscillations in the Receiving Antenna . . . . *L. W. Austin*

158. Some Experiments with Coupled High-Frequency Circuits . . . *L. W. Austin*
159. Some Quantitative Experiments in Long Distance Radiotelegraphy . . . *L. W. Austin*
160. The Behavior of High-Boiling Oils on Heating in the Air . . . *C. E. Waters*
161. The Determination of Vanadium in Vanadium and Chrome-Vanadium Steels . . . *J. R. Cain*
162. On the Computation of the Constant C_2 of Planck's Equation by an Extension of Paschen's Method of Equal Ordinates . . . *Edgar Buckingham and J. H. Dellinger*
163. A Comparison of American Direct Current Switchboard Voltmeters and Ammeters . . . *T. T. Fitch and C. J. Huber*
164. Study of the Current Transformer with Particular Reference to Iron Loss . . . *P. G. Agnew*
165. Thermodynamics of Concentration Cells . . . *Henry S. Carhart*
166. The Capacity and Phase Difference of Paraffined Paper Condensers as Functions of Temperature and Frequency . . . *Frederick W. Grover*
167. The Steam Expansion Line on the Mollier Diagram and a Short Method of Finding the Reheat Factor . . . *Edgar Buckingham*
168. Radiometric Investigation of Water of Crystallization, Light Filters, and Standard Absorption Bands . . . *W. W. Coblentz*
169. Formulas and Tables for the Calculation of Mutual and Self Induction (2d edition, revised and enlarged) . *E. B. Rosa and Frederick W. Grover*
170. The Correction for Emergent Stem of a Mercurial Thermometer . . . *Edgar Buckingham*
171. Thermometric Lag . . . *D. R. Harper*
172. Deflection Potentiometers for Current and Voltage Measurements . . . *H. B. Brooks*
173. Outline of Design of Deflection Potentiometers with Notes on the Design of Moving-Coil Galvanometers . . . *H. B. Brooks*
174. The Determination of Total Sulphur in India Rubber . . . *C. E. Waters and J. B. Tuttle*
175. The Measurement of the Inductances of Resistance Coils . . . *Frederick W. Grover and Harvey L. Curtis*
176. Luminous Properties of Electrically Conducting Helium Gas. II. Reproducibility . . . *P. G. Nutting*
177. Resistance Coils for Alternating Current Work . *H. L. Curtis and F. W. Grover*
178. The Hydrolysis of Sodium Oxalate and Its Influence Upon the Tests for Neutrality . . . *William Blum*
179. Wave-Lengths of Neon . . . *Irwin G. Priest*
180. On the Deduction of Wien's Displacement Law . . . *Edgar Buckingham*
181. The Four-Terminal Conductor and the Thomson Bridge . . . *Frank Wenner*
182. Standardization of Potassium Permanganate Solution by Sodium Oxalate . . . *R. S. McBride*
183. Benzoic Acid as an Acidimetric Standard . . . *G. W. Morey*
184. A Tubular Electrodynamometer for Heavy Currents . . . *P. G. Agnew*
185. Thermometric Lag. . . . *D. R. Harper*
186. Determination of Manganese as Sulphate and by the Sodium Bismuthate Method . . . *William Blum*
187. A New Precision Colorimeter . . . *P. G. Nutting*
188. Instruments and Methods Used in Radiometry, II . . . *W. W. Coblentz*
189. Antenna Resistance . . . *L. W. Austin*
190. Energy Losses in Some Condensers Used in High-Frequency Circuits . . . *L. W. Austin*
191. Selective Radiation from Various Substances, IV . . . *W. W. Coblentz*
192. On a Modified Form of Stability Test for Smokeless Powder and Similar Materials . . . *H. C. P. Weber*
193. Atomic Weight of Bromine . . . *H. C. P. Weber*
194. The Silver Voltameter.—Part I. First Series of Quantitative Experiments . . . *E. B. Rosa and G. W. Vinal*
195. The Silver Voltameter.—Part II. The Chemistry of the Filter Paper Voltameter and the Explanation of Striations . *E. B. Rosa and G. W. Vinal*
196. The Diffuse Reflecting Power of Various Substances . . . *W. W. Coblentz*

197. Density and Thermal Expansion of Ethyl Alcohol and of Its Mixtures with Water . . . *N. S. Osborne, E. C. McKelvy, and H. W. Bearce*
198. A Micropyrometer . . . . *G. K. Burgess*
199. A Simplified Formula for the Change in Order of Interference Due to Changes in Temperature and Pressure of Air . . . . . . *I. G. Priest*
200. New Calorimetric Resistance Thermometers . *H. C. Dickinson and E. F. Mueller*
201. The Silver Voltameter.—Part III. Second Series of Quantitative Experiments and the Preparation and Testing of Silver Nitrate . . . . *E. B. Rosa, G. W. Vinal, and A. S. McDaniel*
202. Note on Cold-Junction Corrections for Thermocouples . . . . *P. D. Foote*
203. The Analysis of Alternating Current Waves by the Method of Fourier, with Special Reference to Methods of Facilitating the Computations . . . . *F. W. Grover*
204. The Constants of Spectral Radiation of a Uniformly Heated Inclosure or So-Called Black Body, Part I . . . . . . *W. W. Coblentz*
205. Melting Points of the Refractory Elements.—I. Elements of Atomic Weight from 48 to 59 . . . . . . *G. K. Burgess and R. G. Waltenberg*
206. High-Frequency Ammeters . . . . *J. H. Dellinger*
207. A Comparative Study of American Direct-Current Watthour Meters . . . . *T. T. Fitch and C. J. Huber*
208. Windage Resistance of Steam Turbine Wheels . . . . . . *E. Buckingham*
209. Latent Heat of Fusion of Ice . *H. C. Dickinson, D. R. Harper 3d, and N. S. Osborne*
210. Observations on Ocean Temperatures in the Vicinity of Icebergs and in other Parts of the Ocean . *C. W. Waidner, H. C. Dickinson, and J. J. Crowe*
211. Accuracy of the Formulas for the Ratio, Regulation, and Phase Angle of Transformers . . . . *P. G. Agnew and F. B. Silsbee*
212. Melting Points of Some Refractory Oxides . . . . *C. W. Kanolt*
213. The Critical Ranges A2 and A3 of Pure Iron . . *G. K. Burgess and J. J. Crowe*
214. Note on the Setting of a Mercury Surface to a Required Height . *M. H. Stillman*
215. Micrometer Microscopes . . . . *A. W. Gray*
216. The Pentane Lamp as a Working Standard . . *E. C. Crittenden and A. H. Taylor*
217. Testing Potential Transformers . . . . *H. B. Brooks*
218. Comparison of the Silver and Iodine Voltameters and the Determination of the Faraday . . . . *G. W. Vinal and S. J. Bates*
219. Production of Temperature Uniformity in an Electric Furnace . *A. W. Gray*
220. The Silver Voltameter.—Part IV. Third Series of Quantitative Experiments and Special Investigations . *E. B. Rosa, G. W. Vinal, and A. S. McDaniel*
221. Influence of Atmospheric Conditions in the Testing of Sugars . . . . *F. J. Bates and F. P. Phelps*
222. Flame Standards in Photometry . . . . . *E. B. Rosa and E. C. Crittenden*
223. The Testing of Potentiometers . . . . . . *Frank Wenner and Ernest Weibel*
224. The Emissivity of Metals and Oxides.—I. Nickel Oxide (NiO) in the Range 600 to 1300° C . . . . *G. K. Burgess and P. D. Foote*
225. Adjustments of the Thomson Bridge in the Measurement of Very Low Resistances . . . . *Frank Wenner and Ernest Weibel*
226. Quantitative Experiments in Radiotelegraphic Transmission . *L. W. Austin*
227. Measurements on Standards of Radiation in Absolute Value . *W. W. Coblentz*
228. An Experimental Study on the Koepsel Permeameter . . *Charles W. Burrows*
229. Various Modifications of Bismuth-Silver Thermopiles Having a Continuous Absorbing Surface . . . . *W. W. Coblentz*
230. Combustion Calorimetry and the Heats of Combustion of Cane Sugar, Benzoic Acid, and Naphthalene . . . . *H. C. Dickinson*
231. Specific Heat of Copper in the Interval 0° to 50° C.; with a Note on Vacuum-Jacketed Calorimeters . . . . *D. R. Harper 3d*
232. Equilibrium in the System: Lead Acetate, Lead Oxide, and Water, at 25° . . . . *Richard F. Jackson*
233. A Watthour Meter Method of Testing Instrument Transformers . *P. G. Agnew*
234. Insulating Properties of Solid Dielectrics . . . . *Harvey L. Curtis*
235. A Direct-Reading Instrument for Measuring the Logarithmic Decrement and Wave Length of Electromagnetic Waves . . *Frederick A. Kolster*
236. Electrical Resistance and Critical Ranges of Pure Iron . . . . *G. K. Burgess and I. N. Kellberg*
237. Absorption, Reflection, and Dispersion Constants of Quartz . *W. W. Coblentz*

238. Characteristic Equations of Tungsten Filament Lamps and Their Application in Heterochromatic Photometry . *G. W. Middlekauff and J. F. Skogland*
239. A Vibration Electrometer . . . . *W. W. Coblentz*
240. Studies on the Silver Voltameter . . . . . *G. A. Hulett and G. W. Vinal*
241. A Wheatstone Bridge for Resistance Thermometry . . . . *C. W. Waidner, H. C. Dickinson, E. F. Mueller, and D. R. Harper 3d*
242. The Emissivity of Metals and Oxides.—II. Measurements with the Micropyrometer . . . . *G. K. Burgess and R. G. Waltenberg*
243. The Emissivity of Metals and Oxides.—III. The Total Emissivity of Platinum and the Relation Between Total Emissivity and Resistivity . . . . *Paul D. Foote*
244. A Comparison of Stellar Radiometers and Radiometric Measurements on 110 Stars . . . . *W. W. Coblentz*
245. Temperature Coefficient of Magnetic Permeability Within the Working Range . . . . *R. L. Sanford*
246. Methods of Measuring the Inductances of Low Resistance Standards . . . . *Frank Wenner, Ernest Weibel, and F. B. Silsbee*
247. An Aneroid Calorimeter . . . . *H. C. Dickinson and N. S. Osborne*
248. Specific Heat and Heat of Fusion of Ice . *H. C. Dickinson and N. S. Osborne*
249. The Emissivity of Metals and Oxides.—IV. Iron Oxide . . . . *George K. Burgess and Paul D. Foote*
250. Characteristics of Radiation Pyrometers . *George K. Burgess and Paul D. Foote*
251. Interference Measurements of Wave Lengths in the Iron Spectrum (2851–3701), with Notes on Comparisons of Lengths of Light Waves by Interference Methods, and Some Wave Lengths in the Spectrum of Neon Gas . . . . *Keivin Burns and W. F. Meggers*
252. Effective Resistance and Inductance of Iron and Bimetallic Wires . . . . *John M. Miller*
253. A Direct-Reading Device for use in Computing Characteristics of Vacuum Tungsten Lamps . . . . *J. F. Skogland*
254. A Study of the Quality of Platinum Ware . *George K. Burgess and P. D. Sale*
255. Calculation of the Maximum Force Between Two Coaxial Circular Currents . . . . *Frederick W. Grover*
256. Construction of Primary Mercurial Resistance Standards . . . . *F. A. Wolff, M. P. Shoemaker, and C. A. Briggs*
257. Note on the Resistance of Radiotelegraphic Antennas . . . . *L. W. Austin*
258. A Method of Measuring Earth Resistivity . . . . *Frank Wenner*
259. A New Relation Derived from Planck's Law . . . . *Paul D. Foote*

CIRCULARS

1. Verification of Standards and Measuring Instruments.
2. Measurements of Length and Area, Including Thermal Expansion.
3. Verification of Standards of Mass.
4. Verification of Standards of Capacity.
5. Testing of Clinical Thermometers.
6. Fees for Electric, Magnetic, and Photometric Testing.
7. Pyrometer Testing and Heat Measurements.
8. Testing of Thermometers.
9. Testing of Glass Volumetric Apparatus.
10. Legal Weights (in pounds) per Bushel of Various Commodities.
11. The Standardization of Bomb Calorimeters.
12. Verification of Polariscopic Apparatus.
13. United States Government Standard Specifications for Incandescent Electric Lamps.
14. Analyzed Irons and Steels—Methods of Analysis.
15. The International Unit of Light.
16. The Testing of Hydrometers.
17. Magnetic Testing.
18. Standard Gauge for Sheet and Plate Iron and Steel.
19. Standard Density and Volumetric Tables.
20. Testing of Electrical Measuring Instruments.
21. Precision Measurements of Resistance and Electromotive Force.

22. Standard Specifications for Transformers, Oil-immersed, Self-cooled, 60-cycle, 2200 Volts.
23. Standardization of Electrical Practice in Mines.
24. Publications of the Bureau of Standards.
25. Standard Analyzed Samples—General Information.
26. Analyzed Iron and Manganese Ores—Methods of Analysis.
27. The Testing and Properties of Optical Instruments.
28. The Determination of the Optical Properties of Materials.
29. Announcement of a Change in the Value of the International Volt.
30. Lime: Its Properties and Uses.
31. Copper Wire Tables.
32. Standards of Gas Service.
33. United States Government Specification for Portland Cement.
34. The Relation of the Horsepower to the Kilowatt.
35. Melting Points of Chemical Elements.
36. The Testing and Properties of Electrical Condensers.
37. Electric Wire and Cable Terminology.
38. The Testing of Mechanical Rubber Goods.
39. Specifications for and Measurement of Standard Sieves.
40. Sodium Oxalate as a Standard in Volumetric Analysis.
41. Testing and Properties of Textile Materials.
42. Metallographic Testing.
43. The Metric Carat.
44. Polarimetry.
45. The Testing of Materials.
46. The Testing of Barometers.
47. Units of Weight and Measure; Definitions and Tables of Equivalents.
48. Standard Methods of Gas Testing.
49. Safety Rules to be Observed in the Operation of Electrical Equipment and Lines.
50. National Standard Hose Couplings and Fittings for Public Fire Service.
51. Measurement of Time and Tests of Timepieces.
52. Regulation of Electrotyping Solutions.
53. The Composition, Properties, and Testing of Printing Inks.
54. Proposed National Electrical Safety Code.
55. Measurements for the Household.
56. Standards for Electric Service.

TECHNOLOGIC PAPERS

1. The Effect of Preliminary Heat Treatment upon the Drying of Clays (53 pp.) . . . . *A. V. Bleininger*
2. The Strength of Reinforced Concrete Beams. Results of Tests of 333 Beams (first series) (200 pp.) . . . . . *R. L. Humphrey and L. H. Losse*
3. Tests of Absorptive and Permeable Properties of Portland Cement Mortars and Concretes, Together with Tests of Damp Proofing and Waterproofing Compounds and Materials (127 pp.) . . . . *Rudolph J. Wig*
4. The Effect of Added Fatty and Other Oils upon the Carbonization of Mineral Lubricating Oils (14 pp.) . . . . *C. E. Waters*
5. The Effect of High-Pressure Steam on the Crushing Strength of Portland Cement Mortar and Concrete (25 pp.) . . . . *R. J. Wig*
6. The Determination of Chromium and Its Separation from Vanadium in Steels (6 pp.) . . . . *J. R. Cain*
7. The Testing of Clay Refractories, With Special Reference to Their Load Carrying Capacity at Furnace Temperatures (78 pp.) . . . . *A. V. Bleininger and G. H. Brown*
8. A Rapid Method for the Determination of Vanadium in Steels, Ores, etc., Based on Its Quantitative Inclusion by the Phosphomolybdate Precipitate (20 pp.) . . . . . . *J. R. Cain and J. C. Hostetter*
9. The Density and Thermal Expansion of Linseed Oil and Turpentine (27 pp.) . . . . *H. W. Bearce*
10. The Melting Points of Fire Bricks (17 pp.) . . . . *C. W. Kanolt*
11. Comparison of Five Methods Used to Measure Hardness (27 pp.) . *Ralph P. Devries*
12. Action of the Salts in Alkali Water and Sea Water on Cements (157 pp.) . . . . *P. H. Bates, A. J. Phillips, and R. J. Wig*

13. The Evaporation Test for Mineral Lubricating and Transformer Oils (13 pp.) . . . . *C. E. Waters*
14. Legal Specifications for Illuminating Gas (31 pp.) . *E. B. Rosa and R. S. McBride*
15. Surface Insulation of Pipes (44 pp.) . . . *Burton McCollum and O. S. Peters*
16. The Manufacture of Lime (130 pp.) . . . . *W. E. Emley*
17. The Function of Time in the Vitrification of Clays (26 pp.) . . . . *G. H. Brown and G. A. Murray*
18. Electrolysis in Concrete (137 pp.) . *E. B. Rosa, Burton McCollum, and O. S. Peters*
19. Physical Testing of Cotton Yarns (31 pp.) . . . . *W. S. Lewis*
20. Determination of Sulphur in Illuminating Gas (46 pp.) . . . . *R. S. McBride and E. R. Weaver*
21. The Dehydration of Clays (23 pp.) . . . *G. H. Brown and E. T. Montgomery*
22. The Effect of Overfiring Upon the Structure of Clays (23 pp.) . . . . *A. V. Bleininger and E. T. Montgomery*
23. The Technical Control of the Colloidal Matter of Clays (118 pp.) . *H. E. Ashley*
24. The Determination of Phosphorus in Steels Containing Vanadium (11 pp.) . . . . *J. R. Cain and F. H. Tucker*
25. Electrolytic Corrosion of Iron in Soils (69 pp.) . *Burton McCollum and K. H. Logan*
26. Earth Resistance and Its Relation to Electrolysis . . . . *Burton McCollum and K. H. Logan*
27. Special Studies in Electrolysis Mitigation.—I. A Preliminary Study of Conditions in Springfield, Ohio, with Recommendations for Mitigation and Control (55 pp.) . . . . *E. B. Rosa and Burton McCollum*
28. Methods of Making Electrolysis Surveys . *Burton McCollum and A. H. Ahlborn*
29. The Variation in Results of Sieving with Standard Cement Sieves (16 pp.) . . . . *R. J. Wig and J. C. Pearson*
30. The Viscosity of Porcelain Bodies (11 pp.) . *A. V. Bleininger and Paul Teetor*
31. Some Leadless Boro-Silicate Glazes Maturing at about 1100° C. (22 pp.) . . . . *E. T. Montgomery*
32. Special Studies in Electrolysis Mitigation.—II. Electrolysis from Electric Railway Currents and Its Prevention—Experimental Test on a System of Insulated Negative Feeders in St. Louis (34 pp.) . . . . *E. B. Rosa, Burton McCollum, and K. H. Logan*
33. Determination of Carbon in Steel and Iron by the Barium Carbonate Titration Method (12 pp.) . . . . *J. R. Cain*
34. Determination of Ammonia in Illuminating Gas (23 pp.) . . . . *J. D. Edwards*
35. Combustion Method for the Direct Determination of Rubber (11 pp.) . *L. G. Wesson*
36. Industrial Gas Calorimetry (150 pp.) . . . *C. W. Waidner and E. F. Mueller*
37. Iodine Number of Linseed and Petroleum Oils (17 pp.) . . . . *W. H. Smith and J. B. Tuttle*
38. Observations on Finishing Temperatures and Properties of Rails (63 pp.) . . *G. K. Burgess, J. J. Crowe, H. S. Rawdon, and R. G. Waltenberg*
39. Analysis of Printing Inks (20 pp.) . . . . . *J. B. Tuttle and W. H. Smith*
40. The Veritas Firing Rings (10 pp.) . . . . *A. V. Bleininger and G. H. Brown*
41. Lead Acetate Test for Hydrogen Sulphide in Gas (46 pp.) . . . . *R. S. McBride and J. D. Edwards*
42. Standardization of No. 200 Cement Sieves (51 pp.) *R. J. Wig and J. C. Pearson*
43. Hydration of Portland Cement (71 pp.) . . . *A. A. Klein and A. J. Phillips*
44. Investigation of the Durability of Cement Drain Tile and Alkali Soils (56 pp.) . . *R. J. Wig and G. M. Williams, in cooperation with S. H. McCrory, E. C. Bebb, and L. R. Ferguson*
45. A Study of Some Recent Methods for the Determination of Total Sulphur in Rubber (16 pp.) . . . . *J. B. Tuttle and A. Isaacs*
46. A Study of the Atterberg Plasticity Method (18 pp.) . . . *Charles S. Kinnison*
47. Value of the High-Pressure Steam Test of Portland Cements (34 pp.) . . . . *R. J. Wig and H. A. Davis*
48. An Air-Analyzer for Determining the Fineness of Portland Cement (74 pp.) . . . . *H. C. Pearson and W. H. Sligh*
49. Emergent Stem Correction for Thermometers in Creosote Oil Distillation Flasks (21 pp.) . . . . *R. M. Wilhelm*
50. Viscosity of Porcelain Bodies High in Feldspar (7 pp.) . . . . *A. V. Bleininger and C. S. Kinnison*
51. Use of Sodium Salts in the Purification of Clays and in the Casting Process (40 pp.) . . . . *A. V. Bleininger*

52. Electrolysis and Its Mitigation (37 pp.) . . *E. B. Rosa and Burton McCollum*
53. An Investigation of Fusible Tin Boiler Plugs (143 pp.) . . . . *George K. Burgess and P. D. Merica*
54. Special Studies in Electrolysis Mitigation.—III. A Report on Conditions in Springfield, Ohio, with Insulated Feeder System Installed . . . . *Burton McCollum and G. H. Ahlborn*
55. Special Studies in Electrolysis Mitigation.—IV. A Preliminary Study of Electrolysis Conditions in Elyria, Ohio, with Recommendations for Mitigation . . . . *Burton McCollum and K. H. Logan*
56. Protection of Life and Property Against Lightning . . . . *O. S. Peters*
57. Difference in Weight Between Raw and Clean Wools . . . . *Walter S. Lewis*
58. Compressive Strength of Portland Cement Mortars and Concretes . . . . *R. J. Wig, G. M. Williams, and E. R. Gates*
59. Standard Test Specimens of Zinc Bronzes, .88 cu, .10 Sn, .02 Zn.—Part I. Preparation of Specifications. Part II. Microstructure . . . . *C. P. Karr and Henry S. Rawdon*
60. Microstructural Changes Accompanying the Annealing of Bronze (Cu .88, Sn .10, Zn .02) . . . . *Henry S. Rawdon*
61. Some Foreign Specifications for Railway Materials . . . . *George K. Burgess and P. D. Merica*

MISCELLANEOUS

International Metric System. (Chart.)
The International Metric System of Weights and Measures. (Pamphlet.)
First Conference on the Weights and Measures of the United States.
Second Annual Conference on the Weights and Measures of the United States.
Third Annual Conference on the Weights and Measures of the United States.
Fourth Annual Conference on the Weights and Measures of the United States.
Fifth Annual Conference on the Weights and Measures of the United States.
Sixth Annual Conference on the Weights and Measures of the United States.
Seventh Annual Conference on the Weights and Measures of the United States.
Eighth Annual Conference on the Weights and Measures of the United States.
Ninth Annual Conference on the Weights and Measures of the United States.
Tenth Annual Conference on the Weights and Measures of the United States.
State and National Laws Concerning the Weights and Measures of the United States (second edition).
The National Bureau of Standards. (Descriptive pamphlet.)
Report of the International Committee on Electrical Units and Standards.

Soc 1520.203

DEPARTMENT OF COMMERCE

Vol. 12 **No. 4**

BULLETIN
OF THE
BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

ISSUED MAY 25, 1916

CONTENTS

Page
"Center of Gravity" and "Effective Wave Length" of Transmission of Pyrometer Color Screens, and the Extrapolation of the High Temperature Scale . . . . *Paul D. Foote* 483
Studies of Instruments for Measuring Radiant Energy in Absolute Value: An Absolute Thermopile . . . . *W. W. Coblentz and W. B. Emerson* 503
Present Status of the Determination of the Constant of Total Radiation from a Black Body . . . . *W. W. Coblentz* 553
Illumination from a Radiating Disk . . . . *Paul D. Foote* 583
Photometry of the Gas-Filled Lamp . . . *G. W. Middlekauff and J. F. Skogland* 589
Life Testing of Incandescent Lamps at the Bureau of Standards . . . . *G. W. Middlekauff, B. Mulligan, and J. F. Skogland* 607

WASHINGTON
GOVERNMENT PRINTING OFFICE
1916

DEPARTMENT OF COMMERCE

Vol. 12 No. 4

BULLETIN
OF THE
BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

ISSUED MAY 25, 1916

CONTENTS

Page
"Center of Gravity" and "Effective Wave Length" of Transmission of Pyrometer Color Screens, and the Extrapolation of the High Temperature Scale . . . . *Paul D. Foote* 483
Studies of Instruments for Measuring Radiant Energy in Absolute Value: An Absolute Thermopile . . . . *W. W. Coblentz and W. B. Emerson* 503
Present Status of the Determination of the Constant of Total Radiation from a Black Body . . . . *W. W. Coblentz* 553
Illumination from a Radiating Disk . . . . *Paul D. Foote* 583
Photometry of the Gas-Filled Lamp . . *G. W. Middlekauff and J. F. Skogland* 589
Life Testing of Incandescent Lamps at the Bureau of Standards . . . . *G. W. Middlekauff, B. Mulligan, and J. F. Skogland* 607

WASHINGTON
GOVERNMENT PRINTING OFFICE
1916

Harvard College Library
July 26, 1916
From the
United States Government.

ADDITIONAL COPIES
OF THIS PUBLICATION MAY BE PROCURED FROM
THE SUPERINTENDENT OF DOCUMENTS
GOVERNMENT PRINTING OFFICE
WASHINGTON, D. C.
AT
25 CENTS PER COPY
SUBSCRIPTION PRICE, $1 PER YEAR

"CENTER OF GRAVITY" AND "EFFECTIVE WAVE LENGTH" OF TRANSMISSION OF PYROMETER COLOR SCREENS, AND THE EXTRAPOLATION OF THE HIGH TEMPERATURE SCALE

By Paul D. Foote, Assistant Physicist

Some 10 years ago Waidner and Burgess[1] called attention to the shift in effective wave length of the color screens used with optical pyrometers, when the temperature of the source sighted upon changes. Pirani[2] has contributed two papers upon this subject, and very recently a paper by Hyde, Cady, and Forsythe[3] has appeared.

Formerly it has been assumed that the effective wave length is the so-called "center of gravity" of the luminosity curve. Hyde, Cady, and Forsythe have given a new definition of the mean effective wave length between any two temperatures, which has a real physical basis. They have defined the effective wave length of the glass used between these two temperatures as that wave length for which the ratio of the two radiation intensities corresponding to the two temperatures exactly equals the ratio of the integral luminosities through the screen for the same two temperatures.

In the following paragraphs the difference between the center of gravity and the true effective wave length is discussed and a very simple and accurate method is derived for obtaining the true effective wave length.

---

[1] Waidner and Burgess, this Bulletin, **3**, p. 175; 1907.

[2] M. v Pirani Verh. d. Phys. Ges., **15**, pp. 836–38; 1913; **17**, pp. 47–62; 1915.

[3] Hyde, Cady, and Forsythe, Phys. R., **6**, pp. 70, 74; 1915; also Astrophys, J., **42**, pp. 294–304; 1915.

Let $J_\lambda = c_1\,\lambda^{-5}\,e^{-\frac{c_2}{\lambda\theta}}$ = Wien's law.
$T = f\,(\lambda)$ = transmission of color screen.
$V = \varphi\,(\lambda)$ = visibility.

T and V are functions of λ only, but the analytical forms of the functions, at least in the case of $T = f\,(\lambda)$, are unknown. Consider two temperatures, θ_1 and θ_2.

$$\begin{cases} \text{for } \theta_1 \quad J_1 = c_1\,\lambda^{-5}\,e^{-c_2/\lambda\theta_1} \\ \text{for } \theta_2 \quad J_2 = c_1\,\lambda^{-5}\,e^{-c_2/\lambda\theta_2} \end{cases} \tag{1}$$

$$J_1/J_2 = e^{\frac{c_2}{\lambda}\left(\frac{1}{\theta_2} - \frac{1}{\theta_1}\right)} \tag{2}$$

Since λ may have any arbitrary value from 0 to ∞ the ratio J_1/J_2 may have any chosen value by properly choosing λ. Let L_1 = luminosity at temperature θ_1 and L_2 = luminosity at temperature θ_2.

$$L_1 = \int_0^\infty JTVd\lambda = \int_0^\infty J\,(\lambda, \theta_1)\,T\,(\lambda)\,V\,(\lambda)\,d\lambda$$

$$L_2 = \int_0^\infty J\,(\lambda, \theta_2)\,T\,(\lambda)\,V\,(\lambda)\,d\lambda$$

$$L_1/L_2 = \frac{\int_0^\infty J\,(\lambda, \theta_1)\,T\,V\,d\,\lambda}{\int_0^\infty J\,(\lambda, \theta_2)\,T\,V\,d\,\lambda} \tag{3}$$

L_1 and L_2, and hence the ratio L_1/L_2, can be determined by graphical integration. Let the ratio be any definite number. It is possible to choose λ in (2) such that $J_1/J_2 = L_1/L_2$. Call this value of $\lambda = \lambda_L$. Substituting in (2):

$$\text{mean } \lambda_L = \frac{c_2\left(\frac{1}{\theta_2} - \frac{1}{\theta_1}\right)}{\log L_1 - \log L_2} \qquad \textbf{(natural logarithms).} \tag{4}$$

This is the definition, expressed analytically, of the mean effective wave length over a given temperature range, proposed by Hyde, Cady, and Forsythe. Instead of referring the effective wave length to a given temperature range it may be referred to a definite temperature by letting the two temperatures approach one another. Let θ_1 and θ_2 approach θ as a limit. The right term of (4) is indeterminate but can be evaluated by the ordinary rule of L'Hospital, observing in the differentiation of the denominator that $L=\int_0^\infty TVJd\lambda$ where only J is a function of θ. Then for the limit $\theta_1=\theta_2=\theta$ the following value of λ_L is obtained:

$$(5) \qquad \lambda_L=\frac{\int_0^\infty TVJ\,d\lambda}{\int_0^\infty \frac{TVJ}{\lambda}d\lambda}$$

This is the true effective wave length of the glass for a temperature θ. Define the wave length corresponding to the line which bisects the area of $\int_0^\infty TVJd\lambda$ as λ_g; that is, the axis of the center of gravity of the luminosity curve, so chosen that $\int_0^\infty TVJd\lambda= 2\int_0^{\lambda_g} TVJd\lambda$. Then by definition of the center of gravity:

$$(6) \qquad \lambda_g=\frac{\int_0^\infty TVJ\,\lambda d\lambda}{\int_0^\infty TVJ\,d\lambda}$$

Having drawn the luminosity curve on any arbitrary scale whatever, paying no attention to actual values of the scale of ordinates (L), only relative values of L needing consideration, λ_g can be determined with a planimeter having two indicating dials, one of area and the other of moments, such as the Amsler integrator No. 1. Surprising accuracy is obtainable in measuring λ_g. This is evident when one considers that the luminosity curve for a good red glass extends only from about $\lambda=0.600$ to 0.750μ. Since all errors are actually based on the difference of these two wave lengths, a low precision in the determination of the center of gravity as far as the curve is concerned still means a very high precision in the determination of the absolute value of λ_g. Four or five significant figures are obtainable.

The method of dertermining λ_L suggested by Hyde involves the log ratio of the areas of two luminosity curves, and when λ_L is expressed by equation (5) its determination requires the ratio of the areas of two curves. In general, the area of luminosity curves can not be measured by a planimeter with an accuracy much better than 0.5 per cent, and since a direct mathematical integration is impossible because $T=f(\lambda)$, certainly, and $V=f(\lambda)$, possibly, are not expressible by any exact usable equation, it is evident that great precision is not readily possible in the determination of λ_L by such a method—probably not better than three significant figures when great care is taken.

Inspection of equation (5), however, shows a very curious relation. Suppose that instead of plotting λ versus JTV as ordinates, the ordinary luminosity curve, we plot JVT/λ as ordinates. The line of the center of gravity of this curve is given by the following expression:

$$\lambda \text{ (center gravity)} = \frac{\int_0^\infty TVJ d\lambda}{\int_0^\infty \frac{TVJ}{\lambda} d\lambda} \tag{7}$$

which is identical with (5).

Therefore, the true effective wave length of the pyrometer glass is the wave length corresponding to the center of gravity of the curve $f(\lambda) = JVT/\lambda$ versus λ, i. e., the luminosity at any wave length divided by the wave length. The true value of λ_L is slightly different from the value of λ_g. The center of gravity of the curve JVT/λ versus λ can be determined with great accuracy so that a curve of $\lambda_L = f(\theta)$ may be easily obtained by this method. Whether such high mathematical accuracy is of value is a point open to question since the transmission of the glass can not be measured extremely accurately and visibility curves appear very different for different observers. But in the case of the transmission, absolute values are not required, merely the form of the curve needs to be known. And in the case of the visibility, only a short range of the curve is important, and over this range the slopes of the visibility curves of various observers are in good agreement.

The fact that the effective wave length of a red pyrometer screen is altered by change of temperature of the source sighted upon is accordingly well recognized, but the effect which this shift may have upon the accuracy of temperature measurements has never been considered. The present paper concerns the type of pyrometer, such as the Holborn-Kurlbaum instrument, in which red glass screens are used in the ocular for producing as monochromatic illumination as possible. This instrument may be calibrated at the so-called fixed points of the temperature scale. On account of deterioration of the filament of the standard lamp, the maximum temperature at which the lamp can be safely operated is not greater than 1400° C and in general 1300° C. Two methods are used for measuring temperatures above this range. One involves decreasing the intensity of the source by means of neutral absorption glasses and the other by use of a sector disk. The effective wave length in either system shifts toward the green with increasing temperature. The best "neutral" absorption glasses are far from being neutral.[4] The transmission coefficient of the best glass obtainable is low in the blue, green, and yellow, and increases very rapidly in the red, with a maximum transmission in the early infra-red.

When strictly monochromatic radiation of wave length λ is used the following relation is obtained from Wien's law, where $\theta =$ true absolute temperature of the source, $S =$ observed absolute temperature when the sector or screen of transmission T is used in front of the source, and $c_2 = 14\,500$ micron degrees.

$$\frac{1}{\theta} - \frac{1}{S} = \frac{\lambda \log T}{c_2 \log e} = A \tag{8}$$

The factor $\lambda \log T/c_2 \log e$ is a constant $= A$. When the above equation is applied for radiation which is not strictly monochromatic A is no longer a constant as generally assumed but is a function of θ, because when a sector disk is employed, λ, the effective wave length of the red glass screen is a function of θ, and when an absorption glass is used, the value of $\log T$ changes rapidly with variations in θ. Experimentally, the quantity A is determined by sighting upon a source at constant temperature and measuring θ and S. But such a calibration can be made over a very limited temperature range only, because the maximum value of θ is determined by the highest temperature at which it is safe to operate the pyrometer lamp, and the minimum value is determined by S, which rapidly becomes so low that no accuracy is possible in matching the pyrometer filament against the source. The allowable range in θ by this method for a transmission factor much less than 1 per cent is not greater than 200° C. Another method of obtaining A directly can be used. This consists in sighting upon a black body immersed in a metal or other material at its melting point. The temperature of the melting point is θ and the apparent or measured temperature is S. Suitable materials having high melting points which are known with sufficient accuracy to justify their use as fixed points are platinum, palladium, and possibly diopside. The construction of a black body to operate at the temperatures of the melting point of platinum and palladium is extremely difficult.

---

[4] Dr. Nutting has informed me that he has succeeded in producing strictly neutral absorption screens from dye materials. These screens should prove of great value for use with optical pyrometers.

The highest fixed temperature which the Bureau has found convenient to use in standardizing absorption glasses is the diopside melting point. The method of obtaining a black body at this temperature has been described elsewhere.[5] However, whether the platinum, palladium, or the diopside melting point is used as the highest working temperature, the range of temperature over which an absorption glass can be calibrated is too small to determine how the quantity A varies with θ.

Two types of glass have been considered, one the Jena red transmission screen 2745, and the other the best obtainable black screen, Jena F 3815 with the laboratory reference number 4248B. A slightly better red filter is the Jena F 4512, but the difference between this and Jena 2745 is not great enough to change materially the conclusions obtained with the somewhat poorer glass. The monochromatic transmission coefficients through a portion of the visible spectrum were carefully measured with a silver strip cube spectrophotometer and a Brodhun adjustable sector disk. The observations are represented graphically in Fig. 1. The accuracy obtained with the low transmission black glass was possible only after eliminating stray light in the spectrophotometer system. For this reason the silver strip cube was found very much superior to the ordinary Lummer-Brodhun or Martens-

---

[5] Kanolt, this Bulletin, **10**, p. 5; 1914.

König spectrophotometer. In order to apply equation (5) to the determination of the effective wave length of the pyrometer system when (*a*) one red glass and (*b*) two red glasses were used in the ocular, the value of J_λ was computed from Wien's law for the absolute temperatures 1000, 1200, 1400, 1600, 1800, 2000, 2500, 3000, 3500. The factors for the visibility were obtained as a mean

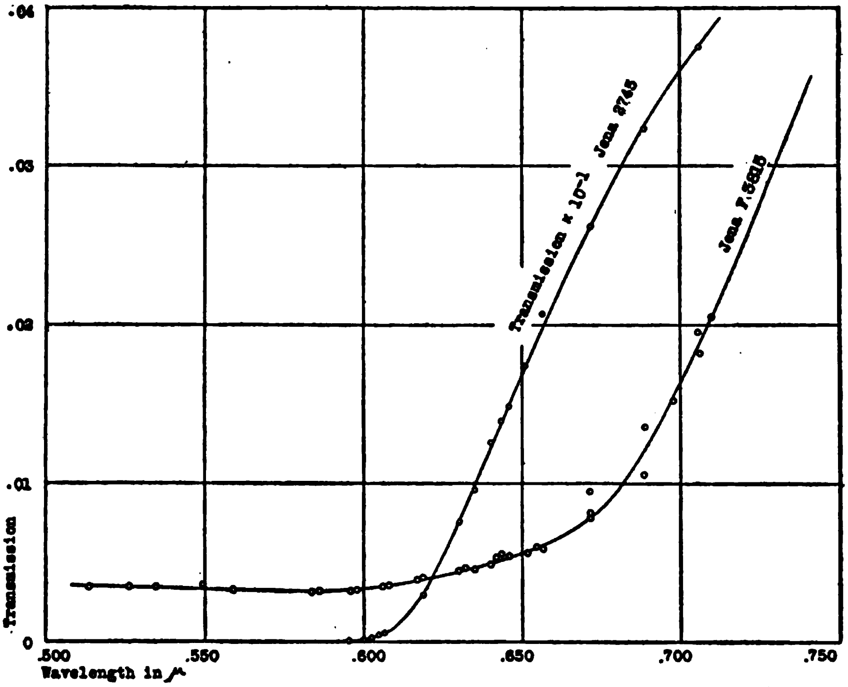


FIG. 1.—*Transmission coefficients of Jena red glass 2745 and Jena "neutral" glass F 3815. Room temperature. Thickness, 2745=3.20 mm. F 3815=0.52 mm*

of the data of Nutting,[6] Ives,[7] and Hyde and Forsythe.[8] The values of $\frac{JVT}{\lambda} = f(\lambda, \theta)$ were computed and the center of gravity λ_L of each of these curves, $f(\lambda, \theta)$ versus λ, was determined directly with an Amsler integrator. The following table illustrates the shift of wave length, λ_L, with change of temperature, and the data are represented graphically by Fig. 2. To show the difference between λ_L and the wave lengths corresponding to the center

---

[6] Nutting, Phil. Mag., **29**, p. 306; 1915.

[7] Ives, Phil. Mag., Dec.; 1912.

[8] Hyde and Forsythe, Phys. R., **6**, p. 70; 1915.

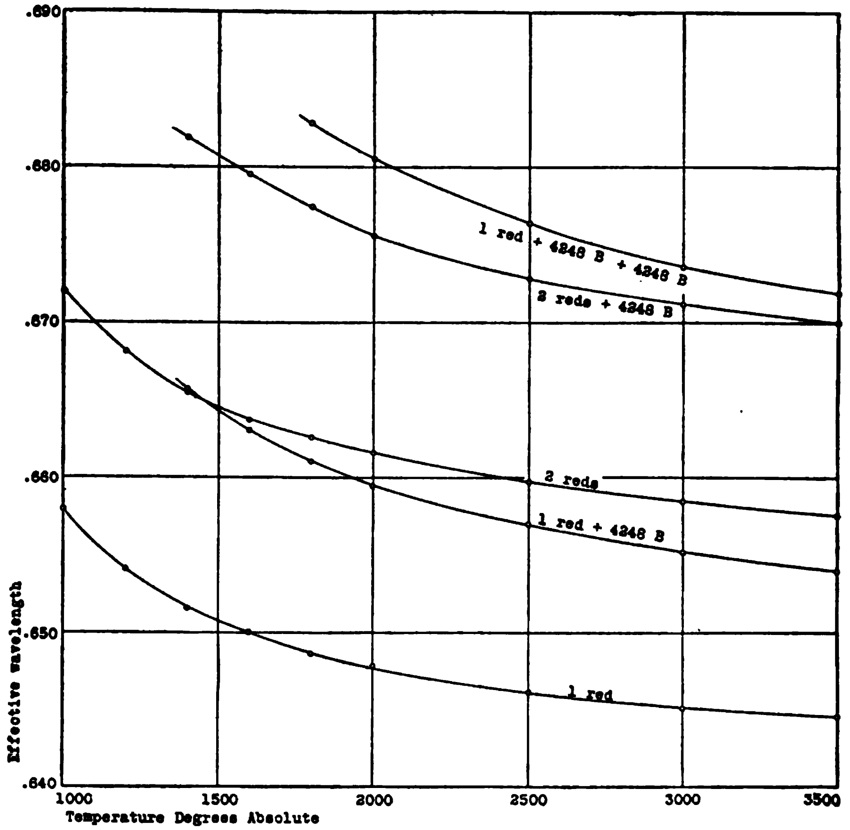


FIG. 2.—*Effective wave lengths of pyrometer glasses as a function of the temperature of the source*

of gravity λ_g of the luminosity curves, *JVT* versus λ, which, incorrectly, have been frequently considered to be the effective wave lengths of the glass, these values also are given in Table 1.

TABLE 1

"Effective Wave Length" and "Center of Gravity" of Red Screen Jena 2745 Used With and Without Neutral Screen Jena F 3815 (Reference No. 4248B)

[Thickness red screen=3.20 mm; thickness black screen=0.52 mm]

Temperature of source	1 red glass		2 red glasses		1 red+4248B		2 reds+4248B	
	λ_L	λ_g	λ_L	λ_g	λ_L	λ_g	λ_L	λ_g
1000 °abs.	0.6578	0.6587	0.6720	0.6729	....	....	....	....
1200	.6541	.6548	.6681	.6690	....	....	....	....
1400	.6515	.6522	.6655	.6661	0.6657	0.6673	0.6818	0.6831
1600	.6500	.6506	.6637	.6644	.6630	.6645	.6796	.6807
1800	.6487	.6490	.6625	.6628	.6610	.6624	.6774	.6783
2000	.6478	.6486	.6615	.6619	.6595	.6609	.6756	.6767
2500	.6462	.6467	.6597	.6600	.6570	.6582	.6728	.6741
3000	.6451	.6456	.6584	.6589	.6552	.6563	.6712	.6721
3500	.6445	.6446	.6574	.6581	.6539	.6552	.6700	.6710

It is seen from Fig. 2 that the curve λ_L versus θ is approximately a hyperbola. An equation of the type $\lambda_L = a + b/\theta + c/\theta^2$ where the term c/θ^2 is exceedingly small fits the observations almost exactly. That the relation is not precisely hyperbolic is evident from the following:

Differentiating equation (5)

$$\frac{d\lambda_L}{d\theta} = \frac{c_2}{\theta^2}\left[1 - \frac{\int_0^\infty JVT d\lambda \int_0^\infty \frac{JVT}{\lambda^2} d\lambda}{\left(\int_0^\infty \frac{JVT}{\lambda} d\lambda\right)^2}\right]$$

$$= \frac{c_2}{\theta^2}(1 - \lambda_L/\lambda_0)$$

where λ_0 is the center of gravity of the curve TVJ/λ^2 versus λ. If λ_L/λ_0 were a constant, then $\lambda_L = a + b/\theta$, the equation of a hyperbola. Actually λ_L/λ_0 shows an extremely small variation with θ. It is interesting to note that values of $\lambda_L = f(\theta)$ suggest the form of an equation representing the luminosity of any system.

Thus,

$$\frac{d \log L}{d\theta} = \frac{c_2}{\theta^2} \frac{1}{\lambda_L}.$$

When $\lambda_L = a + b/\theta$, the equation of luminosity and temperature takes the form derived by Nutting.[9]

If the carbon filament of the lamp of the Holborn-Kurlbaum pyrometer emits approximately black body radiation so that when it is matched against a source at a temperature θ the temperature of the filament is also θ, neglecting reflection losses at the lens surfaces, the match will always be an exact color match as well as intensity match. (Actual deviations from this condition introduce only a second order effect.) Such being the case, as far as temperature measurements are concerned, no errors are introduced by the shift in wave length of the system. It is only when the absorption glass or sector disk is employed that errors occur. When the absorption glass, for example, is used and a photometric match effected, the light from the source at a high temperature passing through the absorption glass and the red glass screen in the ocular is matched against the light from the lamp, at a lower temperature, passing through the red screen only. An exact color match can not be made in this case although the difference in color of the two fields is usually too slight to cause serious trouble. The match is obtained when the luminosity from the source through the black glass and the red glass equals the luminosity from the lamp through the red glass. Let T = transmission coefficient of the red glass, T' the transmission coefficient of the black glass, θ the temperature of the source, S the temperature of the lamp (apparent or measured temperature), L_θ the luminosity of the source, and L_s the luminosity of the lamp. Then

$$L_\theta = \int_0^\infty TT'VJ(\lambda,\theta)d\lambda$$

$$L_s = \int_0^\infty TVJ(\lambda, S)d\lambda$$

and the condition for a match is:

$$L_\theta = L_s$$

Curves were constructed of L_θ and L_s versus θ and S and values of θ and S obtained, by interpolation, for various values of $L_\theta = L_s$.

---

[9] This Bulletin, 5, p. 306; 1908

Hence, knowing θ and S, the quantity A or absorption factor can be determined by the relation $1/\theta - 1/S = A$. It is found, however, that interpolation of these plotted curves is somewhat inaccurate unless the values of L are computed for a great many temperatures. The method discussed below was used and is to be preferred for determining A.

Analogous to the Wien equation (8) for monochromatic radiation, the following equation applies when the condition of monochromatism is not realized.

$$\lambda'_L \log Tr = c_2 \left(\frac{1}{\theta} - \frac{1}{S}\right) \log e = Ac_2 \log e \tag{9}$$

where λ'_L is the effective wave length of the complete system, lamp + red screen and source + black screen + red screen, and Tr is the effective transmission of the black screen. The effective transmission of the black screen will depend upon the temperature of the source and upon the number of red glass screens used in the ocular. Its value is determined by the ratio of the luminosity, L'_θ, of the black body source at a given temperature, through the black glass and the red glass, and the luminosity, L_θ, with the black glass removed, thus:

$$Tr = \frac{L'_\theta}{L_\theta} = \frac{\int_0^\infty TT'VJ(\lambda, \theta)d\lambda}{\int_0^\infty TVJ(\lambda, \theta)d\lambda} \tag{10}$$

Also since $L'_\theta = L_S$

$$Tr = \frac{L_S}{L_\theta} = \frac{\int_0^\infty TVJ(\lambda, S)d\lambda}{\int_0^\infty TVJ(\lambda, \theta)d\lambda} \tag{11}$$

Values of L'_θ and L_θ are obtained by computing and integrating the luminosity curves for any arbitrary temperatures, and hence Tr is found without interpolation of the curves $L = f(\theta)$.

It can be shown that λ'_L, the effective wave length of the complete system is the arithmetical mean of the effective wave lengths of the two components of the system. The effective wave lengths of the system, source + black screen + red screen, were obtained as the centers of gravity of the curves $TT'VJ/\lambda$ versus λ (cf Fig. 2 and Table 1).

The method, then, for computing A is as follows: In equation (9) assume any value of A approximately near the correct value and compute approximate values of S for various values of θ. From curves Fig. 2 obtain the effective wave lengths corresponding to the temperature S of the lamp observed through the red screen, and the temperature θ of the furnace observed through the black glass + the red glass. The mean of these values is λ'_L. The value of $\log Tr$ is obtained from equation (10). Whence from (9) the equation $A = \lambda'_L \log Tr/c_2 \log e$ may be solved for the quantity $A = f(\theta)$. Table 2 shows the variation of A with the temperature of the source for absorption glass 4248B. These values are presented graphically by Fig. 3. Table 2 also includes the effective transmission coefficients of the black screen. The marked variation of the transmission with the temperature of the source is of interest and also the dependence of the transmission upon the number of screens used in the ocular. Thus the addition of 1 red glass in the ocular actually increases the effective transmission of the black glass about 30 per cent. This table emphasizes the importance of a thorough study of the transmission screens used in photometry. A screen calibrated with a lamp burning at a definite watts per candle can not be used without applying corrections to the transmission factor when the lamp is burning at any other efficiency.

TABLE 2

Absorption Glass Jena. F 3815

[$A = 1/\theta - 1/S$ and transmission coefficient as a function of θ]

θ	1400	1600	1800	2000	2500	3000	3500	
$Tr \times 10^6$	696	670	658	636	612	601	589	1 red glass
$-A \times 10^7$	2265	2274	2276	2287	2297	2300	2305	
$Tr \times 10^5$	912	867	849	828	787	768	749	2 red glasses
$-A \times 10^7$	2190	2207	2210	2217	2232	2239	2248	

The above discussion applies equally well when a sector disk is used to cut down the luminosity of the source. In this case the transmission factor is constant, but the effective wave length depends upon the temperature and number of red glasses used in the ocular. Table 3 gives the values of A obtained with a sector having an opening of 2.7°.

TABLE 3

Sector disk with 2.7° opening

[$A = f(\theta)$]

θ	1400	1600	1800	2000	2500	3000	3500	
$-A\times10^7$..................	2205	2201	2196	2192	2187	2183	2181	1 red glass
$-A\times10^7$..................	2254	2248	2243	2239	2233	2228	2226	2 red glasses

The variation of A with the temperature of the source when a sector disk is used is also represented by Fig. 3. For the combination of red glass and black glass, A increases with increasing temperature and for the combination of red glass and sector disk, A decreases with increasing temperature. If the absorption glass and sector disk are calibrated at about the most favorable point possible, the melting point of diopside, and the values of A thus obtained are assumed constant with temperature, as has always been the assumption, measurements upon a source at a temperature of 3500° absolute will be 30° to 40° low when the absorption

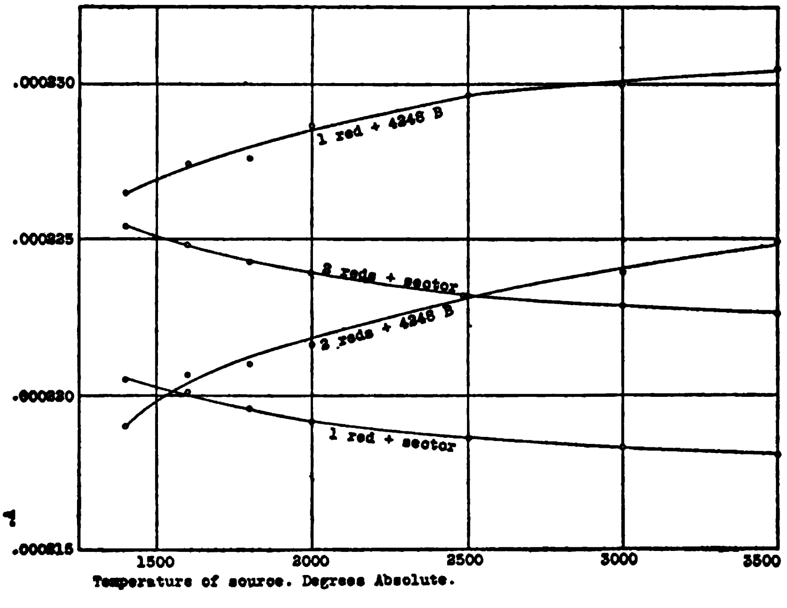


FIG. 3.—*A as a function of θ*

glass is used, and about 20° high when the sector disk is used. The difference in the measurements by the two methods will amount to 50° or 60°.

The errors to be expected in measurements of this kind are shown by Table 4. The values are based on the favorable calibration temperature of about 1650° absolute.

TABLE 4

Absorption Glass 4248B and Sector Disk of Opening 2.7°

[Calibration temperature about 1650° abs.]

Temperature θ° abs.	True temperature minus observed temperature			
	4248B		Sector disk	
	1 red	2 reds	1 red	2 reds
1200	− 5	− 5	+ 2	+ 2
1600	− 1	− 1	0	0
2000	+ 4	+ 5	− 3	− 3
2400	+11	+13	− 7	− 8
2000	+19	+23	−12	−14
3200	+27	+34	−17	−20

Actually it would not be safe to operate the pyrometer lamp at a temperature which, with the absorption factors of the above absorption glass and sector, would permit measurements of the source at a temperature higher than 2500° absolute. A denser absorption glass or a sector of smaller opening must be used. But if the transmission of the glass or sector is much less, it is impossible to calibrate them. The general practice has been to use two absorption glasses, which have been calibrated independently, and to assume that the "constant" A for the two glasses used together is the sum of the constants when calibrated separately. Table 5 and Fig. 4 show the values of A for two absorption glasses of the type 4248B, used together. The values of A for the glasses used separately are added and given by the third line of the table. The "constant" obtained by the separate calibration "constants" of such glasses at the diopside point, added together, is given in the fourth line. This latter value represents

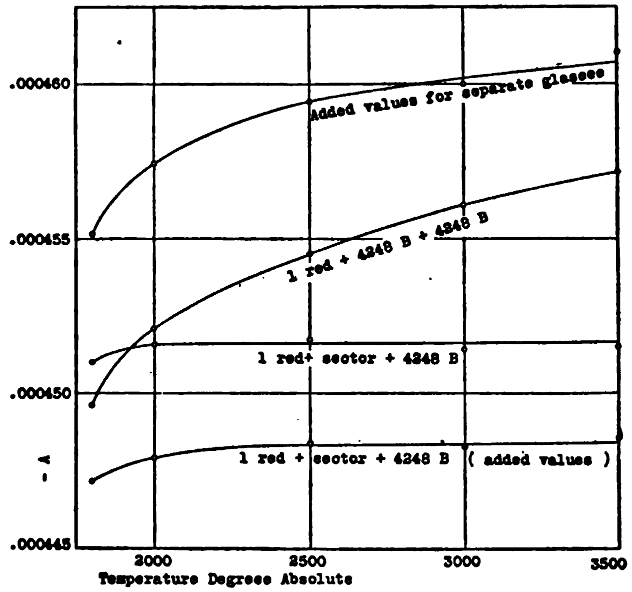


FIG. 4.—*A as a function of θ*

that obtained by the method which has been quite generally heretofore assumed correct.

TABLE 5

Use of Two Black Glasses of the Type 4248B With 1 Red Glass

θ	1800	2000	2500	3000	3500
$-A \times 10^7$ (correct values)	4497	4521	4545	4561	4572
Values obtained by adding A for each glass	4552	4574	4594	4600	4610
Added value of A at $\theta=1650$	4550	4550	4550	4550	4550

The use of a combination of sector disk and absorption glass would very likely prove highly satisfactory for extreme temperatures. Fig. 4 shows the values of *A* for a temperature range 1800° to 3500° absolute, for the combination of one red glass, the black glass 4248B, and a 2.7° opening disk. The variation of *A* is negligible above 2000°, but the calibration of such a system is again difficult. If the sector and absorption glass are calibrated separately and the separate values of *A* added, the values thus obtained are too low, as indicated by Fig. 4. In the present case, however, if the sector is calibrated alone, with two red glasses in the ocular, at any temperature, and the absorption glass is calibrated with one red glass in the ocular, at the same temperature, the sum of the values obtained separately is equal to the value of A for the combination one red glass + sector + black glass, at all temperatures. The addition of the extra red glass in the calibration of the sector is simply to bring the effective wave length of the system nearer to that of the combination one red glass + one black glass.

Experimental confirmation of above discussion:

The difference in the value of $A = 1/\theta - 1/S$ when one red glass is used and when two red glasses are used in the ocular is very pronounced with either a sector disk or absorption glass. This difference may amount to 50° at 2000° if the value of A determined for one red glass in the ocular is used for computing temperatures measured when two red glasses are in the ocular, and vice versa. Or if the pyrometer filament is matched through the black glass or sector against a source at, say, 2000°, using one red glass, the match is completely destroyed when a second red glass is added to the ocular, and vice versa.

The fact that A is a function of the temperature which, with increasing temperature of the source, increases for the absorption glass and decreases for the sector disk, may be easily shown qualitatively. Decreasing the temperature of the source obviously must increase the value of the effective wave length of either system. Instead of increasing the effective wave length by decreasing the temperature, the same effect may be produced by the addition of one red glass in the ocular. Obtain a match at a high temperature with the sector and one red glass. Add one red glass. The match is destroyed and the filament will appear darker than the field. If the same process is repeated with a sector, the filament will appear brighter than the field. A sector disk having one opening of 3.6° was accessible, which could be rotated at 3000 revolutions per minute. But the luminosity is so high when only one red glass is used, at a temperature of the source of 2000°, that 3000 revolutions per minute is below the critical speed and serious flickering prevents accurate matching.

Two Glan nicol prisms nearly crossed were therefore substituted for the sector. Since the Glan nicol is practically nonselectively absorbing, its effect is precisely that of the sector disk. The following table shows the apparent temperatures obtained for a match when the source was about 1700° C:

Ocular	1 red glass	2 red glasses
	°C	°C
Absorption glass	1109	1126
Nicols	1095	1087

Quantitative proof that $A = f\ (\theta)$.

Referring to Table 4, it is seen that the sector disk or nicols should give a higher temperature reading than the absorption glass when all are calibrated at a low temperature and used with a source at a high temperature. The sector disk of 3.6° opening, the nicols, and the absorption glass were calibrated on a source at about 1450° absolute. Measurements throughout were made by two observers working independently. Table 6 shows that the difference in temperatures of a source at about 2000° measured by the two methods is of serious importance. If it had been possible to maintain the temperature of the source at 2500° to 3000°, instead of 2000°, differences of two or three times this magnitude would have resulted.

TABLE 6

Experimental Data on Use of Sector Disk, Nicol Prisms, and Absorption Glass for Determination of High Temperatures

Observer	Calibration temperature	Red glasses	Sector	Nicols	Absorption glass	Sector or nicols-absorption glass
	Deg. abs.		Deg.	Deg.	Deg.	Deg.
COF	1450	2	1828		1810	18
PDF	1450	1	1986		1979	7
PDF	1450	1	1998		1989	9
PDF	1450	2	2002		1984	18
PDF	1450	1		1968	1955	13
PDF	1450	1		1970	1958	12
PDF	1450	1		1974	1961	13
COF	1400	1		1971	1962	9

Use of two and more absorption glasses. That the separate values of A determined for several absorption glasses separately can not be added to obtain the value of A for the glasses when used together has been noted by Waidner and Burgess[10] in their investigation of the temperature of the arc. Recomputing their data for four glasses showing the same absorption the following table is obtained:

Glasses	Observed A	Added values of A
1	−.0001112	−.0001112
2	2209	2224
3	3283	3336
4	4332	4448

The above data were taken without careful reference to the calibration temperatures, and hence they can not be considered quantitatively. In general, however, the value of A for several glasses used together is less than the sum of the values of A for the glasses used separately. This effect has nothing to do with multiple reflection between absorption glasses, as multiple reflection is usually of negligible importance.

The explanation lies in the change of the transmission coefficient of each successively added absorption glass, although all the glasses are identically alike. The quantity A is proportional to $\lambda'_L \log Tr$. The addition of each absorption glass moves the effective wave length of the entire system toward the red. This increases the value of λ'_L and would cause abnormally high values of A. But the transmission coefficient of each successively added absorption glass is greater than that of the glass used immediately preceding. Hence, the value of $\log Tr$ for any definite temperature of the source decreases numerically for each successive glass. The relative decrease in $\log Tr$ is greater than the relative increase in λ'_L, and thus the observed values of A are smaller than the added values of the single glasses.

---

[10] Waidner and Burgess (this Bulletin, 1), p. 116; 1904.

SUMMARY

An equation has been derived representing as a function of the temperature of the black body source sighted upon, the "effective wave length" of any pyrometer color screen or system of color screens, or of color screens used in combinations with sector disk, nicol prisms, or absorption screens. Attention is called to the difference between the "effective wave length" and the "center of gravity" of the luminosity curve, as the so-called center of gravity of the color screen has been in the past sometimes considered the true effective wave length. When an absorption glass, nicol prisms, or sector disk is used with a pyrometer employing light which is not strictly monochromatic, such as the Hoborn-Kurlbaum instrument, for the object of reducing the brightness of the sources sighted upon, the relation between the true temperature θ and the measured temperature S is of the form $1/\theta - 1/S = A$ where A has been heretofore considered a constant. Actually, it is shown, A varies rapidly with change in temperature of the source. The consideration of A as a constant may introduce errors as great as 40° or 50° at 3500° if the absorption device is calibrated in the usual manner. When two or more absorption glasses are used together, it is not permissible to assume that the value of A for the entire system is equal to the sum of the values of A for the glasses used separately. It is shown that the combination of a sector disk and absorption glass gives a value of A which is practically independent of θ.

In conclusion, acknowledgment is due C. O. Fairchild and T. R. Harrison, of this laboratory, for assisting in the measurements and for graphically integrating some 150 curves.

WASHINGTON, September 20, 1915.

STUDIES OF INSTRUMENTS FOR MEASURING RADIANT ENERGY IN ABSOLUTE VALUE:

AN ABSOLUTE THERMOPILE

W. W. Coblentz and W. B. Emerson

CONTENTS

Page.
I. INTRODUCTION 504
II. APPARATUS AND METHODS 505
  1. The radiometer 505
  2. Construction of the receivers 506
  3. Thermopile and galvanometer 511
  4. Water-cooled shutter and diaphragm 513
  5. The radiator 515
  6. The assembled apparatus 516
  7. Method of making observations 518
  8. Method of reduction of data 520
  9. Corrections for diffuse reflection from the receiver 522
  10. Accuracy attainable 525
III. EXPERIMENTAL DATA 530
  Receiver No. 1 542
  Receiver No. 2 542
  Receiver No. 3 542
  Receiver No. 4 542
  Receiver No. 5 543
  Receiver No. 6 543
  Receiver No. 7 544
  Receiver No. 8 545
  Receiver No. 9 545
  Receiver No. 10 546
  Receiver No. 11 546
  Receiver No. 12 547
  Receiver No. 13 548
IV. SUMMARY 548

I. INTRODUCTION

One of the chief needs in the measurement of radiant energy is a reliable radiometer which evaluates the observations in absolute measure. Such an instrument should be a primary one, capable of measuring radiant energy directly. Because of the lack of such a primary instrument, a calibration of radiometers against a standard of radiation is advocated; and such standards of radiation have been prepared[1] by this Bureau.

There are several apparently trustworthy methods for making measurements of radiation in absolute value. One of the problems undertaken in this Bureau is the study of various instruments used in making these absolute measurements; and the purpose of the present paper is to report on the results obtained with one of these instruments.

One of the by-products of this investigation will be the evaluation of the coefficient or the so-called Stefan-Bolzmann constant of total radiation of a uniformly heated cavity or so-called black body. As will be shown in a subsequent paper, the different methods employed by various experimenters in evaluating this constant give somewhat discordant results. It is therefore desirable to emphasize at the very beginning of this paper that, until it can be shown which of these various methods is the most reliable, it is undesirable to depend upon a single numerical value.

In beginning the subject of radiometry in absolute measure, it seemed desirable to study instruments which might prove of use in general radiometric work (e. g., useful as radiation pyrometers), leaving for later investigation the more cumbersome and complicated devices for determining with the highest precision the coefficient of total radiation of a uniformly heated cavity or so-called black body.[2] The first communication[3] on this subject gave the results of a study of the "Radiobalance."[4] This is an ingenious device in which the radiometric receiver consists of a thermojunction which can be heated electrically. In this manner the heat generated in the receiver by absorbing radiant energy is neutralized by the well-known Peltier (cooling) effect, which occurs when an electric current is passed through a thermojunction. The result of the study of this instrument showed that while it has many good points it did not appear to be sufficiently reliable for a primary instrument. The investigation of the instrument used in the present work was undertaken in order to determine its reliability as a precision radiometer, i. e., to determine whether it is a primary instrument capable of measuring the radiation constants directly or whether it is a secondary instrument which must be calibrated by exposing it to a standard of radiation. The first question raised in regard to the use of a receiver of this type is the uniformity of temperature distribution when heated electrically from within and when heated by absorbing radiant energy incident upon its front surface. From the concordant results obtained with metallic strips, differing by 10 times in thickness, and having different kinds of absorbing surfaces, it is evident that whatever errors (if any) are introduced, are much smaller than the variations observed with the various receivers constructed of the same kind of material.

---

[1] This Bulletin, 11, p. 87; 1914.

[2] This Bulletin, 12, p. 553; 1916.

[3] This Bulletin, 9, p. 51; 1912.

[4] Callendar, Proc. Phys. Soc., London, 23, pt. 1; Dec., 15, 1910.

II. APPARATUS AND METHODS

Below are given the essential features of the instruments used in this research. A more complete description of the thermopile, galvanometer, and other accessories is given in a previous paper.[5]

1. THE RADIOMETER

The radiometric receiver,[6] Fig. 1, which was the subject of the present investigation is a very simple device, consisting of a very thin strip of metal, blackened to absorb radiation. Behind this metal strip, at a distance of 2 to 3 mm, is placed a thermopile of bismuth-silver having a continuous receiving surface of tin. The thermopile is connected with an ironclad Thomson or a d'Arsonval galvanometer which serves merely as a null instrument to indicate the rise in temperature of the metal strip.

---

[5] This Bulletin, 11, p. 132; 1914.

[6] For further details, see this Bulletin, 11, p. 157; 1914.

2. CONSTRUCTION OF THE RECEIVERS

The strip of metal serves three purposes—(1) as a receiver for absorbing radiant energy, (2) as a source of radiation which can be produced by heating the strip electrically, and (3) as a standard of radiation to test the galvanometer sensitivity, by heating the strip electrically by a standard current.

The use of a thermopile, separated from the receiver, makes it possible to use various receivers. The radiometric apparatus was therefore designed so that the receivers could be mounted over the thermopile as shown in Fig. 2. Each receiver, as shown in Fig. 2, is a complete unit consisting of an insulating base of slate *B* (cut from an ordinary writing slate), 3 mm in thickness, to which are attached the copper electrodes, *E E*, 0.5 mm in thickness, the receiver *R*, the potential terminals *P P*, the knife-edge slits *S S*, at the front, and the strips of copper, *C C*, at the rear, which are used for the purpose of preventing radiations, not intercepted by

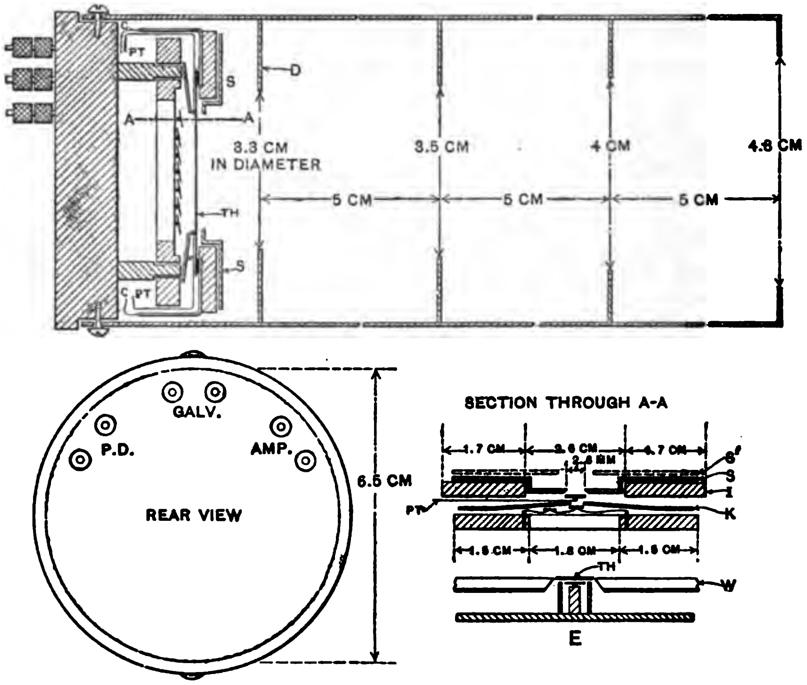
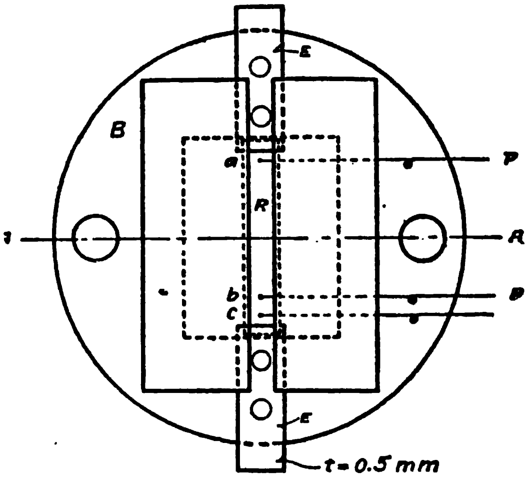
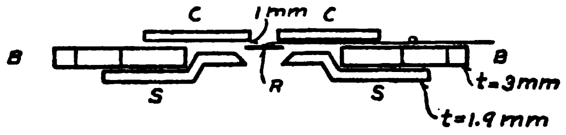


Fig. 1.—*The radiometer*

the receiver, from falling upon the thermopile. The platinum used for receivers was the "platinum in silver" material used for bolometers. This material was cut into strips, by means of a specially constructed cutting device, and soldered to the electrodes, which were then covered with Chatterton compound in order to protect them from the nitric acid which was used in removing the silver from the platinum. The platinum strip was then cleaned electrolytically by dipping it in hydrochloric acid. After attaching the potential wires, the strip was coated electrolytically with platinum black.[7] Incidentally, it may be added that the platinum cholride seems to deteriorate rapidly with usage, giving grayish deposits, so that it is necessary to use freshly prepared solutions.



REAR VIEW OF RECEIVER



END SECTION THROUGH AA

FIG. 2.—*Showing the construction of the receivers*

---

[7] This Bulletin, 9, p. 305; 1913. Kurlbaum, Ann. der Phys. (3), 67, p. 846; 1899.

The potential leads *P P* consisted of No. 36 silk-covered copper wire attached to the holder by means of Chatterton compound. To the end of this copper wire was soldered a fine platinum wire 0.025 mm in thickness, and (for several receivers) to the end of this platinum wire was soldered a short piece of (Wollaston) platinum wire about 0.01 mm in thickness. To the free end of the latter was attached a bead of solder from 0.01 to 0.03 mm in diameter, and this was melted to the platinum strip by means of the small nichrome[8] heater now used in performing such delicate operations. If the bead of solder is dipped in a solution of zinc chloride, there is no difficulty in attaching it to the platinum strip, and the juncture will be only 0.05 to 0.1 mm in diameter. This juncture is covered with shellac to permit repairs in case of breakage, and to protect the solder from the platinum chloride solution when depositing the platinum black. These potential terminals were situated at a distance of 3 to 4 mm from the copper electrodes. In order to test the effect of the potential contacts, an additional wire, *c*, was attached to two of the receivers. The platinum potential wires are shellacked for insulation.

The opening in front of the platinum receiver is defined by means of knife-edged strips of metal, along the sides and across the ends. The latter were placed directly over the potential terminals. When it was found impossible to detect a difference in the radiation constant, as measured with and without these slits (which were made of thin copper) across the ends, they were discarded and the entire length of the receiver was exposed to radiation. This eliminated the conduction of heat from the region between the potential terminals, which were then used to define the length of the receiver. Knife-edge slits *S S* were used over the sides of the receiver, in order to define its effective width which is difficult to determine after the receiver has been covered with platinum black; and, especially so, after smoking it with lampblack when a layer of soot 0.05 mm in width may adhere loosely along the edge. These knife-edged slits were of two kinds: (1) Brass ones 0.5 mm in thickness, and (2) slits of aluminum 1.95 mm in thickness. Both types were bright over a width of about 1 cm along the knife-edge, on the side exposed to the radiator. Measurements were made without the slits and the value of the radiation constant was usually slightly higher than with the slits. In view of the possibility of diffusely reflected radiations from the radiator reaching the thermopile, the slits were used. It would have been desirable to use a hemispherical mirror in front of the receiver, as in previous work,[9] and thus eliminate the correction for the loss of radiation by diffuse reflection, but the uncertainty then arising as to the exact area exposed and as to numerous adjustments, etc., appeared to introduce greater errors than would arise from a lack of proper correction for reflection, which correction was determined directly. The receiver of the thermopile had an area of 1.8 by 19 mm, the length being somewhat less than the distance between the potential terminals on the receiver *R*. The thermopile was covered permanently with a piece of cardboard, about 0.4 mm in thickness, which had an opening, about 2.5 mm in width and 20 mm in length, to admit radiations from the receiver. This cardboard shield was used to exclude the radiations coming from that part of the receiver which extends from the potential terminals to the copper electrodes. In addition to this shield, each receiver mounting was covered on the rear side with heavy (0.8 mm) strips of copper (or sheet iron folded as in receiver No. 6), *C C*, Fig. 2, to protect the thermopile from possible stray radiations which might not be intercepted by the receiver. These strips of metal were painted on both sides with lampblack, then blackened by holding them in the flame of a sperm candle. This method of construction places the thermopile surface at a distance of about 3.5 mm from the receiver, which greatly reduces the sensitivity. However, it eliminates the question of convection currents produced by the warming of the receiver. In the first part of the work the shields, *C C*, were not used, and consequently the thermopile was much closer to the receiver, which increased the sensitivity. The same numerical value of the constant of radiation was obtained as with the greater separation of the receiver and the thermopile.

---

[8] This Bulletin, 9, p. 7; 1912.

The method of blackening the receivers with soot is of some importance. The thick metal plates used as shields were painted with lampblack and smoked by drawing them through the tip of the flame of a sperm candle. This produces a deposit of soot which reflects but little more than 0.5 per cent.[10] However, as indicated in the previous paper relating to this subject, the cold deposits of soot from an acetylene flame or sperm candle reflect a little over 1 per cent. In order to obtain the blackest deposit from a sperm candle, the wick must be free from old, charred material which produces a "hard" bluish smoke, having a high reflecting power. On the other hand, the wick must not be trimmed too short. The best results are obtained by using a freshly trimmed candle in which the flame is burning at its normal height. The smoke is produced with a sheet-iron cone about 8 cm long and 4 cm at its base, flattened at the top, leaving an opening 1.5 by 10 mm for the escape of the soot. When the candle is burning properly this funnel is held over the flame by means of a crucible tongs, and the object which is to receive the soot is passed back and forth, at a distance of 2 to 5 cm, over the top of the metal cone, which is not permitted to become hot. As to the thickness of the deposit of lampblack, that seems to be a matter of guesswork. The paint is a mixture of lampblack in alcohol and turpentine which is thoroughly shaken and allowed to stand a few minutes so that the coarse agglomerations may settle to the bottom. The coating of paint applied to a receiver is of such thickness that in bright sunlight the metal underneath is barely perceptible through the thin spots in the paint. This surface of lampblack paint is then exposed in the smoke of a sperm candle until it is thoroughly covered with soot. Some layers of soot were, no doubt, as much as 0.1 mm in thickness, although the deposit is usually just sufficient to cover the paint.

---

[9] This Bulletin, 10, p. 2; 1913.

The receivers made of manganin or therlo and painted with lampblack, Fig. 3, do not have an appreciable (to 1 part in 2650) temperature coefficient of resistance. On the other hand, the temperature coefficient of resistance of platinum, which is positive when unblackened, becomes negative after it is given a coating of platinum black. This is illustrated in Fig. 3, which shows the change in resistance of the strip when heated to different temperatures, as indicated by the current which was passed through the strip in order to raise it to the same temperature as was produced by the absorption of radiant energy.

---

[10] This Bulletin, 9, p. 283; 1913.

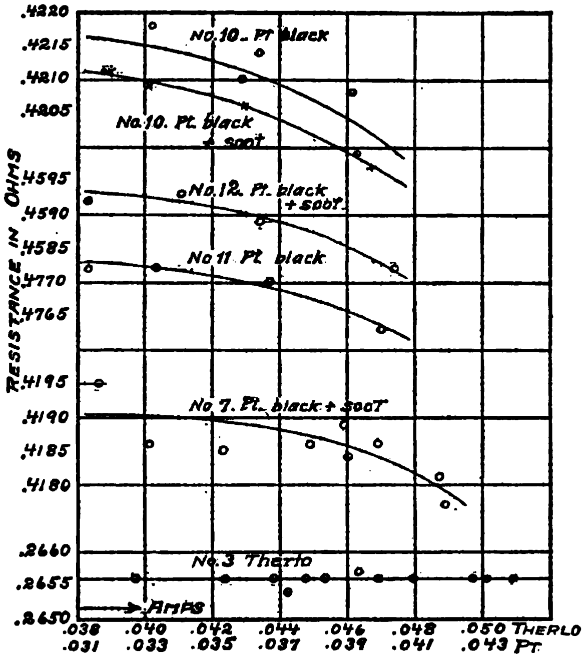


FIG. 3.—*Showing the change in resistance with temperature (heating current) of receivers of therlo, No. 3, and platinum*

3. THERMOPILE AND GALVANOMETER

These two instruments have been described elsewhere,[11] and it will be sufficient to add that the thermopile with its auxiliary receiver required, at a minimum, from 12 to 15 seconds (depending upon the receiver used) to produce a maximum galvanometer deflection. This long period requires uniform conditions in order to obtain accurate measurement. No observations were made, therefore, on very windy days when the thermopile was affected by air currents. The sensitivity was such that it would have been possible to observe the rise in temperature of the thermopile by means of a d'Arsonval galvanometer. Two such instruments were available, but they happened to respond to vertical tremors in the laboratory so that readings could not be made with an accuracy higher than 0.2 mm in a maximum deflection of 60 to 70 mm. A deflection of this size (75 per cent larger than normal) was obtained by reducing the critical damping resistance of 100 ohms to 50 ohms. This increase in sensitivity was, of course, at the expense of the period. The d'Arsonval galvanometer was therefore used simply as a check on the observations made with an ironclad Thomson galvanometer of 5.3 ohms, which was operated on a single swing of 1.5 to 2 seconds. But even on this short period the instrument was too sensitive, so that an external resistance of 60 to 120 ohms was constantly used in the circuit. The galvanometer deflections were then from 150 to 200 mm, and under these conditions it was the usual experience to repeat the readings to 0.3 to 0.5 mm.

---

[11] This Bulletin, 11, p. 132; 1914.

The galvanometer being completely shielded magnetically, the unsteadiness observed was found to be caused by air currents in the thermopile. On rare occasions the galvanometer was momentarily deflected, which was attributed to radiotelegraphic disturbances. Great steadiness in the galvanometer is necessary because of the long wait (about 30 seconds at a minimum) to obtain the complete scale reading. No difficulty was experienced with the galvanometer, and in the present work plenty of time could be allowed for the receiver and the thermopile to attain a steady temperature. Moreover, after the deflection had attained a maximum it would stay at that point whether the stimulus was applied for 20 or 30 seconds or for a much longer period. This is an important point in radiometric work of this type. The galvanometer was not completely damped on a swing of less than 2.5 seconds, and, because of the quickness of the response of the thermopile, especially when the receiver was of thin bolometer platinum, the maximum steady deflection was attained by executing one large throw and one or two small vibrations before the needle came to rest. This large first throw of the galvanometer needle was slightly different when the receiver was heated radiometrically and electrically, especially for the thick manganin receivers, but there was no marked difference in the time of attaining the final deflection. In view of the fact that observations made with the damped and undamped galvanometer showed no difference in the final result (and it should not do so, since plenty of time was given to obtain a steady deflection), no errors are to be expected from this source.

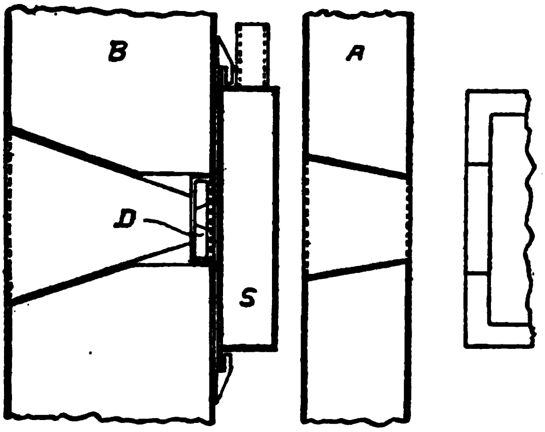
4. WATER-COOLED SHUTTER AND DIAPHRAGM

The water-cooled shield employed consisted of a tank, *A*, 25 cm in diameter and 1.5 cm in thickness, which faced the radiator, and a tank, *B*, 20 cm in diameter and 3 cm in thickness, which faced the radiometer. The water-cooled shutter, *S*, Fig. 4, consisting of a thin metal box 3.5 by 3.5 by 0.8 cm, was operated in vertical ways between these two shields. A mercurial thermometer, *T*, having its bulb in the water flowing through this shutter, was used to measure the temperature, *T*, used in equation (1). A more detailed illustration of a similar shutter is given elsewhere,[13] and some of the parts are shown in Fig. 6. The side of the shutter facing the radiometer was smoked in a sperm candle, and in connection with the conical-shaped opening (painted black and smoked) in the water-cooled diaphragm, *B*, it formed a miniature black body, the temperature of which remained constant throughout a series of measurements. The temperature of the water varied, of course, from day to day, being as low as 10° C in the winter, and averaged 15° C during this investigation. The temperature of the shutter was easily kept constant to within 0.5° C. The opening in the diaphragm, *B*, which must be accurately known, was defined by means of a series of small perforated disks of brass 8 mm in diameter and 1.5 mm in thickness, which were mounted in a recess provided for the purpose. These brass disks were provided with accurately cut, knife-edged holes having the following diameters: No. 1 = 5.410, No. 2 = 4.939, No. 3 = 3.987, and No. 4 = 2.005. This design enabled one to easily change the size of the opening which admits radiation upon the receiver and thus study the effect upon the radiation constant.

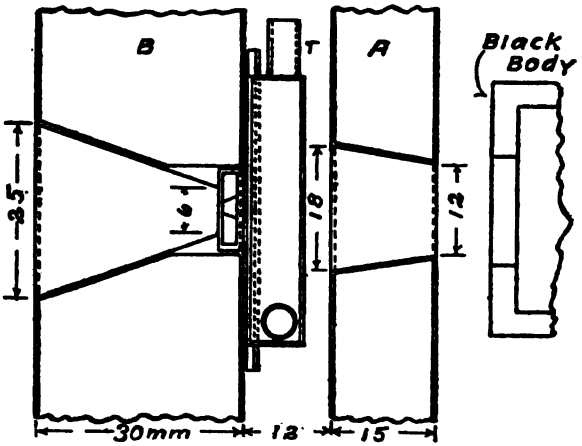
---

[13] This Bulletin, 10, p. 30; 1913.

The amount of radiation emitted through the smallest diaphragm, for the temperatures used, was small. This increased the errors of observation. However, from the data given in Table 1



HORIZONTAL SECTION



VERTICAL SECTION

FIG. 4.—*Water-cooled shutter and diaphragms*

(receiver No. 4), series XXXVII to XLVI, it appears that the size of the opening used did not affect the numerical value of the results, which have a range in variation of about 1 per cent.

TABLE 1

Data Showing that the Size of Water-Cooled Diaphragm Has No Appreciable Effect Upon the Radiation Constant

[Receiver No. 4; slits entirely removed; length of receiver, 26.565 mm; width, $\begin{smallmatrix}5.536?\\5.575\end{smallmatrix}$]

Diaphragm No. 2.—d=4.939 mm

Series	Distance	Temperature		Coefficient of radiation $\sigma \times 10^{12}$					
		t_0	t_1	a	b	c	d	e	f
	mm	°C	°C						
XXXVII	343.3	10.0	1005.0	5.45	5.56	5.52	5.53	5.55	5.51
XXXVIII	496.8	11.8	1004.5	.52	.51	.50	......	......	......
XXXIX	522.7	14.5	1022.3	.50	.50	.47	.48	.48	.50
XL	440.3	14.7	1022.2	.48	.48	.48	......	......	......

Mean value $\sigma = 5.49 \times 10^{-12}$ watt cm^{-2} deg^{-4}

Diaphragm No. 4.—d=2.005 mm

XLI	420.7	19.8	1055.5	.56	.51	.48	......	......	......
XLII	422.0	16.5	1056.5	.41	.41	.41	......	......	......
XLIII	503.1	16.5	1056.4	.43	.40	.35	.36	.40	.42

Mean value $\sigma = 5.44 \times 10^{-12}$ watt cm^{-2} deg^{-4}

Diaphragm No. 3.—d=3.987 mm

XLIV	451.0	18.5	1055.7	.37	.38	.39	.42	.41	.42
XLV	497.3	16.3	1052.1	.46	.49	.47	.50	.49	.51
XLVI	421.2	19.5	1051.0	.48	.46	.48	.49	.48	.50

Mean value $\sigma = 5.46 \times 10^{-12}$ watt cm^{-2} deg^{-4}

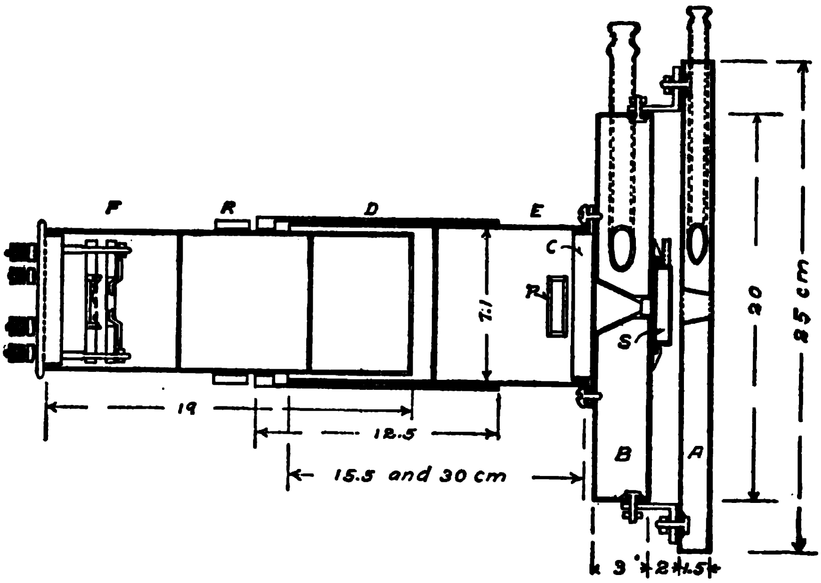
5. THE RADIATOR

The radiator, or so-called black body, was the one (painted Marquardt porcelain) used in a previous investigation[13] of the constant of spectral radiation. The device was operated as in the previous work, to which reference may be made for details of construction. The temperature measurements were made with thermocouples as heretofore, the radiator being operated at a sufficiently high temperature (1000° C) to produce sufficient radiant energy for measurement, and yet not at a sufficiently high temperature to affect the calibration of the thermocouples.

---

[13] This Bulletin, 10, p. 2; 1913. (See Fig. 4 B.)

Furthermore, at this temperature the radiator is easily held constant to 0.°1, which is necessary in view of the fact that under certain conditions a change in temperature of 0.°1 was perceptible with the receiver. This radiator was painted with a mixture of cobalt and chromium oxides. A series of measurements was made upon an exactly similar radiator which was unpainted. This white porcelain radiator gave a value of the radiation constant which is about 1 per cent less than that obtained from the painted tube. This will be discussed in a subsequent paper, in view of the fact that it has nothing to do with the study of the radiometer which was under investigation.



HORIZONTAL SECTION

FIG. 5.—*Assembled apparatus.*

6. THE ASSEMBLED APPARATUS

In Fig. 5 is shown a drawing of the assembled apparatus, excluding the radiator, which, of course, would face the water-cooled diaphragm, as shown in Fig. 6. The construction and operation of the radiator, *R*, Fig. 6, has been described elsewhere,[14] and it will

---

[14] This Bulletin, 10, p. 24; 1913.

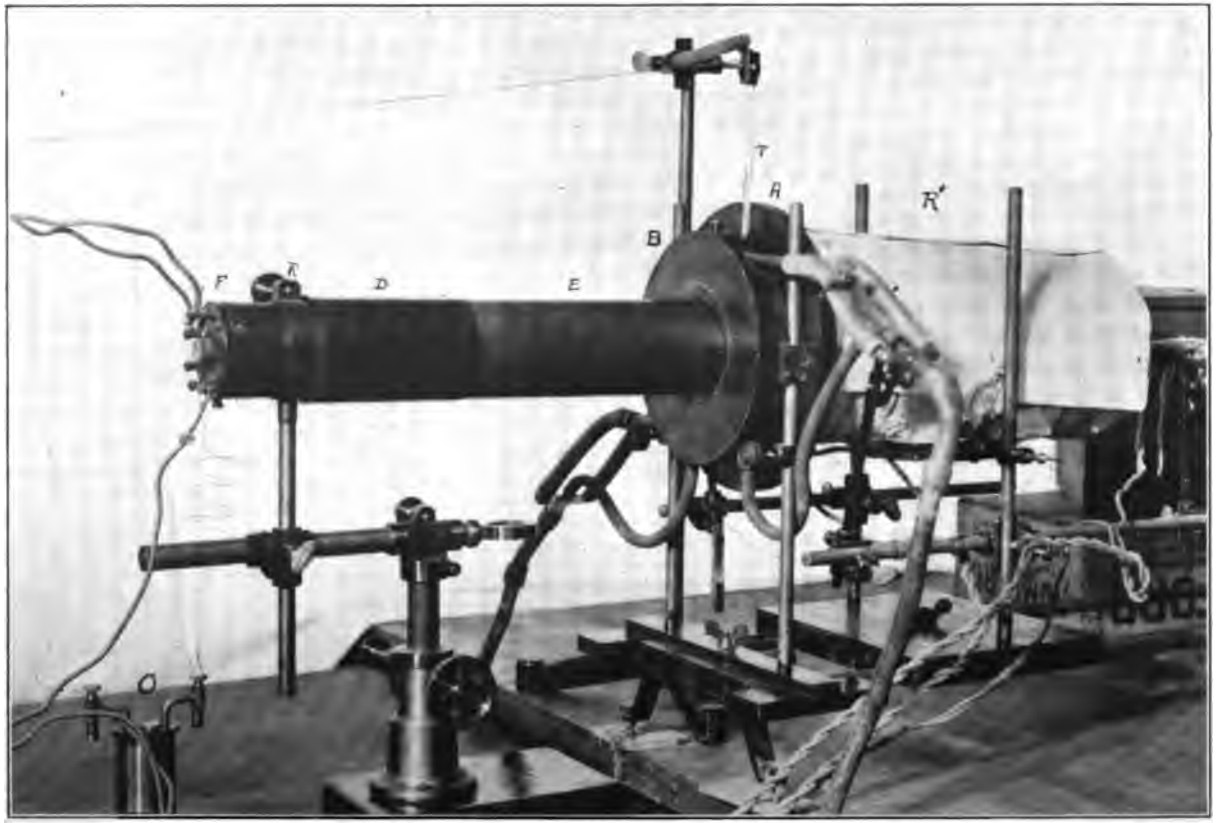


FIG. 6.—*Photograph of the assembled apparatus used in evaluating radiant energy in absolute measure*

be sufficient to add a few of the essential details concerning the method of adjusting the apparatus.

Instead of using an optical bench, the apparatus was kept in alignment and the optical path was varied by means of a series of telescoping brass tubes, *D E*, as shown in Figs. 5 and 6. For this purpose a collar, *C*, was permanently attached to the water-cooled diaphragm, *B*, over which one end of the brass tube, *E* (15 or 30 cm long), was slipped, making a tight-fitting juncture. This tube had diaphragms and, in order to further prevent reflections, the inside surface was smoked in an acetylene flame. Resting within the collar, *C*, and well below the line of the incoming radiations, was a small aluminum box (dimensions 4 by 2 by 1 cm) containing phosphorus pentoxide to absorb the moisture. The whole was quite air-tight, so that the pentoxide would last for several days.

An outer brass tube, *D*, having a guard ring, permitted telescoping the well-diaphragmed radiometer tube, Fig. 1, into the brass tube, *E*, without injuring the lampblack surface of the latter. The wide ring, *R*, which was permanently attached to the radiometer tube insured accurate alignment by always bringing it in contact with the ring on the tube, *D*. The cone of rays coming through the opening in the water-cooled shutter was not sufficiently wide to strike the side of the tube, *E*, and, as described in the previous paper, the radiometer tube was constructed so as to reduce stray radiations to a minimum.

A reference line, *F*, was cut upon the radiometer tube at a distance of 28 mm from the base, and the variable distance, *D′*, between this reference line and the face of the water-cooled shutter, *B*, was measured with a meter stick. The complete distance, *D*, between the surface of the split jaws of the receiver and the water-cooled diaphragm is determined from this variable distance, *D′*, the distance (29.2 mm) from the surface of the water-cooled tank, *B*, to the surface of the water-cooled diaphragm, Fig. 4, *D*, and the distance of the surface of the slit jaws from the base of the receiver. These dimensions are easily measured and they need not be determined with the greatest accuracy. For example, an error of 1 mm would affect the final result by 1 part in 200, when as a matter of experience it is believed that this distance was measured with an accuracy of 0.2 to 0.3 mm or, say, 1 part in 700.

The question of the alignment of the radiometer, the water-cooled diaphragm, and the center of the radiating wall of the black body may be of considerable importance, although experimentally no difference in the radiation constant could be detected by rotating the telescoping tube about the water-cooled shutter as an axis, thus exposing the radiometer to the side wall of the radiator. The alignment of this apparatus was made by illuminating the interior of the radiator by means of an especially constructed Nernst glower, about 5 mm long, placed within the radiator and operated on a transformer, which enabled one to sight upon the center of the radiating wall and then (by changing the focus) upon cross wires temporarily stretched over the outer opening in the radiator. With the telescope thus aligned, the opening in the water-cooled diaphragm was adjusted to its proper position in front of the radiator and centrally on the cross hairs. On placing the radiometer in position in the telescoping tubes and sighting upon its rear face, the axis of the radiometer was found displaced only about 1 mm from the central cross hairs, which was a far more accurate adjustment than was required.

The length of the receiver and the size of the opening of the water-cooled diaphragm were such that when the distance between the receiver and the diaphragm was less than 30 cm, there was a possibility (as shown by geometrical construction) that radiations coming from the edges of the opening in the first diaphragm within the radiator might fall upon the receiver. However, the whole interior for a distance of 8 to 10 cm from the radiating wall is uniformly heated, and no marked difference was found in the values of the radiation constant as determined at very short distances from the water-cooled diaphragm.

7. METHOD OF MAKING OBSERVATIONS

The method of operation is very simple. The receiver is exposed to a source of radiation, say, a uniformily heated cavity or so-called black body (heated to a difinite temperature) and its rise in temperature is noted by reading the deflection produced on the galvanometer needle. The galvanometer sensitivity having been determined, an electric current of sufficient strength is passed through the strip to heat it, and thus cause a deflection which is the same size as that obtained when the strip was exposed to radiation. The energy expended in heating the receiver is determined by measuring the drop in potential between two terminals (which define the length of strip exposed to radiation) and the electric current through the strip. The current is determined by measuring the drop in potential across a standard resistance of 1 ohm, *O*, Fig. 6. These emfs were measured with a Leeds and Northrup type K potentiometer having a range of 1.5 volts and provided with shunts having the following ratios: 1:1, 1:0.1, and 1:0.01.

The complete outfit was designed to be used with the potentiometer and galvanometer equipment which obtains in the average laboratory.

In practice the radiator was heated to a uniform temperature, and the radiometer was exposed to radiation for a few minutes in order to establish temperature uniformity within the connecting apparatus. Two observers made the required readings. The one observer made the galvanometer readings and operated the switch for electrically heating the radiometer. The other observer recorded the galvanometer deflections, maintained a constant temperature in the radiator (which was operated on a storage battery) while radiometric observations were in progress, and made the potential measurements each time the first observer applied a heating current to the radiometer. This insured high accuracy, for sometimes the (portable) storage cell used was found to vary. It was therefore desirable to make the potential measurements for each heating of the receiver.

A complete set of measurements consisted in exposing the receiver from 5 to 10 times to the black body radiations, and noting the galvanometer deflections. The receiver was then heated electrically a similar number of times by closing and opening a knife switch and noting the galvanometer deflections which could be very easily made the same size (to within 1 per cent.) as those obtained by exposing the receiver to the black body. From such a set of measurements the constant of the radiometer (or the constant of radiation) may be determined. However, to make certain of the readings another set of observations was made on the black body. In Table 2 is given a typical set of observations, first on the radiator, then for electrical heating, followed by a repetition of the observations on the black body. When conditions were more unsteady a greater number of galvanometer readings were taken. Usually from three to four such pairs of determinations were made for a given distance between the receiver and the water-cooled diaphragm, the order of heating the receiver, electrically or radiaometrically, being interchanged. This eliminated some of the very small changes in galvanometer sensitivity which (in the course of an hour, the time required for making the measurements) occurred in the galvanometer. The change in galvanometer sensitivity, if any, was easily made apparent by using the same heating current for the three to four pairs of determinations. The appended tables show the precision of the single pairs of determinations, each determination being the mean of the "pair" just mentioned.

Since the receivers were mounted upon a piece of slate there was no chance for appreciable leakage, and none could be detected by insulation tests. The various receivers, after mounting them in front of the thermopile, when heated electrically produced the same deflection on reversal, showing that the thermopile was thoroughly insulated from the receiver.

TABLE 2

Illustrating Observations

[Receiver No. 3; series CXL; temperatures, $t_0=15°$, $t_1=1040.8° C$; voltage=0.010797; amperage=0.046436; resistance in series with the galvanometer=60 ohms; requires 15 seconds to attain maximum deflection.]

Stimulus	Galvanometer deflections
Radiation from black body	15.48 .42 .43 .46 .46 cm
Electrical heating	15.36 .38 .36 .40 .38 M=15.38 cm
Radiation from black body	15.46 .46 .46 .43 .46 Mean=15.45 cm

8. METHOD OF REDUCTION OF DATA

The evaluation of the observed data is an easy matter. The galvanometer deflection produced by electrically heating the receiving strip was found to be proportional, over a considerable range, to the energy input. It is, therefore, an easy task to observe the galvanometer deflection produced by exposing the receiver to the radiator, and to determine the electrical power put into the receiver in order to produce closely (say within 1 per cent) the same deflection. The exact amount of energy necessary to cause the same deflection as that produced by absorbing radiant energy is obtained by multiplying the observed energy input by the ratio of the galvanometer deflections. This gives the "constant" of each receiver. In order to reduce all the measurements to a common basis, and at the same time obtain a value of the coefficient, or the so-called Stefan-Bolzmann constant, of total radiation, the custom of previous experimenters is followed in reducing the present data. For this purpose it is necessary to know the area, A_1, of the water-cooled diaphragm, the area, A_2, of the receiver which is exposed to the radiation emanating from, A_1, the distance, D, between these two surfaces, the absolute temperature, T_1, of the shutter, and the absolute temperature, T_2, of the radiator. The electrical energy input necessary to produce the same (deflection) rise in temperature that was produced by exposing the receiver to the radiator is EI, both of which quantities are measured with the potentiometer as already described. Under these conditions the energy consumed by the electrical stimulus may be equated to the energy emanating from the radiator, or,

$$EI \doteq \frac{\sigma}{\pi}(T_2^4 - T_1^4)\frac{A_1A_2}{D^2} \tag{1}$$

where σ is the coefficient of total radiation, E is the voltage and I is the current.

Usually the distance between the two surfaces A_1 and A_2 are so close that experimenters have had to apply a second term correction. In Gerlach's[15] work the correction applied was

$$\frac{A_1A_2}{D^2}\left(1 - \frac{1}{6}\,\frac{a^2+b^2+a_1^2+b_1^2}{D^2}\right)$$

where the sides of the openings in the rectangular diaphragms were a, b, a_1, b_1. A shorter correction factor is

$$\frac{A_1A_2}{D^2}\left(1 - \frac{1}{4}\,\frac{a^2+b^2}{D^2}\right)$$

---

[15] Gerlach, Ann. Der Phys, (4), 38, p. 1; 1912.

which results from neglecting the effect of the first diaphragm, which is very small in the present work. In either case this second term correction is quite negligible in the present investigation, since it usually amounts to less than 2 parts in 1000.

9. CORRECTIONS FOR DIFFUSE REFLECTION FROM THE RECEIVER

The diffuse reflecting power of lampblack (soot) and electrolytically deposited platinum black, for different spectral regions was previously investigated.[16] However, in view of the different values of the radiation constant obtained with the various receivers, it seemed desirable to make reflectivity measurements on some of the receivers used in the present work for a spectral energy distribution similar to that of a black body at the temperature (1000°) used in this investigation. This question was therefore undertaken anew, using the apparatus and methods previously employed. The main difficulty experienced was in the warming of the sample when exposed to radiation and the consequent reradiation upon the thermopile. This was aggravated by the warming of the glass window (by absorbing long waves) which had to be used to prevent this reradiation falling upon the thermopile. This warming (and cooling) effect would continue for 1.5 minutes, when using radiations which were absorbed by the glass window. On the other hand, using the radiations from an acetylene flame, transmitted by an absorption cell of water 1 cm in thickness and hence but little absorbed by the glass window, the same measurements could be made in 30 to 40 seconds, which was the period for obtaining temperature uniformity in the thermopile.

As a source of radiation having a spectral energy distribution closely that of a black body, a strip of thin nickel, covered with nickel oxide, was used. The coating of oxide was heated to 900 to 1000°.

The thermopile was covered with a glass window 0.88 mm in thickness, which transmits radiations to about 4.5 μ. The absorption cell of water, used in producing more homogeneous radiations, was 1 cm in thickness; and it was opaque to all wave lengths greater than 1.4 μ. The wave length of maximum emission when using such a cell is about 0.95 μ. When this absorption filter was not in the path of the rays the maximum radiation when using an acetylene flame was at 1.2 μ, and when using the nickel strip heated to about 900° the maximum emission was at about 2.4.

---

[16] This Bulletin, 9, p. 283; 1913.

Measurements were made on samples previously investigated and they were found in fair agreement with the values previously obtained. The samples of lampblack Nos. 1 and 2 were painted on the brass blocks previously used, then smoked with soot from a sperm candle. The sample of "Platinum black No. 20" was previously examined, and was found to have a diffuse reflectivity of 1.29 per cent for homogeneous radiations of wave length $\lambda = 4.4\ \mu$.

In order to examine these receivers they were mounted upon the brass blocks used in the previous investigation. The mixture of oxides of cobalt and chromium, mentioned in Table 3, was similar to that used in coating the inside of the radiator. A second sample was painted upon a piece of porcelain and heated in a blast flame. It had a more grayish appearance than that which obtains in the radiator. These samples, and the one of Marquardt porcelain, were examined in order to obtain data for the discussion of the blackness of the radiator.

TABLE 3

Diffuse Reflecting Power of the Receiver

Reflecting surface tested	Acetylene flame		Nickel oxide—no absorption cell, $\lambda_m = 2.4\mu$
	No absorption cell, $\lambda_m = 1.2\mu$	Using cell, $\lambda_m = 0.95\mu$	
	Per cent		
Lampblack paint covered with soot, No. 1	1.26		
Lampblack paint covered with soot, No. 2	1.30		
Platinum black No. 20	1.41	0.83	
Receiver No. 7.—Platinum black, resmoked	1.22		1.65
Receiver No. 13.—Platinum black, smoked	1.27		
Receiver No. 11.—Platinum black	1.31		1.78
Receiver No. 9.—Platinum black, smoked	1.45	1.13	
Mixture of oxides of chromium and cobalt			7.1
Mixture of oxides of chromium and cobalt after heating to 900° C			12.3
Marquardt porcelain			58.5

In applying these corrections for loss by reflection from the radiator it is to be noted that, of the total amount of radiation emitted by a black body at a temperature of 1050° C, only a small part is of wave lengths greater than 4.5 μ, so that even though the reflecting power for these longer wave lengths may be greater (by several parts in 1000) than the values here used, the effect upon the final value is negligibly small in comparison with other experimental errors which are of greater magnitude.

It is of interest to note that there is but very little difference in the reflection from the various receivers, which, of course, one could perceive by comparing them side by side in diffuse daylight. The different values of the radiation constant, obtained with different receivers, is therefore not attributable to a difference in the reflecting power of the absorbing surfaces.

It is to be noted that there is a considerable difference in the observed reflecting power, depending upon the source of radiation. However, as noted above, there is a good reason for believing that this is caused by the absorption of the infra-red rays, of wave lengths greater than 4 μ, which are far more abundant in the spectrum of nickel oxide than in that of acetylene. Any slight warming of the glass window, caused by absorbing these infra-red rays, will affect the thermopile just as obtains in the apparatus used for measuring the constant of radiation, and an erroneous value will be obtained. It is difficult to eliminate this warming effect, especially when using radiations of such heterogeneity as obtains in the nickel-oxide spectrum. This matter was discussed in the previous paper to which reference may be made. The values of the reflecting powers, obtained using homogeneous radiations of wave length $\lambda = 4.4\ \mu$, are in close agreement with those obtained in the present work using the acetylene flame. The correction adopted for loss of energy by reflection from the receivers used in the present investigation is $R = 1.4$ per cent for lampblack and $R = 1.5$ per cent for platinum black. This probably overcorrects rather than undercorrects the observations, but it will assist in eliminating a small absorption by atmospheric water vapor and carbon dioxide for which no correction has been made. From the experiments with receivers Nos. 6 and 10, the difference in the reflecting power of lampblack and platinum black is not as great as here given, the difference being only from 1 to 2 per cent. From the experience gained in the present work, as well as from previous work, it is concluded that the preparation of a good deposit of platinum black is a rather difficult task. The platinum-chloride solutions seem to deteriorate rapidly on standing after being used, which tends to produce grayish deposits. It is therefore desirable to prepare fresh solutions.

Numerous experiments have been made by various observers showing that the reflecting power of lampblack and platinum black is of the order of 2 per cent. Only one recent experimenter is in disagreement, his[17] value being of the order of 18 per cent, which is in agreement with the early observations by Melloni. However, there are good reasons for questioning the experimental procedure. It seems obvious that if, with the crude radiometers obtainable half a century ago, Melloni could find a diffuse reflection of 18 per cent from lampblack, then it should be still easier to prove the existence of such a high reflectivity by the employment of the highly sensitive instruments now available. But as a matter of fact, the modern instruments must be operated at a very high sensitivity in order to detect and to measure the diffuse reflection of lampblack and platinum black, which would not be the case if the diffusely reflected radiation were one-fifth the intensity of the incident radiation.

10. ACCURACY ATTAINABLE

It is difficult to estimate the accuracy attained in the final results. The various receivers (especially Nos. 8 and 9) gave values which have a range of over 3 per cent, although measurements made with a given receiver usually have a range of only about 1 per cent, which is considerably smaller than hitherto attained. It is important to note that the value of the radiation constant is based upon observations with a far greater number of receivers than were heretofore employed. The wide range in values seems to be due to some peculiarity in the receivers, which has not yet been located; for the various recognized sources of error do not account for this variation. Among these possible sources of error are the following:

---

[17] Fery, Compt., Rend., 148, p. 777; 1909.

(1) An incorrect evaluation of the galvanometer deflection. In the preliminary work, an account of which was given elsewhere,[18] the galvanometer was not thoroughly shielded from magnetic perturbations and the tendency was to read the deflection as quickly as possible. The data (Table 4) are therefore not

TABLE 4

Data on the Radiation Constant obtained in 1914

Series	Receiver		Distance	Temperature	Correction applied for reflection	σ		Remarks
	Length	Width				a	b	
	No. 2							
	mm	mm	mm	°C	Per cent			
VIII	24.356	3.192	356.1	922.3	1.4	5.73	5.72	Slit on side and ends.
IX	24.354	3.192	356.1	922.3	....	.72	.73	End jaw slits off.
	No. 1							
XI	23.092	2.150	463.3	896.5	1.3	.71	......	Slit on side.
XII	....	....	316.0	894.3	....	.60	.60	Do.
XIII	....	....	330.0	894.4	....	.73	.71	Do.
XIV	....	....	309.3	880.1	....	.70	.70	Do.
XV	23.092	2.545	310.1	879.5	....	.80	.80	No slit.
	No. 3							
XVI	24.910	4.990	314.0	944.5	1.3	.56	.52	Slit; battery changing.
XVII	....	....	326.8	943.8	....	.51	.52	Slit.
XVIII	....	....	268.3	943.2	....	.56	.49	Slit; battery changing.
XX	24.910	5.051	270.6	867.9	....	.61	.59	No slit; battery changing.
XXI	....	....	309.8	866.8	....	.63	.64	No slit.
XXII	....	....	255.3	867.0	....	.57	......	Do.
XXIII	....	....	255.3	867.0	....	.57	......	Do.
XXIV	24.910	4.992	283.3	881.4	....	.52	.52	Slit; battery changing.
XXV	....	....	307.4	881.5	....	.58	.60	Slit.
XXVI	24.910	4.435	307.4	901.4	....	.73	.73	Slit narrower.
XXVIIa	....	....	324.0	901.5	....	.73	......	Slit; battery changing.
XXVIIb	....	....	281.4	900.6	....	.63	.62	Slit.
	No. 4							
XXVIII	26.565	5.508	292.5	938.0	1.8	.51	.53	Slit; battery changing.
XXIX	....	....	320.5	938.0	....	.49	.52	Slit; battery reset.
XXX	....	....	298.1	937.8	....	.53	.54	Slit.
XXXI	....	....	323.2	937.4	....	.53	.56	Do.
XXXII	26.565	5.536	324.5	938.8	....	.61	.62	No slit.
XXXIII	....	....	324.5	920.9	1.3	.62	.63	No slit; smoked.
XXXIV	....	....	289.3	920.5	....	.63	.64	Do.
XXXV	26.565	5.490	288.5	930.4	....	.57	.55	Slit; smoked.
XXXVI	....	....	320.3	930.0	....	.57	.57	Do.

Mean value $\sigma = 5.61 \times 10^{-12}$ watt cm^{-2} deg^{-4}

---

[18] Coblentz, Phys. Zs., 15, p. 762; 1914.

included in the final result. In the present work there was no need of haste for, as already mentioned, the galvanometer was not subject to disturbances and this possible source of error was eliminated.

(2) Errors may be introduced by improper evaluation of the electric current used in heating the receiver. These heating currents could easily be kept constant to one division on the potentiometer dial, which represented 0.00001 ampere, the actual current used in some cases being as high as 0.06 ampere. Under certain conditions this would represent a variation of about 0.2 mm in the observed galvanometer deflection or an error of 1 part in 500 to 900.

(3) The variations in the reflecting power of the absorbing surfaces, as already mentioned, do not account for the different values of the radiation constant. This was abundantly demonstrated throughout the work. For example, using receiver No. 10, which had an excellent coating of platinum black, the radiation constant differs but little (see series CXV to CXX, Table 5), from the value obtained (see series CXXI to CXXII) after smoking this receiver with soot from a sperm candle. Another example is receiver No. 7 which underwent various treatments (see series LXXXII, LXXXIX, and XCVII). For the latter series this receiver had been given an unusually heavy coating of soot, which easily peeled off. Nevertheless, the radiation constant is practically the same as previously observed.

(4) The effect of the potential terminals was tested by attaching an additional terminal, *c*, Fig. 2, at a distance of about 3 mm from the one used in this work. This test was applied to receivers Nos. 10 and 12. For receiver No. 10, in which the connections were poor, the resistance per millimeter length (0.021145 ohm) was 0.6 per cent less, and for receiver No. 12 it was 0.4 per cent less for the longer distance. Part of this decrease in resistance observed for the longer distance (*a* to *c*) is to be attributed to a lowering of the resistance by the solder at the juncture *b*, Fig. 2, so that the actual error introduced by the uncertainty of the location of the juncture of the potential terminal is probably less than 0.3 per cent. The length of the receiver was defined by

measuring the distance between the point of emergence of the potential wires from the juncture with the receiver.

(5) The width of the receiver exposed to radiation was defined by the distance between the knife edges of the slits. These distances could easily be measured to 0.001 mm by means of a Zeiss comparator. The effect of shadowing different widths along the edges of the receiver (series XXVI to XXVII, also X to XV and LXXXII) was tested, but no certain results were observed. Furthermore, the slit edges could easily be adjusted so that they did not overlap the edges of the receiver by more than 0.1 mm, which is negligible as regards cooling by conduction from the exposed surface. This question was also investigated by Gerlach,[19] who likewise found no effect for overlapping of the edges of the slits and the receiver.

(6) The slits at the side of the receiver might become warmed by exposure to radiation and, in turn, might radiate to the receiver. One test to determine this effect was to cover the opening between the slits with a strip of sheet aluminum 0.5 mm in thickness and exposing it to the radiator, which was heated to about 1000°. The effect of warming, if any, was estimated to be of the order of 1 part in 500 to 1000. From the concordant values obtained with the brass slits (thickness, $t = 0.5$ mm), and the aluminum slits ($t = 1.95$ mm) already mentioned, it seems evident that if there is any warming of the slits it is negligible.

However, it is important to have the receiver properly protected, as is evident from the following test on receiver No. 4, which was about 5.6 mm in width. The slits were removed and a strip of copper 6 mm wide and 0.3 mm in thickness, painted with lampblack, was placed close (1 mm) over the receiver. An exactly similar shield of copper was placed at a distance of about 3 mm in front of this plate and the whole was exposed to the radiator for 10 seconds, after which the shutter was closed. On first exposing this device there was no response until after the lapse of about 10 seconds (shutter was then closed), when there was a small galvanometer deflection which increased rapidly to about 7 mm. at the end of 30 seconds, then decreased to zero at the end of 75 seconds. On removing the lampblack from the front strip of copper (which was not polished), there was but little response at the end of 10 seconds and the maximum deflection was only about 2.5 mm. This was about 1 per cent of the whole deflection for the incident radiation. The slowness of the initial response was due to the warming of the first strip, which then radiated to the second one. It is, of course, possible that some of the radiations incident upon the receiver came from warming of the blackened shields (*C C*, Fig. 2), which were unavoidably exposed to the radiator. The whole, while emphasizing the importance of protecting the receiver, also indicates that when using highly polished surfaces on the front of the knife-edged slits the errors, caused by warming, are negligibly small.

---

[19] Gerlach, Ann der Phys. (4), 38, p. 1; 1912.

(7) An uncertainty of 1° in the temperature measurements at 1000° C represents an error of 0.3 per cent and at 2200° C it represents an error of 0.16 per cent in the radiation constant. On using the radiometer to measure temperatures an error of 1 per cent in the radiation measurements is equivalent to an uncertainty of 3°.2 at 1000° and 6°.2 at 2200° C. While these errors appear to be small, from the nature of the scale reading, it is necessary to keep the temperature of the radiator constant to 0.°1, for it frequently was observed that a variation of 0.°1 in the temperature of the radiator had a perceptible effect upon the galvanometer deflection, which affected the result by perhaps 0.1 per cent. Under certain conditions a change in temperature of 0.°1 produced a change of 0.5 to 1 mm in the galvanometer deflection.

(8) As already mentioned, the uncertainty in measuring the distance between the water-cooled diaphragm and the receiver is negligibly small.

(9) Absorption of radiation by water vapor and carbon dioxide was reduced by having the optical path inclosed and absorbing the moisture with phosphorous pentoxide. That the atmospheric absorption occurs was shown by direct tests, using receiver No. 11. For this purpose measurements were made with and without the brass tubes *D E*, Fig. 5, in place. The first test (series CLV and CLVI), with the optical path (53 cm) constant,

showed that the presence of the tube did not affect the measurements within the errors of observation. These errors were considerably increased by unsteadiness caused by air currents when the tube was not in place. With the aligning tubes in place, but using no drying material, the value of the radiation constant increased from $\sigma = 5.43$ (series CLVI) to $\sigma = 5.5$ (series CLVII and CLVIII) for a decrease of 25 cm in the optical path, or an actual increase of about 1.5 per cent. On the other hand, using drying material to remove the moisture, all the determinations made with this receiver are very much higher (by 3 to 4 per cent) than those observed without the drying material, and the inverse square law holds with greater exactness. It is probable that the values would have been still more constant for different distances if greater care had been taken in drying the air. For example, when in any series of observations the distance is changed by lengthening the optical path, the whole column of air is not free from moisture at the start. It would have been better to have set the apparatus with a long air path and allow it to stand overnight in order to remove the moisture, then make the series of observations by decreasing the air path, i. e., the distance, which would be accomplished by the expulsion of dry air and the maintenance of constant conditions. Further data on atmospheric absorption are given in a subsequent paper.

(10) As already mentioned, black-body conditions within the radiator were obtained by painting the interior of the porcelain tube with cobalt oxide. Tests were made by exposing the receiver to different parts of the interior of the radiator. This was accomplished by tilting the whole radiometric outfit (see Fig. 5) about the water-cooled diaphragm as an axis of rotation, thus exposing the receiver to the side walls of the radiator. This test showed no definite variation in the resulting galvanometer deflection, thus indicating that the interior of the radiator was uniformly heated.

III. EXPERIMENTAL DATA

The data given in the appended tables were obtained with a large assortment of receivers, differently prepared and operated under various conditions as to width and thickness of the metal strip, distance between potential terminals, kind and thickness of the absorbing layer, etc. The receivers Nos. 1, 3, and 5 were constructed of strips of manganin or "therlo," which were from 0.008 to 0.011 mm in thickness. The others were constructed of platinum used for bolometers, the thickness being 0.001 mm or less in thickness. The distance between the heavy copper electrodes was from 30 to 31 mm. In Table 4 data are given showing that the value of the constant is not affected by the distance of the receiver from, or the temperature of, the radiator. In Table 5 the data were obtained under a great variety of working conditions. The individual receivers will now be described.

TABLE 5

Values of the Constant of Radiation as Determined with Various Receivers

[The mean values (uncorrected for reflection from receiver) are based on the data which are free from doubt as to water vapor, blackness of the radiator, stray light resulting from the absence of slits, etc. The doubtful data are marked with an asterisk, thus (*).]

Receiver No. 5.—Diaphragm No. 3; d=3.987 mm

Series	Length of receiver	Width of receiver	Distance	Temperature		Coefficient of radiation $\sigma \times 10^{12}$	Remarks
				t_0	t_1		
	mm	mm	mm	°C	°C		
XLVII..........	18.561	5.640	380.5	25.5	1056.5	5.734	With brass slits; painted and smoked.
					.6	.745	
					.2	.758	
					.1	.731	
XLVIII..........	....	....	457.9	16.5	1056.2	5.749	
					.0	.732	
					5.9	.735	
XLIX..........	....	....	421.1	17.0	1056.1	5.720	
					5.9	.744	
					.9	.743	
					.9	.743	
					4.6	.693	
					.6	.740	
L*..........	18.561	5.662	424.0	17.0	1057.4	5.771	Slits drawn back.
					.4	.793	
					.4	.830	
					.2	.807	
					6.5	.794	
LI*..........	....	....	....	25.0	1056.3	5.827	No slits.
				.3	5.9	.871	
Mean value..	....	....	....	....	....	5.736	

TABLE 5—Continued

Receiver No. 6.—Diaphragm No. 3; d=3.987 mm

Series	Length of receiver	Width of receiver	Distance	Temperature		Coefficient of radiation $\sigma \times 10^{12}$	Remarks
				t_0	t_1		
	mm	mm	mm	°C	°C		
LII	20.061	5.979	422.0	18.3	1057.2	5.504	With brass slits. Pt. black; poor coating.
					6.2	.529	
					.2	.515	
					.2	.582	
LIII*	20.061	6.025	423.9	23.5	1056.3	5.845	No slits
					.2	.883	
LIV	20.061	5.983	422.0	17.5	1059.4	5.678	With slits; smoked. Thermopile is shielded with sheet iron; two layers of paper between. Metal shields are smoked.
					60.0	.672	
					.2	.682	
LV	....	....	402.3	25.5	1060.2	5.648	
					58.8	.706	
LVI	....	....	394.6	10.0	1059.5	5.721	d'Arsonval galvanometer.
					60.0	.696	
					59.5	.696	
LVII	....	....	394.6	10.0	1059.5	5.692	Thomson galvanometer.
					.1	.723	
					8.6	.712	
LVIII	....	....	387.1	13.0	1076.2	5.642	
					8.6	.704	
					8.1	.717	
					7.5	.670	
LIX	....	....	417.7	22.0	1078.3	5.723	
					.2	.709	
					.0	.709	
					.0	.657	
LX	....	....	390.0	11.0	1078.1	5.663	
					.1	.701	
					.1	.696	
					7.8	.659	
LXI	....	....	419.2	11.3	1077.4	5.695	
					.3	.722	
					.3	.740	
					.2	.747	
LXII	....	....	374.9	12.0	1075.6	.645	Rear of furnace is 2° too high.
					.4	.646	
					.4	.645	
Mean value	....	....	....	....	....	5.672	

TABLE 5—Continued

Receiver No. 5.—Diaphragm No. 3; d=3.987 mm

Series	Length of receiver	Width of receiver	Distance	Temperature t_0	Temperature t_1	Coefficient of radiation $\sigma \times 10^{12}$	Remarks
	mm	mm	mm	°C	°C		
LXIII..........	18.561	5.638	399.8	17.5	1069.8	5.730	Brass slits; repainted and smoked. P. D. terminals shellacked and insulated.
					70.6	.696	
					68.7	.742	
					.6	.714	
LXIV..........	....	....	375.5	17.5	1068.2	5.714	
					7.9	.734	
					8.0	.704	
					8.0	.710	
LXV..........	....	....	441.0	19.2	1068.3	5.687	
					7.8	.747	
					7.2	.746	
LXVI..........	....	....	379.5	16.0	1077.9	5.747	
					.9	.721	
LXVII..........	....	....	465.6	17.0	1078.1	5.611	Poor series. Galv. unsteady.
					7.9	.642	
LXVIII..........	....	....	432.5	17.0	1078.0	5.762	
					7.7	.727	
					.0	.751	
LXIX..........	....	....	376.5	17.0	1044.1	5.712	
					38.3	.749	
					36.8	.748	
LXX..........	....	....	375.5	18.4	1072.2	5.687	
				22.5	69.2	.701	
					8.6	.755	
					.3	.745	
LXXI..........	....	....	410.6	22.0	1066.5	5.780	Galv. unsteady; windy.
					.1	.778	
					.0	.755	
Mean value..	....	....	....	....	....	5.725	

Receiver No. 3.—Diaphragm No. 3; d=3.987 mm

LXXII..........	24.910	5.000	377.8	16.5	1053.0	5.603	Aluminum slits reset; receiver resmoked.
				18.0	60.0	.581	
					59.9	.586	
					.3	.581	
LXXIII..........	....	....	424.1	22.0	1058.2	5.577	
					7.3	.564	
LXXIV..........	....	....	417.0	16.5	1068.5	5.628	
					.1	.617	

TABLE 5—Continued

Receiver No. 3—Continued

Series	Length of receiver	Width of receiver	Distance	Temperature		Coefficient of radiation $\sigma \times 10^{12}$	Remarks
				t_0	t_1		
	mm	mm	mm	°C	°C		
LXXV	....	....	442.8	12.0	1080.8	5.675	
					79.9	.625	
					7.7	.598	
					6.7	.677	
LXXVI	....	....	419.9	11.0	1068.3	5.612	
					8.5	.631	
LXXVII	....	....	418.2	13.0	1073.8	5.608	
					2.2	.575	
				12.5	4.7	.642	
					0.4	.624	
					69.7	.616	
LXXVIII	....	....	397.0	12.0	1062.4	5.575	
					54.9	.528	Poor series. Temp. changing.
					2.3	.564	
					46.4	.530	
LXXIX	....	....	397.2	12.0	1067.7	5.662	Excellent series.
					.8	.645	
					.9	.655	
LXXX	....	....	376.1	12.5	1067.6	5.624	
					.5	.645	
					.3	.623	
LXXXI	....	....	459.5	12.6	1066.8	5.613	
					4.2	.596	Temp. suddenly changed.
					.0	.645	
					.2	.627	
Mean value	....	....	....	....	....	5.607	

Receiver No. 1.—Diaphragm No. 3; d=3.987 mm

Series	Length of receiver	Width of receiver	Distance	t_0	t_1	Coefficient of radiation $\sigma \times 10^{12}$	Remarks
LXXXII	22.985	2.537	376.7	14.0	1068.6	5.831	Brass slits. Repainted and smoked.
					.5	.831	
				13.5	.0	.828	
					7.4	.834	
					.1	.838	
LXXXIIa	....	....	395.3	19.0	1065.7	5.751	Poor series. Very unsteady.
Mean value	....	....	....	....	....	5.819	

TABLE 5—Continued.

Receiver No. 7.—Diaphragm No. 3; d=3.987 mm

Series	Length of receiver	Width of receiver	Distance	Temperature		Coefficient of radiation $\sigma\times10^{13}$	Remarks
				t_0	t_1		
	mm	mm	mm	°C	°C		
LXXXIII	24.659	7.952	396.1	11.5	1071.0	5.547	Brass slits. Coated with Pt. black; surface washed with alcohol and lampblack paint; coated with soot.
					69.8	.577	
					.0	.559	
					.5	.563	
				12.0	.1	.565	
					8.8	.579	
					.8	.577	
					.7	.606	
LXXXIV	....	....	391.3	19.7	1068.7	5.622	
					9.1	.598	
					.0	.560	
					.3	.541	
					.2	.559	
					.9	.586	
					.8	.590	
					.2	.580	
LXXXV	....	....	459.5	20.0	1068.3	5.557	Probably slightly low, due to water vaper.
					.2	.574	
					.4	.555	
LXXXVI	....	....	459.5	16.0	1069.3	5.553	
					8.9	.571	
					9.2	.547	
LXXXVII	....	....	428.6	16.0	1069.1	5.561	
					8.7	.551	
					.6	.564	
					.2	.561	
					.1	.577	
LXXXVIII	....	....	391.2	16.0	1067.4	5.621	
					.9	.612	
					.9	.606	
					.7	.588	
LXXXIX	24.659	7.966	391.2	25.2	1073.0	5.578	Painted black and smoked on both sides. Rear shields overlap more. No drying material.
					.6	.597	
XC	....	....	409.2	16.0	1072.3	5.565	Drying material inserted.
					.1	.572	
					.3	.605	
					.3	.619	
					.4	.601	
					.3	.609	
XCI	....	....	474.6	19.6	1068.7	5.588	Good series.
					.5	.591	
					.6	.582	
XCII	....	....	375.8	27.0	1070.4	5.554	Temp. t_0 is high because of slow water circulation.
					.1	.563	

TABLE 5—Continued

Receiver No. 7—Continued

Series	Length of receiver	Width of receiver	Distance	Temperature		Coefficient of radiation $\sigma \times 10^{12}$	Remarks
				t_0	t_1		
	mm	mm	mm	°C	°C		
XCIIb			431.1	27.0	1068.1	5.548	
					7.7	.544	
					.7	.560	
XCIII			453.6	27.0	1065.7	5.550	
					6.5	.523	
XCIV			375.6	11.0	1072.4	5.600	d'Arsenval galv. Vertical tremors.
XCV			400.5	13.0	1071.2	.625	
					.1	.616	
					0.7	.629	
XCVI			400.5	13.0	0.8	5.617	Thomson galv.
					.8	.599	
					.7	.608	
XCVII	24.659	7.964	381.2	17.5	1076.7	5.576	Resmoked; given a thick, soft layer of soot.
					.4	.600	
					.2	.587	
					.1	.561	
XCVIII			433.9	17.0	1075.6	5.590	
					4.8	.594	
Mean value						5.580	
Receiver No. 8.—Diaphragm No. 3; d=3.987 mm							
XCIX	23.989	7.074	378.6	11.2	1075.6	5.876	Brass slits. Coated with Pt. black; washed with sol. of lampblack; smoked.
					.6	.873	
C			404.8	11.5	1075.4	5.946	
					.0	.930	
					4.5	.946	
CI			436.4	11.5	1075.3	5.864	
					6.6	.904	
					.0	.894	
CII	23.989	7.085	377.0	12.0	1074.5	5.880	Resmoked; washed with mixture of alcohol and turpentine; smoked.
					3.3	.885	
					2.8	.878	
CIII			405.0	13.5	1071.2	5.913	
					.0	.915	
					0.9	.900	
CIV			432.4	13.0	1070.3	5.887	
					.1	.848	
					9.9	.849	
					9.4	.870	
CV			376.8	11.5	1076.0	5.791	Galv. unsteady.
					5.4	.856	

TABLE 5—Continued

Receiver No. 8—Continued

Series	Length of receiver	Width of receiver	Distance	Temperature		Coefficient of radiation $\sigma \times 10^{12}$	Remarks
				t_0	t_1		
	mm	mm	mm	°C	°C		
CVI...........	23.989	6.857	379.4	12.5	1075.0	5.934	Resmoked rear surface. Reset metal guard plates at rear. P. D. terminals shellacked. Reset slits farther from receiver. Galv. unsteady.
					0.3	.999	
					68.7	.998	
					.6	6.007	
CVII..........	....	....	379.4	12.0	1068.4	5.949	
					.7	.927	
					9.3	.934	
Mean value..	....	....	....	....	....	5.906	
Receiver No. 9.—Diaphragm No. 3; d=3.987 mm							
CVIII..........	24.917	6.205	380.3	12.0	1072.3	5.794	Brass slits. Strips across ends. Small hole in receiver. Pt. black; painted; smoked.
					.4	.837	
					.3	.855	
					.0	.872	
CIX...........	....	....	413.4	12.0	1072.3	5.921	
					.6	.909	
					.3	.910	
CX............	....	....	396.6	12.5	1072.0	5.903	Good series.
					.1	.889	
					.2	.898	
					.3	.895	
CXI...........	....	....	377.7	13.2	1078.0	5.791	
					.0	.849	
					7.8	.835	
CXII..........	....	....	409.3	13.5	1077.7	5.896	
					.1	.926	
					.3	.924	
Mean value..	....	....	....	....	....	5.877	
Receiver No. 3.—Diaphragm No. 3; d=3.987 mm							
CXIII..........	24.910	5.005	378.6	13.2	1105.5	5.632	Aluminum slits. Receiver resmoked.
					1.5	.628	
					.6	.626	
					.5	.628	
CXIV..........	....	....	413.8	13.2	1098.4	5.638	Distance is questioned.
					7.4	.634	
					.8	.660	
Mean value..	....	....	....	....	....	5.635	

TABLE 5—Continued

Receiver No. 10.—Diaphragm No. 3; d=3.987 mm

Series	Length of receiver	Width of receiver	Distance	Temperature		Coefficient of radiation $\sigma \times 10^{12}$	Remarks
				t_0	t_1		
	mm	mm	mm	°C	°C		
CXV	19.947	6.721	376.4	21.5	1076.9	5.681	Brass slits. Excellent coating of Pt. black. One P. D. terminal has but little solder; other has solder spread out in a thin layer.
					7.0	.688	
CXVI			406.2	14.0	1081.9	5.730	
					.7	.718	
					.3	.757	
CXVII			440.1	14.0	1079.3	5.712	
					.3	.717	
CXVIII			378.0	12.0	1080.1	5.719	
					.0	.705	
CXIX			411.3	13.0	1079.4	5.732	Sensitivity changing.
					.5	.757	
					.0	.779	
					8.8	.752	
CXX			461.3	13.2	1079.5	5.722	
					8.7	.710	
					.3	.739	
CXXI			378.1	13.0	1085.0	5.775	Smoked.
					4.8	.789	
CXXIa			412.8	13.5	1082.8	5.739	
					.4	.756	
					.2	.763	
					.0	.767	
CXXII			442.0	14.0	1084.0	5.682	
					3.6	.734	
					.3	.712	
					2.7	.743	
Mean value						5.734	

Receiver No. 7.—Diaphragm No. 3; d=3.987 mm

CXXIII	24.659	7.959	376.6	13.6	1071.9	5.669	Brass slits. Paint removed; coated with Pt. black; smoked on both sides. Thermopile broken and repaired. Strip is now 1 mm nearer pile.
					2.3	.662	
CXXIV			412.9	13.8	1082.4	5.684	
					.3	.691	
CXXV			472.8	14.0	1083.8	5.717	
					3.0	.693	
CXXVI			448.7	14.5	1082.3	5.688	
					.2	.653	
					.7	.679	
Mean value						5.682	

TABLE 5—Continued.

Receiver No. 3.—Diaphragm No. 3; d=3.987 mm

Series	Length of receiver	Width of receiver	Distance	Temperature		Coefficient of radiation $\sigma \times 10^{12}$	Remarks
				t_0	t_1		
	mm	mm	mm	°C	°C		
CXXVII........	24.910	5.005	376.7	14.0	1079.6	5.618	Aluminum slits. Smoked.
					.3	.600	
					80.6	.633	
					.4	.626	
CXXVIII......	....	....	429.2	15.0	1078.6	5.618	Good series.
					.0	.610	
CXXIX........	....	....	412.5	14.2	1077.6	5.648	
					6.8	.629	
CXL............	21.620	4.953	377.0	15.0	1040.7	5.662	Serial number should be CXXX. Brass slits. Removed paint; repainted; smoked. Reset P. D. terminals. Radiater heating coil burned out; heated on outer coil.
					.7	.666	
					1.1	.654	
CXLI..........	....	....	408.7	15.0	1059.9	5.675	
					60.3	.683	
					.6	.677	
CXLII.........	....	....	471.1	15.0	1062.5	5.613	
					.5	.615	
					.5	.622	
CXLIII........	....	....	444.9	14.0	1063.2	5.649	
					.0	.652	
					2.6	.662	
Mean value..	....	....	....	....	....	5.641	

Receiver No. 11.—Diaphragm No. 3; d=3.987 mm

CXLIV.........	22.389	6.481	376.2	15.5	1077.5	5.649	Slits. Excellent coating of Pt. black. Ends of slits not covered. Radiator heated on outer coil.
					.6	.663	
					.5	.646	
CXLV..........	....	....	410.5	15.0	1076.0	5.670	
					5.5	.698	
					.5	.692	
CXLVI.........	....	....	479.4	15.0	1074.3	5.659	
					3.9	.661	
					.8	.660	
CXLVII........	....	....	450.1	15.5	1075.1	5.658	
					.0	.654	
					4.9	.656	
					.6	.657	

TABLE 5—Continued

Receiver No. 11—Continued

Series	Length of receiver	Width of receiver	Distance	Temperature		Coefficient of radiation $\sigma \times 10^{12}$	Remarks
				t_0	t_1		
	mm	mm	mm	°C	°C		
CXLVIII*			377.0	15.5	1078.0	5.572	Unpainted radiator. Bare front couple; see this Bulletin, 10, p. 34, Fig. 4 A. Heated on outer coil. Ends of slits of receiver covered. Mean value=5.576.
					.1	.566	
					.3	.572	
CXLIX*			419.4	16.0	1078.7	5.592	
					.7	.591	
					.7	.603	
CL*			475.7	16.0	1078.6	5.557	
					.4	.560	
					.2	.565	
CLI*			442.9	16.0	1078.1	5.569	
					7.9	.585	
CLII			379.8	17.0	1082.0	5.610	Painted black body. Heated on outer coil. Couple repaired (broken at junction) and covered as shown in this Bulletin, 10, Fig. 4 B. Mean value=5.622.
					1.2	.620	
					.0	.637	
CLIII			417.6	17.0	1084.0	5.637	
					.0	.625	
					3.7	.627	
CLIV			446.5	17.0	1084.5	5.605	
					.5	.614	
CLV*			523.9	20.0	1075.6	5.377	No inclosing tube to radiometer. Heated by both coils; humid air.
					.9	.479	
					.7	.430	
CLVI*			523.9	20.3	1089.7	5.432	With inclosing tube. No drying material. Mean value=5.467.
					90.0	.424	
					89.9	.453	
					90.0	.435	
					89.5	.453	
CLVII*			272.3	20.5	1088.5	5.479	
					.7	.495	
					.4	.513	
CLVIII*			336.4	20.5	1088.2	5.521	
					7.9	.523	
					.6	.525	
CLIX			376.8	20.0	1076.2	5.623	With drying material. Mean value=5.621.
					.0	.631	
					.0	.624	
CLX			420.8	21.3	1074.1	5.634	
					.0	.609	
					.0	.602	
CLXI			484.2	20.4	1071.5	5.567	Rear of furnace was 4° too high.
					.7	.558	
					.7	.559	
CLXII			446.1	20.8	1071.0	5.581	
					0.8	.596	
					.8	.574	
Mean value						5.629	

TABLE 5—Continued

Receiver No. 13.—Diaphragm No. 3; d=3.987 mm

Series	Length of receiver	Width of receiver	Distance	Temperature		Coefficient of radiation $\sigma \times 10^{12}$	Remarks
				t_0	t_1		
	mm	mm	mm	°C	°C		
CLXIII	21.418	5.340	376.4	21.0	1076.2	5.584	Aluminum slits. Coated with Pt. black; smoked on both sides.
					.2	.582	
					.0	.599	
CLXIV	....	....	424.5	21.5	1075.3	5.596	
					.3	.617	
					.3	.607	
CLXV*	21.418	5.340	480.6	21.0	1076.1	5.561	Low values due to humidity.
					.2	.568	
					.2	.568	
CLXVI	....	....	455.8	21.0	1076.2	5.579	
					.0	.586	
					5.8	.588	
Mean value	....	....	....	....	....	5.593	

Receiver No. 12.—Diaphragm No. 3; d=3.987 mm

CLXVII	22.385	6.365	376.7	21.0	1078.3	5.651	Brass slits. Poor deposit of Pt. black; smoked on both sides.
					7.4	.660	
					.1	.657	
CLXVIII	....	....	418.6	21.0	1075.9	5.687	
					6.4	.675	
					6.6	.679	
CLXIX	....	....	486.2	20.0	1077.8	5.656	
					8.1	.671	
					.3	.668	
CLXX	....	....	448.3	20.0	1078.0	5.699	
					7.9	.686	
					.8	.688	
CLXXI	22.385	6.326	377.1	21.5	1075.3	5.605	Aluminum slits. Soot brushed off and resmoked.
					.6	.614	
					.5	.631	
					.6	.624	
CLXXII	....	....	478.8	21.5	1085.3	5.626	
					.1	.608	
					4.9	.604	
CLXXIII	....	....	440.0	21.4	1084.5	5.623	
					.8	.621	
					.7	.620	
CLXXIV	....	....	401.5	215.	1084.8	5.660	
					.7	.651	
					.4	.640	
Mean value	....	....	....	....	....	5.647	

RECEIVER No. 1 was made of "therlo," 0.007 to 0.008 mm in thickness, painted with lampblack and smoked with soot from a sperm candle; potential terminals 0.025 mm diameter; aluminum slits 2 mm in thickness. For a subsequent series of measurements LXXXII, Table 5, the paint was removed, the potential terminals were reset, the strip was repainted, and brass slits 0.5 mm in thickness were used. The strip being narrow the sensitivity is greatly reduced, which increases the errors of observation. For this reason the apparently higher values of the radiation constant, when using a very narrow receiver, are not considered to be attributable to the narrow slits. However, the error due to shadowing of the receiver by the jaw slits would be larger for a narrow strip than for a wide one, thus causing a higher value.

RECEIVER No. 2 was made of platinum less than 0.001 mm in thickness, covered electrolytically with platinum black. The potential wires were of platinum 0.0055 mm in diameter. The aluminum slits were 2 mm in thickness, and the edges were about 1.5 mm away from the receiver.

RECEIVER No. 3 was made of "therlo"; thickness about 0.008 mm, painted with lampblack and smoked. The slits were the ones of aluminum used with the preceding receivers. Subsequently the receiver was resmoked (the dust having been brushed off with a fine, especially prepared brush) and the measurements were repeated (series LXXII to LXXXI). To check the measurements on receivers No. 7 and No. 9, further measurements (CXIII to CXIV and CXXVII to CXXIX) were made. On examination with the rear metal shields removed, it appeared as though these aluminum slits had not overlapped sufficiently, which might have produced a low value. The work was therefore repeated (CXL to CXLII) after having removed the old paint, reset the potential terminals (to eliminate a possible conduction from the ends), repainted and resmoked the receiver, and attached the brass slits which were used with the other receivers. A slightly higher value of the constant was then found.

RECEIVER No. 4 was made of platinum. It had an excellent coating of platinum black. The potential terminals, of platinum 0.025 mm in thickness, lay flat upon the receiver for a length about 1.5 mm instead of making contact at the ends of the potential wires. In determining the distance between the potential terminals an error of 1.5 to 2 per cent was inadvertently made. The distance should have been measured at the point where the wire is free from the receiver instead of along the central axis of the receiver. This receiver was used in various subsidiary tests (e. g., on the effect of using water-cooled diaphragms having different openings) where the measurements were relative, and, because of the defect in the potential terminals, it is not used in considering the absolute measurements.

When using such a receiver without the jaw slits it is difficult to define the effective width exposed to radiation. For example, the mean value of the width of this strip before blackening, with platinum black, was 5.515 mm and after blackening the width was 5.536 mm. After smoking this receiver the width was 0.05 mm greater than before smoking. A similar example is receiver No. 6, which was 0.06 mm narrower after wiping the soot off the edges.

RECEIVER NO. 5 was made of manganin, 0.011 mm in thickness, which was painted with lampblack and smoked. The slit jaws were of brass, 0.5 mm in thickness, situated at a distance of 1.3 mm from the receiver. This particular set of brass slits (previously used) was used with the various receivers subsequently investigated. The potential terminals were of platinum wires, 0.025 mm in diameter, the tips of which were soldered to the strip of metal. This method was used in attaching the potential leads to all of the receivers subsequently constructed.

For series XLVII to LI, each shield *C C*, Fig. 2, overlapped the receiver by about 2.5 mm. After repainting and smoking the receiver the brass slits were reset and the shields, *C C*, overlapped the edges only about 1 mm. However, there is no marked difference in the results. (See series LXIII to LXXI.)

RECEIVER NO. 6 was made of platinum, with potential leads 0.025 mm in thickness and brass slit jaws. The shields *C C*, Fig. 2, at the rear of the receiver were of sheet iron, 0.3 mm in thickness, folded double, with a layer of thick paper intervening.

Measurements were made with this receiver when covered with platinum black (series LII to LIII), and when smoked and soot removed from the edges (series LIV to LXII); with a Thomson galvanometer and with a d'Arsonval galvanometer (series LVI); with and without slits. The results show that there is but little difference between the diffuse reflecting power of lampblack and platinum black. Similar tests were made with receiver No. 10, giving results in agreement with the above data.

Receiver No. 7 was made of platinum, with potential leads 0.025 mm in diameter and brass slits. The coating (deposited four minutes) of platinum black was very poor. The front surface was therefore washed with a very dilute solution of lampblack in alcohol. This made an excellent surface, which was then given a light coating of soot from a sperm candle. (See series LXXXIII to LXXXVIII.) The blackening being unsymmetrical, both surfaces of the receiver were given a thicker coat of lampblack and then smoked (series LXXXIX to XCVI).

In order to determine whether a still thicker coating of soot would affect the value of the radiation constant, the front surface of the receiver was given an additional coating of soot, which was deposited in a thick soft layer which could be easily brushed off in large patches (series XCVII to XCVIII). The thick layer of soot has no marked effect upon the results obtained. The supposed effect of such a thick coating of soot on the front surface is to reduce the value of the constant. This is due to the fact that in the electrical heating a smaller energy input would be required when the surface layers are unsymmetrical and the rear one is the thinner. A d'Arsonval galvanometer was used (series XCIV–XCV) with no marked effect upon the results. The soot and paint were then removed and an attempt made to blacken the surface with platinum black. The surfaces were rough and not very black, so that both sides were smoked (series CXXIII to CXXVI). The value of the constant is perhaps 1.0 to 1.5 per cent larger, instead of being smaller than that previously observed. This last blackening of the receiver increased the heat capacity to such an extent that it required 35 seconds to establish temperature equilibrium after exposure to radiation. It may be added that all of these platinum receivers attained temperature in 8 to 10 seconds, and that an exposure to radiation for 15 seconds was necessary because the thermopile required that length of time to attain temperature equilibrium.

RECEIVER No. 8 was made of platinum which was heated electrically in order to anneal it and especially to clean the surfaces which had a discolored appearance. In the annealing process the solder at the ends alloyed with the platinum strip, which then sagged considerably. The potential terminals were of platinum 0.025 mm in diameter, and the slits were of brass. The surface of the platinum was discolored and did not take a good deposit of platinum black. A thin coating of lampblack paint was therefore applied to both surfaces, which were then smoked (series XCIX–CI). The surfaces were resmoked and painted with a mixture of alcohol and turpentine, after which they were smoked (series CII–CV). This did not change the high value of the radiation constant. The slits were reset, being placed at a greater distance (1.8 mm instead of about 1.3 mm) from the receiver. The rear surface of the receiver was resmoked, which should increase the value of the constant; the potential wires were given an additional coating of shellac, and the guard plates at the rear of the receiver were reset. In spite of all of these manipulations the value of the constant (series CVI–CVII) remained the same, and no clue has been found to account for the high values.

RECEIVER No. 9 was constructed of platinum, which was annealed as was No. 8. The potential terminals were of platinum 0.025 mm in diameter, and brass slits were used. The alloying with the solder made the ends brittle, one end of the receiver having a small hole near the copper electrode. Like receiver No. 8 this receiver sagged as a result of this alloying with the solder. The surfaces of both of these receivers were wavy and uneven, and slightly curved at the edges. The surfaces did not take an even coating of platinum black. They were, therefore, given a thin coating of lampblack paint and then smoked. The value of the radiation constant is high as obtained with receiver No. 8 (series CVIII to CXII). To show that this value was due to something connected with these receivers, the observations were continued, using receiver No. 3 (series CXIII to CXIV), which gave its customary value of the constant which represents more nearly the "constant of the receiver" than the "constant of radiation." The general appearance of receivers 8 and 9 was such that one would hardly use them in accurate determinations.

Receiver No. 10 was constructed of platinum, which was cleaned by making it the anode in depositing platinum black upon a strip of platinum. A freshly prepared solution of platinum chloride was used, which gave an excellent (six minutes) deposit of platinum black

Three potential leads (*a*, *b*, *c*, Fig. 2) were applied, the extra one being used to test the effect of the potential terminals. The terminal wires were of platinum 0.01 mm in diameter and about 3 mm long, attached to similar wires 0.025 mm diameter. After attaching the wires to the receiver the diameter of the soldered connection was: $a = 0.1$ mm, $b = 0.08$ mm, $c = 0.06$ mm. Unfortunately two of these connections (*a*, *b*) became detached (by the solution) while cleaning and blackening the receiver. They were reattached by soldering them upon the platinum black, which caused the connection *a* to be somewhat larger in diameter than that just quoted.

The brass slits of previous experiments were used (series CXV–CXX) and to test the loss by reflection the receiver was smoked (series CXXI–CXXII) without disturbing the slits, the edges of which were covered to prevent them from becoming blackened with soot. The value of the radiation constant is somewhat higher, corresponding with the smaller reflecting power of soot.

The distance between the potential terminals *a–b* was 19.947 mm and from *a–c* it was 22.157 mm. The resistance per millimeter length between the terminals $a\text{–}b = 0.021145$ ohm and $a\text{–}c = 0.021020$ ohm, or an apparent decrease of about 0.6 per cent for the longer distance.

Receiver No. 11 was constructed of platinum. Before blackening it the surface was cleaned electrolytically by using the receiver as an anode, then dipping it in hydrochloric acid, which removed a whitish tarnish that is frequently found on platinum from which the silver has been removed. The platinum black was deposited for 6.5 minutes in a freshly prepared solution. It was the most uniform deposit, and in all appearances the best coat of platinum black in the whole series. At the completion of the work its reflecting power was determined as described elsewhere in this paper.

The potential terminals of platinum 0.01 mm in diameter (as described under receiver No. 10) were attached after cleaning the receiver and before applying the platinum black. The two soldered connections with the receiver were only 0.072 and 0.055 mm in diameter.

This receiver was used (with brass slits) in determining the radiation of a blackened porcelain radiator (CXLIV to CXLVII) and also of an unpainted radiator (CXLVIII to CLI). It was used also in determining the effect of moisture (CLV to CLVIII), which appears to produce a definite lowering of the value of the constant.

For the series of observations extending from CXLIV to CLIV the radiator was heated by the outer heating coil, which no doubt produced a different temperature distribution within the radiator. If this had any effect upon the constant, it was not appreciable.

RECEIVER No. 12 was constructed of platinum cleaned in hydrochloric acid. There were three potential terminals, of platinum wire 0.01 mm in diameter, to determine the effect of the terminal. The diameters of the soldered connections were: $a = 0.07$ mm, $b = 0.11$ mm, $c = 0.90$ mm.

The platinum black was deposited for 6.5 minutes. The front surface had a brownish appearance and the rear surface had a grayish appearance. Hence, both surfaces were smoked with a thin coat of soot from a sperm candle. Brass slits (series CLXVII to CLXX) and aluminum slits (series CLXXI to CLXXIV) were used, but in view of the fact that for the latter series the soot had been brushed off the front surface of the receiver, which was then resmoked, the difference of about 1 per cent in the radiation constant is probably as much attributable to the smoking as to the different slits. The distance between the potential terminals a to b was 22.385 mm and from a to c it was 24.740 mm. The resistance per millimeter length between the terminals $a-b = 0.020486$ ohm and $a-c = 0.020400$ ohm, or a decrease of about 0.4 per cent for the longer distance.

RECEIVER NO. 13 was constructed of platinum cleaned in hydrochloric acid. The platinum black was deposited 6.5 minutes and the surfaces not being uniformly black, especially on the rear side, both sides were given a thin coat of lampblack soot from a sperm candle. The total length of this receiver was 31 mm, the potential terminals, of platinum 0.01 mm in diameter, being separated 21.418 mm. The soldered contacts of the potential terminals were 0.05 mm in diameter. Aluminum slits were used. The value of the radiation constant (series CLXIII to CLXVI) is practically the same as that obtained with a majority of the other receivers. It is probably somewhat too low, due to incomplete removal of atmospheric water vapor.

A summary of the results obtained with the various receivers is given in Table 6.

IV. SUMMARY

The present paper gives the results of an investigation of the behavior of a bismuth-silver thermopile suitably modified to measure radiant energy in absolute value. Instead of exposing the thermopile directly to the incident radiation a blackened metal strip intervenes. This metal strip functions (1) as a receiver for absorbing radiant energy; (2) as a source of radiation (by heating it electrically) which can be evaluated in absolute measure, and by using a constant current for heating the strip; (3) as a standard source of radiation for testing the sensitivity of the radiometer, which includes both galvanometer and thermopile.

The present investigation pertains to 13 receivers, made of manganin, "therlo," and platinum, differing in width from 2.5 to 8 mm, and in thickness from less than 0.001 mm for platinum to 0.011 mm for manganin. The manganin and "therlo" receivers were painted with a thick coat of lampblack, then smoked. The platinum receivers were covered with platinum black and afterwards smoked. In this manner (the same values being obtained in the two cases) it was shown that there is but little difference in the reflecting power of these two kinds of absorbing surfaces. (See receiver No. 10, Table 6.)

TABLE 6

Summary of the Dimensions of the Receivers and of the Slits, the Kind of Absorbing Surfaces, and the Results Obtained with Each Receiver

[The value of the radiation constant for each receiver (uncorrected for diffuse reflection) is the mean of the values not marked with an asterisk (*) in Table 5. Receivers 1, 3, and 5 are of manganin or "therlo"; the others are of platinum.]

Receiver No.	Length between potential terminals	Width of strip	Width of slit	Serial number of test	Value of radiation constant $\sigma \times 10^{12}$	Remarks
	mm	mm	mm			
1........	23.095	2.545	2.150	X to XIV (inc.)..........	5.60	Repainted and smoked. Aluminum slits.
	22.985	....	2.537	LXXXII.................	5.819	Repainted and smoked. Brass slits.
2........	24.354	3.584	3.192	IV to IX.................	5.67	Platinum black. Aluminum slits.
3........	24.910	5.035	4.990	XVI to XXV.............	5.50	Painted and smoked. Aluminum slits.
			5.000	LXXII to LXXXI........	5.607	Resmoked.
	21.620	....	4.953	CXL to CXLIII..........	5.653	Repainted and smoked. Brass slits.
5........	18.561	5.667	5.640	XLVII to XLIX..........	5.736	Painted and smoked. Brass slits.
			5.638	LXIII to LXXI...........	5.725	Repainted and smoked.
6........	20.061	6.025	5.979	LII.......................	5.533	Platinum black. Brass slits.
			5.983	LIV to LXII.............	5.691	Platinum black. Smoked.
7........	24.659	7.965	7.952	LXXXIII to LXXXVIII..	5.574	Platinum black; painted; smoked on front. Brass slits.
			7.966	LXXXIX to XCVI.......	5.586	Repainted; smoked on both sides.
			7.964	XCVII to XCVIII........	5.585	Front resmoked, thick layer.
			7.959	CXXIII to CXXVI......	5.682	Paint removed; coated with Pt. black; smoked on both sides.
8........	23.989	7.10	7.074	XCIX to CV.............	5.885	Platinum black; painted; smoked. Brass slits.
			6.857	CVI to CVII.............	5.964	Resmoked rear surface.
9........	24.917	6.25	6.205	CVIII to CXII...........	5.877	Platinum black; painted; smoked. Brass slits.
10........	19.947	6.8	6.721	CXV to CXX............	5.726	Platinum black, excellent coat. Brass slits.
				CXXI to CXXII..........	5.746	Platinum black. Smoked.
11........	22.389	6.5	6.481	CXLIV to CXLVII......	5.663	Platinum black, best coat of all. Slits.
				CXLVIII to CLI.........	5.576	Unblackened radiator.
12........	22.385	6.4	6.365	CLXVII to CLXX.......	5.673	Platinum black; smoked on both sides. Brass slits.
			6.326	CLXXI to CLXXIV......	5.625	Platinum black. Resmoked. Aluminum slits.
13........	21.418	5.5	5.340	CLXIII to CLXVI.......	5.593	Platinum black; smoked on both sides. Aluminum slits.

The method of operation is unsymmetrical in that when the receiver is exposed to radiation the heating is produced in the lampblack surface, while in passing an electrical current through the strip the heat is generated within the receiver. However, from the data obtained with receivers differing 10 times in thickness, and covered with different kinds and thicknesses of absorbing material, it appears that the manner of heating the receiver has but little effect upon the final result.

For any one receiver, operated under different conditions, the precision attained is usually much better than 1 per cent. For the different receivers the maximum range in the value of σ, which was caused by two receivers giving high values, is of the order of 3.5 to 4 per cent. Excluding these two receivers (Nos. 8 and 9), the range of values for the different receivers is of the order of 1.5 to 2 per cent. This seems to be independent of the length and width of the receiver, and of the kind of slits used. The accuracy attained with this method of evaluating energy in absolute measure, as estimated by the departure of individual determinations from the mean value, appears to be of the order of 1 per cent. To this extent one can consider the present device a primary instrument for evaluating radiant energy in absolute measure.

The device commends itself as an instrument of precision because of its quickness of action, its freedom of surrounding conditions, its high sensitivity, and its simplicity of operation. It can be designed to suit the equipment of the average laboratory. The device can be much simplified and used as a radiation pyrometer. In practice it is advisable to calibrate the receiver by exposing it to a black body heated to about 1000° when it is desired to make refined radiation measurements. The loss of energy by diffuse reflection from the blackened surface of the receiver was investigated and it was found that there was but little difference in the reflecting power of the different surfaces examined.

In view of the fact that this instrument has proved to be applicable for precise measurements, the complete radiometric outfit, including the radiator, has been placed in an evacuated inclosure. The radiation measurements in absolute value will be made at certain fixed temperature points as defined by the melting points of metals such as gold, silver, palladium, and platinum. In this manner it is hoped to eliminate the effect of atmospheric absorption and otherwise improve in the reliability of the measurements.

WASHINGTON, July 2, 1915.

PRESENT STATUS OF THE DETERMINATION OF THE CONSTANT OF TOTAL RADIATION FROM A BLACK BODY

By W. W. Coblentz

CONTENTS

Page.
I. Historical summary 553
  1. Introduction 553
  2. The bolometric method of electric compensation 554
  3. The thermometric method with "black" receivers 560
  4. Indirect and substitution methods 564
II. The present determination of this constant 568
  1. Correction for lack of blackness of the radiator 570
  2. Correction for atmospheric absorption 573
  3. Summary 577
NOTE I: On standards of radiation 580

I. HISTORICAL SUMMARY

INTRODUCTION

In the present summary of the work on radiation constants it seems desirable to inquire rather critically into the various methods employed in order to find the causes of the discrepancies in the results obtained. The past few years have been very productive of researches, employing a variety of methods for determining the constant, σ, of total radiation.[1] Each method has some defect, but in some cases this is negligible in comparison with the accidental errors of experimentation. The evaluation of radiant energy in absolute measure is accomplished by substitution methods; and the fault to be found with the various methods thus far employed is that they are unsymmetrical in their application. What is needed is some sort of calorimeter in which the energy recovered can be compared with the energy supplied. The Abbot pyrheliometer, with proper modifications, seems the most feasible method; or a bolometer fashioned somewhat after the methods employed by Puccianti so that it can be used also as the inner parts of a thermometer, thus making the two methods interchangeable. In view of the fact that some of these suggestions are modifications of previous methods which are in the process of development by the writer, and have not been thoroughly tested as to their general applicability, no further comment need be made thereon.

---

[1] The Stefan-Boltzmann law states that the total radiation emanating from a uniformly heated enclosure, or so-called black body, is proportional to the fourth power of the absolute temperature. That is to say, the total radiation emitted is $R = \sigma T^4$, where σ is the coefficient or "Stefan-Boltzmann constant" of radiation. The numerical value of this coefficient is of the order $\sigma = 5.7 \times 10^{-12}$ watt per cm^2 per deg^4. In the text only the numerical value is quoted, e. g., $\sigma = 5.32$.

It will be noticed that in many experiments the radiators were operated at the temperatures which were too low to properly eliminate the correction for temperature (radiation) of the shutters, diaphragms, etc.; and also for the losses by air conduction in the receiver. On the other hand, some experimenters used radiators operated at a high temperature, thus obviating these difficulties; but then the receiver was a sluggishly-acting instrument which could not be properly calibrated. To Féry is due the credit for abandoning the idea of using a radiator heated to 100° C., and adopting a radiator heated to 1000° C. or higher. In this manner the radiation to be measured is far in excess of the small disturbing factors, such as radiation from shutters, diaphragms, etc.

THE BOLOMETRIC METHOD OF ELECTRIC COMPENSATION

Kurlbaum[2] was the first to employ a method for determining the constant of total radiation which is free from the gross experimental errors that are inherent in the work.

The principle of the method is as follows: Three branches of a Wheatstone bridge (bolometer) consist of thick manganin wires which are not affected by a change in current through the bridge. The fourth branch consists of thin bolometer platinum, the resistance of which is affected by a change in current in the bridge. Starting with the bridge balanced, the bolometer branch of thin platinum is exposed to the radiation from a black body at 100° C, and the change in resistance (or galvanometer deflection) is noted. With the bolometer branch shielded from radiation the bridge current is varied until the change in resistance in the bolometer branch is equal to that attained when exposed to radiation. In other words, the bridge current is increased by a sufficient amount to produce the same galvanometer deflection as was caused by radiation falling on the bolometer strip. From a knowledge of the change in the bridge current, the bolometer resistance, etc., Kurlbaum was enabled to compute the energy input. He used two very different surface bolometers and found very concordant values. He found a value of $\sigma = \mathbf{5.32}$. In a recent communication[3] he makes a correction of 2.5 per cent for loss by reflection from the platinum black surfaces of the bolometer. This gives a value of $\sigma = \mathbf{5.45}$. Recently the reflecting power of platinum black[4] has been examined, and it was found that, using the blackest obtainable deposits, the reflecting power is almost 2 per cent for wave lengths at 8 to 9 μ. An examination of six surface bolometers (12 branches or 24 surfaces of about 3 by 4 cm area), which were recently purchased abroad, showed that all of them had microscopically small bright patches of bare platinum, and that none of them were as black as the samples examined. From this it appears that Kurlbaum's correction to his work is none too large.

---

[2] Kurlbaum, Wied. Ann., 65, p. 746; 1898.

Valentiner[5] undertook the work anew and found quite different values, depending upon the blackness (kind) of radiators employed, which were heated as high as 1450°. For the two blackest radiators (the steam-heated radiator "W. S. K" and the electrically heated unblackened porcelain tube radiator "G. S. K") he found a value of $\sigma = \mathbf{5.36}$.

In a note replying[6] to Paschen's criticisms of the bolometric method he made a correction of about 4 per cent for various causes (e. g., 1 per cent for lack of blackness of the radiator, 2.5 per cent for reflection from the bolometer, etc.) which gives him a value of $\sigma = \mathbf{5.58}$.

It is rather singular that the correction for atmospheric absorption is entirely overlooked by all those who entered into the, rather controversal, discussion of these measurements. The numerical values of the various determinations made by Valentiner, upon any one radiator, have a range of about 3 per cent, which was usually higher than allowable in order to ascertain the effect of atmospheric absorption in a space of 30 to 50 cm. However, when using a larger porcelain tube radiator [7], "G. S. K.," the distances were rather longer than usual, being, respectively, 89 and 125 cm, and the effect of absorption seems unmistakable. The value of the radiation constant is about 2 per cent smaller for the greater distance, i. e., an increase of 35 cm in optical path introduced an absorption of about 2 per cent. It is difficult to say how great a correction should be applied to Valentiner's values in order to eliminate atmospheric absorption. The observations were made in the winter, when the humidity was, no doubt, much lower than for the tests given on a subsequent page, where an absorption of over 5 per cent was found in an air path of 124 cm, the temperatures of the radiator being of the same order as used by Valentiner.

---

[3] Kurlbaum, Verh. Phys. Gesell., **14**, p. 576; 1912.

[4] Coblentz, J. Franklin Institute, p. 549, Nov.; 1912. This Bulletin, **9**, p. 283; 1913.

[5] Valentiner, Ann. der Phys., (4), **31**, p. 272; 1910.

[6] Valentiner, Ann. der Phys., (4), **39**: p. 489, 1912; **41**, p. 1059; 1913.

From the experiments to be described presently, it appears that the correction of 1 per cent for lack of blackness of the radiator seems to be well taken, while the correction of 2.5 per cent for reflection from the bolometer is probably 1 per cent too high. For the steam-heated radiator "W. S. K." the correction for atmospheric absorption would, no doubt, be higher than for the porcelain radiator "G. S. K." A conservative estimate of the total correction to Valentiner's original data is 5 to 6 per cent, of which amount about 3 per cent is the estimated absorption by atmospheric water vapor and carbon dioxide. This increases Valentiner's value to $\sigma = 5.65$ with a possibility of the value being as high as $\sigma = 5.7$, which is in remarkably close agreement with the various determinations made by other methods.

The following method is not exactly bolometric in principle but it may as well be considered here since its virtues have been under rather animated discussion in connection with the results obtained by the bolometric methods just mentioned. Gerlach [8] working in the laboratory of Prof. Paschen has recently determined the constant of total radiation, σ, by a modification of Ångström's electric compensation pyrheliometer. In the original pyrheliometer of Ångström[9] there are two thin, narrow, sheets of manganin to each of which is attached one junction of a thermoelement which is joined through a galvanometer. One of these manganin strips is exposed to solar radiation. Through the other manganin strip is passed an electric current of such strength that the manganin strip is heated to the same temperature as is the strip which is exposed to solar radiation. The equality of temperature is indicated when there is no current flowing through the galvanometer. Various instruments which have been tested show wide variations (15 to 30 per cent) in the value of the solar constant. Recent improvements, however, have produced a more reliable instrument.

---

[7] Valentiner, Ann. der Phys., (4), **31**, p. 304; 1910.

[8] Gerlach, Ann. der Phys., (4), **38**, p. 1; 1912.

In the instrument as used by Gerlach there is but one strip of manganin which is 0.01 mm in thickness, and 2 to 4 mm wide (length 2.7 to 3 cm). At the back of this manganin strip and close to it was a thermopile of 45 elements (joined through a galvanometer) which is heated by radiation from the manganin strip, thus differing from Ångström's device in which a single thermojunction was heated by conduction from the manganin strip. The idea of using many junctions was to integrate over the whole strip, thus eliminating any irregular heating due to inequalities in the thickness of the manganin strip which was blackened electrolytically with platinum black. This manganin strip was heated electrically to the same temperature as that attained by the strip when exposed to the radiation from a black body, which was heated by steam. From a knowledge of the resistance of the strip and the electric current it was possible to determine the watt input, and hence the value of the radiation constant.

Gerlach experienced some difficulty in determining when he had exact compensation when heating the receiver electrically and radiometrically. Covering the sides of the receiver with knife-edged slits had no effect upon the radiation constant; but shielding the ends of the manganin strip from radiation caused the value of σ to increase to a constant value, when the shields covered 1.5 mm. or more of the ends of the strips. This is caused by heat conduction from the receiver to the heavy copper electrodes. In practice he exposed the whole length of the receiver to radiation, claiming that the heat conducted from the ends is the same when the metal strip is heated radiometrically and electrically. In the instrument used in the first part of the present investigation[10] this difficulty was eliminated by placing the potential terminals upon the receiver, and at some distance from the heavy electrodes.

---

[9] Ångström, Phys. Rev., 1, p. 365; 1893. Ann. der Phys., (3), 67, p. 663; 1899.

The mean value obtained by Gerlach is $\sigma = 5.803$. He made no correction for losses by reflection (2 per cent). This was done by Paschen,[11] who gives the value of $\sigma = 5.9$.

Gerlach's value is based upon the determinations made with four receivers, the individual values of which depend somewhat upon the receiver, as was observed in the present work. No attempt was made to remove the atmospheric water vapor from the space between the radiator and the receiver. There is a systematic decrease of 1 to 2 per cent in the radiation constant for an increase of 32 to 42 cm, in the distance of the receiver from the radiator. An appreciable absorption by atmospheric water vapor is to be expected in view of the fact that the radiator was heated to only 100° C, which produced radiations easily absorbed by water vapor.

The work of Gerlach aroused considerable discussion among experimenters previously engaged on this same problem. This brought forth a very considerable amount of laborious experimental work by Gerlach[12] in defense of his method, in which he showed that his apparatus gave the same values as when he operated it by the bolometric method employed by Kurlbaum.

Puccianti[13] proceeded to determine the constant of total radiation by a method which is just the opposite from the usual mode of operation; and, because of the small radiant energy exchanges involved, in comparison with the transfer of energy by heat conduction and by convection, this may be the weak point in his method.

---

[10] This Bulletin, 12, p. 503; 1916.

[11] Pashen, Ann. der Phys., (4), 38, p. 30; 1912.

[12] Gerlach, Ann. der Phys., (4), 40, p. 701; 41, p. 99; 42, pp. 1163, 1167; 1913.

[13] Puccianti, Nuovo Cimento, (6) 4, p., 31; 1912.

He constructed a bolometer in the form of a black body, which is kept at room temperature. This black body really is the radiator. The other black body (which is really the receiver), instead of being at a higher temperature, as usually is the case, is at the temperature of carbon dioxide snow or of liquid air. He measured the electric power which had to be supplied to the first black body to keep the temperature constant in order to compensate for the energy lost by radiation to the second black body. He very ingeniously constructed two black-body bolometer branches exactly alike, the one to be exposed to the cold receiver and the other to be protected from it. Each of these bolometer branches consisted of a vessel of 0.1 mm sheet copper having the form of a cone and a frustrum of a cone united at the bases. The lengths were 12 cm; the maximum internal diameter was 4 cm. The internal surface was smoked. The external surface was polished and upon it was wound two thin insulated wires. One of these wires, of iron, formed the bolometric branch, and the other wire, of manganin, was used as a heating resistance. The other two branches of the bolometer bridge were formed of resistance coils and the whole was connected with a galvanometer and storage battery as in any ordinary bolometer. The two sensitive black-body branches of the bolometer were in an evacuated vessel which was kept in a tank of water.

The receiver was a blackened glass bulb immersed in liquid air, and the bolometer was allowed to radiate to this receiver. Puccianti measured the (electrical) energy necessary for compensation to prevent the bridge from being unbalanced when one branch was exposed to the receiver. He obtained a value of $\sigma = 5.96$.

The method is an ingenious variation from the usual procedure. The apparatus should have been constructed so that both bolometer branches could have been used as radiators. From the illustrations it appears that radiation from one branch could fall upon the other which would introduce errors. Tests might have been made to determine whether the bolometer remained balanced when a heating current was applied to both branches, without allowing one branch to radiate to the receiver. Furthermore, to repeat the herein oft-mentioned device, a heating coil should have been inserted temporarily within the radiator to determine the energy input as compared with the energy input required in the outer heating coils in order to maintain a balance.

3. THE THERMOMETRIC METHOD WITH "BLACK" RECEIVERS

In order to eliminate the question of reflection from the surface Féry [14] made a series of determinations of the radiation constant, σ, by means of a thermojunction which is formed into a long conical-shaped metal receiver which is blackened on the inside. On the outside of the cone, and insulated from it, was wound a heating coil of known resistance for calibrating the receiver. This was done by noting the temperature rise (galvanometer deflections) with the energy input, in watts. The receiver was then exposed to an electric furnace which was heated to various temperatures from 500 to 1200° C, and the corresponding galvanometer deflections were observed. He obtained a value of $\sigma = 6.30$.

The work was undertaken anew by Féry and Drecq,[15] the receiver being a cone of brass having an opening of 30° placed within a large sphere of brass which was surmounted by a glass tube with a capillary 1 mm in diameter. The large brass sphere was filled with alcohol, the whole forming a thermometer in which 1 mm rise in height of the column in the capillary indicated a rise in temperature of 0°.005. Surrounding the outside of the brass case was a coil of wire through which an electric current was passed and the watt input noted to cause the same temperature as that produced by exposing the opening of the cone to the radiator. They found a mean value of $\sigma = 6.51$. This is the highest value yet obtained. The relative values obtained with the instrument show a fair constancy over the whole temperature range which extended to the melting point of gold. However, the same is true of Valentiner's values determined by the bolometric method. This indicates that there is some constant factor which is not eliminated and which causes a high or a low value. Various suggestions have been made as to the cause of their excessively large values. It appears that part of the difficulty lies in the unsymmetrical way in which the instrument is operated. It is calibrated by a heating coil which is in contact with the alcohol (or air, in the first case) and can warm the latter by conduction and by absorption of radiant energy. On the other hand, the incoming radiation must be transformed by absorption in the cone and then reaches the alcohol principally by conduction. As frequently mentioned in this paper, the weak point lies in not having the heating coil within the receiver, which should be so constructed that but little, if any, of the entering radiation, or the energy radiated from the heating coil, can escape through the opening in the receiver. This might easily have been done by forming the receiver into a double-coned receiver, such as was used by Puccianti.

---

[14] Féry, Compt. Rendus, 148, p. 915; 1909.

[15] Féry and Drecq, Jour. de Phys. (5), 1, p. 551; 1911.

A further determination of this constant was made by Féry and Drecq [16] by a method which is somewhat similar to the one used in the present paper. The radiations from an electric furnace fell upon a strip of platinum (area 36 by 55 mm and 0.03 mm in thickness), blackened electrolytically with platinum black. Instead of measuring the rise in temperature by means of a thermopile, as was done in the present paper, they sighted upon the front and the rear surfaces of the platinum strip by means of a Féry pyrometer, the angle of incidence being 48°. Their new value of the radiation constant is $\sigma = 6.2$. The measurements upon the posterior surface lead to the value $\sigma = 5.57$, which is said to correspond with the measurements on the anterior surface made with plane receivers. If we correct the latter value by 2 per cent for reflection,[17] we obtain the value $\sigma = 5.68$, which is of the same magnitude as observed by the writer. This new determination of σ by Féry and Drecq appears to have been made defective by their reduction of the original observations; for example, they claim that the coefficient of absorption of the receiver was only 0.82 to 0.84, which seems impossible from numerous and diverse experiments on the diffuse reflecting power of platinum black made by others. Attention was called to this fact by Bauer,[18] who places their value of σ between 5.1 and 5.8, and, by making a correction of 2 per cent for reflection, deduced a value of $\sigma = 5.68$. In all three methods the data are meager as to elimination of the various errors which may occur. For example, as noted in a previous paper,[19] a source of error may arise in determining the power put into the platinum strip used in the last method. Another important source of error lies in the manner of operation of the water-cooled shutter, which should be placed between the water-cooled diaphragm and the radiator.

---

[16] Féry and Drecq, Compt. Rend., **155**, p. 1239; 1912.

[17] This Bulletin, **9**, p. 283; 1913.

In order to obviate the difficulties encountered by Féry and Drecq in calibrating their conical-shaped receivers, Bauer and Moulin[20] calibrated their receiver (which was a Féry pyrometer) by sighting it upon a strip of platinum which was heated to different temperatures by an electric current. In order to determine the amount of radiant energy falling upon the receiver it was necessary to eliminate the losses from the strip by conduction and convection. For this purpose the platinum strip was heated in air and in a vacuum, the power consumed being determined for a definite length of platinum, defined by potential terminals as used in the present paper. Having calibrated the pyrometer by noting the galvanometer deflections for the various amounts of energy (in watts) put into the platinum strip, they sighted the pyrometer upon a black body heated to various temperatures and noted the galvanometer deflections. Their first announced value was $\sigma = 6.0$. However, as in numerous other cases herein cited, they introduced errors in the final reduction of their data. They had observed the radiation emitted from the platinum strip at an angle of 13° from the normal and applied a correction[21] of about 12 per cent which reduced their value to $\sigma = 5.3$. This correction is recognized to be much too large,[22] so that their value of the radiation constant lies between $\sigma = 5.3$ and 6.0, which indicates the close grouping of all the determinations about the value $\sigma = 5.7$. They made no correction for atmospheric absorption which would increase their value from $\sigma = 5.3$

---

[18] Bauer, Soc. Franc. de Phys., Proces Verbeaux, No. 41; 1913.

[19] This Bulletin, 12, p. 503; 1916.

[20] Bauer and Moulin, Compt. Rend., 149, p. 988; 1909. Bull., Seances Soc. Fr. de Phys., p. 58; 1910.

[21] Bauer and Moulin, Compt. Rend., 150, p. 167; 1910.

[22] Bauer, Soc. Fr. de Phys., Proces Verbaux, No. 41; 1913.

to $\sigma = 5.6$ or 5.7. In a previous communication on the solar constant Bauer and Moulin,[23] using an Ångström pyrheliometer, found the value $\sigma = 5.7$.

Puccianti[24] gives a determination in absolute measure of the radiation of a black body in which the temperature change is measured by means of a toluene thermometer, the bulb of which is formed into a hollow cone that is allowed to radiate to a black-body receiver, which is at the temperature of liquid air. The measurement is made by compensating the heat lost by the thermometer by the application of an electric current.

The apparatus not being differential in construction, the temperature of the water bath had to be kept rigorously constant in order to have the meniscus of the thermometer move slowly and regularly. The response of the apparatus was, of course, slow and sluggish, which is a common property of this type of receiver (radiator), so that it required from four to eight minutes to obtain a measurement.

The same criticisms apply to this instrument that have been mentioned in the cruder form of thermometer used by Féry.[25] The energy for compensation is supplied by a heating coil which is in contact with the liquid (a good scheme in so far as it applies to heating the liquid) and on the side of the wall of the receiver opposite to that upon which the incoming radiations impinge. The arrangement is therefore unsymmetrical. The heat of compensation should have been supplied by a coil inserted within the receiver, provision being made that little or none could escape by reflection and by direct radiation through the opening. In this manner the condition of heat interchanges would have been the most closely fulfilled. In the instrument as used the opportunity for escape of energy seems greater, so that in compensation there is a tendency to produce a value which is higher than the true value for radiation unaccompanied by conduction. By placing the heating coil within the receiver and using a high temperature radiator, Puccianti's device should be applied in the manner recently used by Keene.

---

[23] Bauer and Moulin, Compt. Rend., 150, p. 1658; 1910.

[25] Féry, J. de Phys. (5), 1, p. 551; 1911.

[24] Puccianti, Nuovo Cimento (6), 4, p. 322; 1912.

Puccianti considered the precision of this method as high as that of the bolometric apparatus, but the sensitivity of the thermometric apparatus was very much inferior to the bolometer. Nevertheless, he seems to prefer the thermometric method in spite of its small sensitivity. He succeeded in assigning a value of $6.00 < \sigma < 6.3$, and his intermediate value is $\sigma = 6.15$.

The most recent determination of the constant, σ, is by Keene.[26] The radiator consisted of an electric furnace which could be heated to 1000° C. The receiver consisted of a hollow spherical double-walled thermometer bulb provided with a small aperture in its side to admit the radiation to be measured. The space between the walls is filled with aniline, which served as thermometric substance, its expansion being observed in a capillary tube in the usual way, 1 mm division = 0.0005° C.

In order to eliminate the effect of the variation of room temperature, two such thermometers were used differentially, radiation being admitted into one of them, the differential effect giving a measure of the energy supply. The interior of the bulb receiving the radiation was provided with an electric heating coil for the purpose of calibration. His value of the radiation constant is $\sigma = 5.89$.

4. INDIRECT AND SUBSTITUTION METHODS.

Shakespear[27] obtained the constant, σ, by a method which is based upon the principle that a heated body in air, in surroundings at a lower temperature, losses heat (*a*) by conduction, (*b*) by convection, and (*c*) by radiation. If the rate of loss by a body be observed in two cases, the only difference being that the emissivity of the radiating surfaces varies, other conditions remaining the same, it is quite correct to assume that the losses (*a*) and (*b*) will be the same and that the difference between the observed rate of loss of energy in the two cases is due only to the difference in the losses by radiation. If, now, these two different surfaces, at the temperature of boiling water, be exposed in turn to a radiomicrometer at the room temperature, we get the ratio of the rates of the energy radiated by the two surfaces.

---

[26] Keene, Proc. Roy. Soc., 88 A., p. 49; 1913. London Electrician, 70, p. 541; 1912.

[27] Shakespear, Proc Roy. Soc. London, 86 A, p. 180; 1912.

In the experiments a plate of metal with a silvered surface was heated electrically to 100° C and close to it was another plate, blackened with soot, which was cooled with water. Between the plates was air at atmospheric pressure. Shakespeare measured the watt input in order to maintain the plate at 100° when (1) the surface of the plate was highly polished, and (2) when it was blackened. He measured also the ratio of the emissivities of the plate under these two conditions, using a radiomicrometer for the purpose. From this he obtained a value, σ', in absolute units. He then compared the emissivity of the lampblack surface at 100° with that of a black body at the same temperature by means of a radiomicrometer. From this latter comparison combined with the value of σ' he found a value of $\sigma = 5.67$.

From this description it may be noticed that the essential parts of the method differ from that of Westphal in that the radiator was a flat metal plate which was used in air instead of a vacuum and the black body, with which the emissivity of the plate had to be measured, was separate from it; while in Westphal's instrument the black body was self-contained within the metal (in the form of a cylinder) of which the emissivity of the surface had to be measured.

In this connection is to be considered the work of Todd[23] who, working on the thermal conductivity of gases, obtained a value of the constant of total radiation which is somewhat lower than that of Shakespear. In his experiments on the thermal conductivity of gases, Todd used a layer of air inclosed between two parallel, good conducting plates, which were maintained at different temperatures. The colder plate, of course, receives heat by radiation and by conduction through the air from the hotter plate which is above it. Communication from the surrounding air is shut off by an insulating ring, and the two plates being close together in comparison with their linear dimensions the convection currents are eliminated. He determined the energy lost by radiation by varying the distance x between the two plates and noting the corresponding variation in the quantity, Q, of heat passing from the upper to the lower plate. These values of x and

---

[23] Todd, Proc. Roy. Soc., 83 A, p. 19; 1909.

Q when plotted form a rectangular hyperbola and the horizontal asymptote gives the value, *R*, of the radiation. The energy input was determined by a calorimetric method. In order to determine the constant, σ, he had simply to find the ratio of emissivities of the blackened plate to that of a black body at the same temperature, for which purpose a radiomicrometer was employed. The value of this ratio and the constants obtained in the main part of his experiment enabled him to compute the radiation constant which he found to be $\sigma = 5.48$.

An important determination of the Stefan-Boltzmann constant, σ, has recently been made by Westphal[29] in the laboratory of Prof. Rubens. The experiment consisted in comparing the emissivity of a cylindrical block of copper, when it was highly polished and when it was blackened, with the emissivity of a black body at the same temperature. The novelty involved in the method is in having the black body contained within the cylinder. The copper cylinder was heated electrically, and to reduce the energy losses by gaseous heat conduction this copper cylinder was suspended in a glass flask from which the air could be exhausted to 1 mm pressure. The outer surface of the cylinder was either highly polished to give it a low emissivity or painted with lampblack to give it a high emissivity. The end surfaces remained unchanged. The heat losses by conduction and convection were therefore the same throughout the experiment and the difference in energy input, when the surface of the cylinder had a high emissivity and when it had a low emissivity, was a measure of the energy lost by radiation.

Using the surface of the cylindrical body when highly polished, Westphal proceeded to find the curve of watt input of the body between the temperatures 350° and 425° absolute as a function of the temperature. Then the surface of the body was brought to a high emissivity by applying in succession different blackening materials, and the energy input was measured at different temperatures.

The emissivities of the surface and of the interior of the copper cylinder were compared by means of a thermopile. The numerous details need not be discussed. Suffice it to say, that the work appears to have been thoroughly done, and from the nature of the method it seems free from gross systematic errors. His mean value is $\sigma = 5.54$.

---

[29] Westphal, Verh. Phys. Gesell.. 14, p. 987; 1912.

He modified the original apparatus and extended the observation over a wider range of temperature with a view to increasing the accuracy. The new value[30] agrees well with the earlier determination, being $\sigma = 5.57$.

From this summary (see Table 1) it is of interest to note that the predominating values of the coefficient of total radiation cluster about a mean value of $\sigma = 5.7$. In this connection it is of interest to include in this paper a theoretical computation by Lewis and Adams[31] based upon the elementary electric charge, E, the gas constant, R and the Faraday equivalent, F, which gives a value of $\sigma = 5.7$.

TABLE 1

Observed Value and the Most Probable Value of the Radiation Constant after Correcting for Reflection, Atmospheric Absorption, etc.

Observer	Date	$\sigma \times 10^{12}$	Probable value of $\sigma \times 10^{12}$	Method
Kurlbaum	1898	5.45	?	Bolometer.
Féry	1909	6.3	?	Thermometer.
Bauer and Moulin	1909	5.30	5.7	Thermopile.
	1910	5.7	5.7	Pyrheliometer.
Todd	1909	5.48	5.48	Gas conduction expts.
Valentiner	1910	5.58	5.65–5.7	Bolometer.
Féry and Drecq	1911	6.51	?	Thermometer.
Do	1912	6.2	5.68	Calibrated pyrometer.
Shakespear	1912	5.67	5.67	Ratio of emissivities, metal: Black body.
Gerlach	1912	5.9	5.9	Modified Ångström pyrheliometer.
Puccianti	1912	5.96	?	Bolometer.
		6.15	?	Thermometer.
Westphal	1912	5.54	5.57	Ratio of emissivities, metal: Black body.
	1913	5.57	....	
Keene	1913	5.89	5.89	Thermometer.
Coblentz	1915	....	5.72	Modified Ångström pyrheliometer.

---

[30] Westphal, Verh. d. Deutsch. Phys. Gesell , 15, p. 897; 1913.

[31] Lewis and Adams, Physical Review, (2), 3, p. 92; 1914.

II. THE PRESENT DETERMINATION OF THIS CONSTANT

In the foregoing pages the various methods for evaluating radiant energy in absolute measure have been summarized. Each method appears to have some merits not possessed by others. The numerical value of the coefficient of total radiation can not therefore be selected by using a method which may give a value which happens to fit closely a given theory; but it must be based upon a proper weighing of all the data at hand. It has, of course, been realized all along that the "value[32] of the radiation constant is open to criticism until it has been determined with apparatus in which both the emitter and the receiver fulfill black body conditions." However, the work already done can not be brushed aside by pedantic statements of this kind. It is an easy matter to propound the conditions of operation. It is an entirely different matter to realize these conditions in practice because of the mechanical difficulties in construction. Nearly all receivers hitherto employed, which embodied the black-body principle, have been unwieldy devices, slow in response to the stimulus; and they have been inapplicable as pyrometers for routine laboratory work. The bolometer and the thermopile, by proper calibration, can be used as a pyrometer, but one is then forced to rely upon the galvanometer unless one devises a special potentiometer outfit for measuring the heating effect, and uses a special device for a standard of radiation for testing the sensitivity of the receiver.

In the instrument used in the present investigation[33] the primary receiver and the standard of radiation which is used for testing the sensitivity of the thermopile and galvanometer are incorporated in one piece. The electric power consumed in producing the same temperature within the strip as was produced by the absorption of radiant energy is measured by ordinary laboratory instruments. The loss of radiant energy by reflection from the surface of the receiver (likewise the loss by conduction from the ends, and the effect of thickness) can be determined experimentally. Conduction and convection losses of energy from the surface are assumed to be the same when the strip is heated internally as when it is heated by the absorption of radiant energy upon one surface.

---

[32] Keene, Proc. Roy. Doc., 91, p. 190; 1915.

[33] This Bulletin, 12, p. 503; 1916.

From the nature of the construction of the receiver one would expect to find that it would require a smaller expenditure of energy when the strip is heated internally than when it is heated to the same temperature by heating one surface by the absorption of radiant energy. One would therefore expect that, barring experimental errors, the value of the radiation constant, determined by the present method, is smaller than the true value.

A very thick layer of lampblack, especially when present upon the front surface, should give a smaller value for the constant radiation than that observed with a thin absorbing layer. In a previous paper[34] it was shown that the infra-red rays of long wave lengths penetrate the absorbing layer to a considerable depth, so that, for complete absorption, the layer of platinum black must be of considerable thickness. However, from the experiments[35] given in the first part of the present investigation, it is evident that the observed value of the radiation constant can not be much too low, excepting in the case noted where the front surface had a much thicker layer of lampblack than the rear surface. An excellent example is receiver No. 7, in which the heat capacity was increased to such an extent that it required 35 seconds to attain temperature equilibrium. The thin receivers of platinum attained temperature equilibrium in 8 to 10 seconds (the delay of 15 seconds was due to the lag in the thermopile), and although it required about four times as long to attain temperature equilibrium in receiver No. 7, the value of the constant of radiation is not affected in a marked degree by the thick layer of material. One can therefore conclude from the experiments upon receivers varying 10 times in thickness that the difference in thickness of the metal strips and of the absorbing surfaces has only a minor effect upon the radiation measurements. Although the various receivers give different values, there is a remarkably close agreement among the determinations made with any one receiver. As was observed also by Gerlach, the amount of shadowing of the receiver by the slits had an inappreciable effect upon the value of the constant.

Having eliminated the atmospheric water vapor the absorption of radiant energy was reduced. Furthermore, the distance of the receiver from the radiator was short, and it was varied from 30 to 50 cm. Throughout this range the value of the radiation constant was not affected by the absorption of atmospheric water vapor. However, when the water vapor was not removed the value of the constant σ was decreased as a result of absorption of infra-red radiations of long wave lengths, as will be described in a subsequent page. Kurlbaum and Valentiner used much greater distances between the receiver and the radiator, which no doubt affected their results, as already described.

---

[34] This Bulletin, 9, p. 280; 1913.

[35] This Bulletin, 12, p. 503; 1916.

1. CORRECTION FOR LACK OF BLACKNESS OF THE RADIATOR

There have been discussions as to the proper dimensions of the interior of the radiator to insure completeness of emission as defined theoretically. The main difficulty with large-sized radiators, heated to a high temperature, is that they are clumsy, unwieldy devices which are slow to reach a steady temperature; and it is difficult to establish a uniform temperature throughout their interior. It has not yet been shown that there is a marked difference (if any) in the radiation constant as determined from measurements upon a large-sized radiator (heated by a steam bath) and a small-sized, blackened, tubular, porcelain radiator of the Lummer-Kurlbaum type, heated to 1000° C or higher. The work of Valentiner seems quite conclusive on this point. For the calibration of optical pyrometers and for radiation work in general the tubular form of radiator is the most applicable instrument yet devised. It is therefore desirable to determine the correction for lack of blackness of such a radiator. In a previous paper [36] it was found that the constant of spectral radiation from painted and unpainted porcelain radiators was the same, indicating that the radiator had the properties of a "gray body" even if it did not fulfill the condition of a theoretically complete radiator. The experiments, about to be described, show that the radiations from a blackened porcelain tube radiator approach very closely to the theoretically complete radiator.

The coefficient of total radiation was determined for a new unpainted Marquardt porcelain radiator and for the blackened radiator, prepared as described in the paper just quoted. Both radiators were heated radiometrically by means of the outer heating coil described in the paper just quoted. In this manner the new unpainted porcelain radiator did not require winding with platinum ribbon. In the unblackened radiator the front thermocouple was bare along the line of contact across the radiating surface which had a reflecting power of about 58 per cent. (See Table 3 [37].) The value of the constant differs, if at all, by less than 1 per cent from that observed with the blackened radiator, as may be seen from Table 2.

---

[36] This bulletin, 10, p. 2; 1913.

TABLE 2

Difference in Emissivity of an Unblackened and a Blackened Radiator

[Receiver No. 11 (length=22.389 mm, width=6.481 mm); diaphragm No. 3 (d=3.987 mm); excellent coating of platinum black]

UNPAINTED RADIATOR

Series	Distance	Temperature		Coefficient of radiation, $\sigma \times 10^{12}$
		t_0	t_1	
	mm	°C	°C	
CXLVIII	377.0	15.5	1078.0	5.572
			.1	.566
			.3	.572
CXLIX	419.4	16.0	1078.7	5.592
			.7	.591
			.7	.603
CL	475.7	16.0	1078.6	5.557
			.4	.560
			.2	.565
CLI	442.9	16.0	1078.1	5.569
			7.9	.585

Mean value $\sigma = 5.576 \times 10^{-12}$ watts cm^{-2} deg^{-4}.

PAINTED RADIATOR

Series	Distance	t_0	t_1	$\sigma \times 10^{12}$
CLII	379.8	17.0	1082.0	5.610
			1.2	.620
			.0	.637
CLIII	417.6	17.0	1084.0	5.637
			.0	.625
			3.7	.627
CLIV	446.5	17.0	1084.5	5.605
			.5	.614

Mean value $\sigma = 5.622 \times 10^{-12}$ watts cm^{-2} deg^{-4}.
$\Delta\sigma = 0.046$.

---

[37] This bulletin, 12, p. 523; 1916.

The lack of blackness of the radiator has been discussed by Wien and Lummer,[38] who give a method for computing the correction for the opening in the radiator, on the assumption that the inclosure is spherical and diffusely reflecting. The amount of energy that can escape by diffuse reflection through the opening is determined practically by the size of the opening as compared with the total area of the interior of the radiator. In the porcelain tube radiator used in the present work the interior is uniformly heated over a length of 8 to 10 cm. However, for the purpose of the present computation, a length of only 2.5 cm, which is defined by the first diaphragm, is considered. The diameter of the tube is 3 cm and that of the hole in the inner diaphragm is 2 cm. The area of the inclosure is 37.6 cm^2 and that of the opening is 3.1 cm.^{2} On the basis that the reflecting power of the interior of the radiator is 7 per cent (see Table 3[38a]) the loss of energy by diffuse reflection through the inner diaphragm is 0.63 per cent. Using the unpainted Marquardt porcelain having a reflecting power of 58 per cent (see Table 3[38a]), the loss of energy by diffuse reflection through the opening in the black body would be about 11.5 per cent. In other words, the values of the radiation constant as determined on the blackened and the unblackened radiator should differ by about 11 per cent on account of the lack of "blackness" of the unpainted radiator. This is entirely at variance with the experimental determinations, which show a difference in blackness of less than 1 per cent. These computations are, of course, open to criticism because the inclosure is not spherical. However, the diffuse reflection and emission within a cylindrical inclosure can hardly differ from that of a spherical inclosure by an amount indicated by the computations. Furthermore, these computations are based upon reflecting power data obtained at room temperature. There are good reasons for believing that the reflecting power of porcelain and especially of (the black) oxides decreases with rise in temperature; for it is well known[39] that their total emissivity increases with rise in temperature, that they become better electrical conductors, that their emission spectra become more continuous, etc. All these factors[40] indicate a decrease in internal reflection and hence a decrease of the diffuse reflection approaching a value of about 4 per cent, which is the average reflectivity of a plane surface of a vitreous substance. The measusements made by Rubens[41] on a gas mantle, when cold and when heated to incandescence, show a marked decrease in reflecting power with rise in temperature. This is an extreme case in which but little change in reflectivity is to be expected in view of the fact that the electrical conductivity of the gas mantle increases but little with rise in temperature.

---

[38] Wien and Lummer, Ann. der Phys., (3), **56**, p. 450; 1895.

[38a] This bulletin, **12**, p. 523; 1916.

[39] See recent papers by Randolph and Overholzer, Phys. Rev., (2), **2**, p. 144, 1913; by Burgess and Foote, this bulletin, **11**, p. 41, 1914; and by Foote, this Bulletin, **11**, p. 607, 1915.

From the foregoing it appears that the correction, if any, for lack of blackness of the radiator used in this work is not greater than 0.5 per cent. In other words, the mean value of the radiation constant is to be increased by 0.03 per cent, which is less than the range in the values of the various measurements due to experimental errors and due to variation humidity.

2. CORRECTION FOR ATMOSPHERIC ABSORPTION

The amount of carbon dioxide in the atmosphere is sufficient to produce an appreciable absorption in the spectral regions at 2.7, 4.2, and 14 μ. Water vapor has a much greater absorption than CO_2 throughout the spectrum from 1.4 to 100 μ. A radiator heated to 1000° emits more strongly radiations of all wave lengths throughout the infra-red spectrum to 100 μ, and consequently the effect of atmospheric absorption becomes appreciable even at a distance 30 to 40 cm from the radiator.

In the present investigation the air path between the receiver and the radiator was inclosed and the water vapor was removed by means of phosphorous pentoxide. Most of the observations were made in February and March, when the humidity was low. The drying material was placed in the inclosure the evening preceding the day the observations were made, and the following morning it would be found that but little of the phosphorous pentoxide had been affected by the moisture, showing that the moisture had been eliminated. However, the tests now to be described were made in June, when the vapor pressure was high, and the phosphorous pentoxide placed in the inclosure in the evening preceding the test was found entirely dissolved the following morning. The radiation constant determined then with a certain receiver (No. 12, series CLXXIV et seq., Table 3) was much lower than observed when the humidity was lower. It is therefore concluded that although fresh drying material was provided about an hour before beginning observations there was sufficient moisture present to affect the value of the radiation constant. This is well illustrated in Table 3, series CLXXX (and in Table 5, series CLV to CLXII, given in this bulletin, 12, p. 540).

---

[40] This bulletin, 4, p. 533; 5, p. 159, 1908.

[41] Rubens, Bull., des Seances Soc. Franc. de physique; 1906.

As already mentioned, when the moisture was removed from the intervening space between the radiator and the receiver the radiation constant was independent of the distance. This is true especially in cases where the measurements were first made at the longest distance (after the air had been dried overnight) and then the distance reduced. In cases where the measurements were made at the shortest distance, moist air would enter when increasing the distance, and the constant might be lower, owing to the fact that sufficient time had not elapsed in order to remove all the moisture.

TABLE 3

Effect of Atmospheric Moisture on the Radiation Constant

[Diaphragm No. 3; dia.=3.987 mm; receiver No. 12; area receiver [22.385×6.326]=141.608 mm^2]

Series	Date	Distance mm	Temperature		Coefficient of radiation $\sigma\times10^{12}$	Remarks
			t_0	t_1		
			°C	°C		
CLXXIV..........	June 5,1915	1238.9	20.2	1077.9	5.328	No tube; vapor pressure 11.21 mm.
				.6	.325	
				5.4	.359	
CLXXV..........	June 7,1915	1238.7	21.0	1081.0	5.104	No tube; vapor pressure 14.02 mm.
				.2	.114	
				.3	.213	
				.3	.213	
				.2	.209	
				0.9	.239	
CLXXVI..........	.....do	1238.7	21.1	1077.7	5.178	No tube; Thermopile lowered 1 cm below optic axis.
				.9	.186	
				.8	.218	
				.4	.180	
				.4	.189	
				.2	.213	
				.2	.196	
				.3	.198	

TABLE 3—Continued

Series	Date	Distance mm	Temperature		Coefficient of radiation $\sigma \times 10^{12}$	Remarks
			t_0	t_1		
			°C	°C		
CLXXVII........	June 7, 1915	503.8	21.1	1076.1	5.423	No tube.
				5.9	.407	
				.8	.441	
				.7	.438	
				.8	.453	
				.8	.437	
CLXXVIII......	.....do.......	356.6	21.0	1075.3	5.509	Do.
				.1	.499	
				4.7	.512	
				.7	.512	
CLXXIX.........	June 9, 1915	521.2	25.5	1081.7	5.496	With tube; drying material (in 30 minutes).
				.7	.512	
				.6	.526	
				.7	.531	
				.7	.537	
				.9	.538	
CLXXX..........	.....do.......	521.2	25.2	1082.4	5.535	With tube; drying material (in 2 hours).
				.3	.543	
				.4	.544	
				.1	.564	
				.1	.532	
				1.9	.558	
				2.0	.550	
				.1	.534	
CLXXXI.........	.....do.......	521.2	25.2	1082.3	5.419	No tube.
				.5	.419	
				.5	.415	
				.5	.421	
CLXXXII........	.....do.......	521.2	25.2	1073.2	5.408	With tube; no drying material.
				.4	.448	
				.3	.412	
				2.8	.396	
				.6	.391	
				3.0	.379	
CLXXXIII.......	.....do.......	521.2	25.2	1072.6	5.445	No tube.
				2.7	.420	
				3.0	.399	
				.0	.415	
				.1	.411	

The measurements of the radiation constant through humid air are given in Table 3. The receiver was aligned in front of the water-cooled diaphragm by means of a long, straight, especially prepared brass tube, as described in the previous paper.[42] However, fearing that the vapors from the blackened diaphragms within this tube might affect the results, the connecting tube was removed after making the alignment. The measurements are therefore somewhat affected by air currents on the thermopile. On June 5 (see Table 3) the air temperature was 21°4 C and the vapor pressure was 11.21 mm. The weight of water was, therefore, 11 grams per cubic meter. The next day the air temperature was 22°6 and the vapor pressure was 14.05 mm, equivalent to 13.7 grams of precipitable water per cubic meter. For this higher vapor pressure the radiation constant (series CLXXV) is about 2 per cent lower than that observed through a lower vapor pressure. The same result was obtained after lowering the receiver 1 cm below the axis of the rays showing that the alignment was not at fault. Decreasing the distance from 124 cm to 36 cm increased the radiation constant from $\sigma = 5.21$ to $\sigma = 5.51$, i. e., almost 6 per cent. Further tests were made with and without the inclosing tube (series CLXXXI to CLXXXIII) to show that there was no reflection from the inclosure. This showed that the lower value for the radiation constant, observed when the moisture was not removed, is due to the presence of water vapor. This is well demonstrated in series CLXXIX, which gives the value of $\sigma = 5.496$, after the air had been dried for about one-half hour and the gradual increase in this value, with each new determination, to $\sigma = 5.538$ at the end of another half hour. After two hours drying of the air, the value increased to $\sigma = 5.55$ or an increase of 2 per cent ($\sigma = 5.41$ in air) by removing the greater part of the moisture from a column of air 52 cm. in length. The highest value observed is no doubt still too low, because of the presence of moisture as already explained, for, in earlier observations made with this same receiver, the value was $\sigma = 5.65$, or a 2 per cent higher value, which is not attributable entirely to experimental errors in readjusting the apparatus. Further data, showing the great effect of the presence of water vapor upon the value of the radiation constant, are to be found in Table 5, receiver No. 11, series CLV to CLXII, given in this Bulletin 12, p. 540, in which there is a variation of 2 to 3 per cent, depending upon the presence of moisture.

---

[42] This bulletin, 12, p. 517; 1916.

The data at hand indicate conclusively the serious effect of the presence of water vapor upon radiation measurements. This is, of course, not entirely new, for Tyndall (and recently Fowle) had made similar measurements. However, in the measurements of the coefficient of total radiation summarized on a preceding page, experimenters heretofore did not make any correction for atmospheric absorption which was about as large as the range (2 to 3 per cent) in their radiometric measurements. In the present investigation, the moisture having been removed from the optical path, the correction for absorption is negligible; although there are indications that some of the variations in the observations may be attributable to the presence of moisture in the air. From this subsidiary investigation it is quite evident that it is desirable to perform the whole radiation experiment in a vacuum. In a future paper it is hoped to contribute further to this subject, in view of the fact that the data on the radiation constant will be obtainable incidentally from measurements upon a vacuum furnace the temperature of which will be measured by means of an absolute thermopile of the type just investigated. For, as stated in the beginning of the preceding paper, the object of the investigation was the production of a radiation pyrometer suitable for refined radiometric investigations.

3. SUMMARY

The first part of this paper gives a summary of the methods used and the values of the constant of total radiation obtained by various observers. Corrections for losses by reflection, for atmospheric absorption, and for lack of blackness of the radiator were applied. After making these corrections to the most reliable measurements the large variations in the radiation constant have disappeared, giving a value of the coefficient which is of the order of

$$\sigma = 5.7 \times 10^{-12} \text{ watt cm}^{-2} \text{ deg.}^{-4}.$$

This paper gives also the correction (about 1 per cent) for lack of "blackness" of the radiations from an unpainted porcelain tube radiator and for the atmospheric absorption in 1.2 m of air containing a determined amount of water vapor, the radiator being at about 1050° C. The data obtained in a previous paper have been corrected for losses by diffuse reflection from the receiver and combined to give a value of the coefficient (the so-called Stefan-Boltzmann constant) of total radiation from a black body, as determined by means of an absolute thermopile of the type previously described. The correction applied to all the data for diffuse reflection is taken as being 1.2 per cent. This correction may be 0.1 to 0.2 per cent too high for lampblack, but it will eliminate, in part, atmospheric absorption and the possible lack of blackness of the radiator.

Of the 13 receivers used, 3 were known to be defective, and receiver No. 4 (which was one of these 3) was used mainly for relative measurements on the effect of using water-cooled diaphragms having different openings. The mean value of the radiation constant (corrected by 1.2 per cent for reflection) as determined by 12 receivers and representing 348 pairs of measurements is

$$\sigma = 5.75 \times 10^{-12} \text{ watt cm}^{-2} \text{ deg.}^{-4}.$$

If we exclude receivers Nos. 8 and 9, which is permissible in view of the fact that they were known to be defective, the value of the coefficient of total radiation, representing the mean of 304 pairs of determinations, is

$$\sigma = 5.72 \times 10^{-12} \text{ watt cm}^{-2} \text{ deg.}^{-4}.$$

which is but little smaller than the value obtained for all the receivers. This value is based upon the mean of all the data tabulated in Table 5 of this Bulletin, Vol. 12, page 531, excepting the measurements (marked with an asterisk) in which there was no slit in front of the receiver, in which moisture was present, and in which an unblackened radiator was used. All the determinations are given the same weight in finding the mean value. Each tabulated value represents two sets of measurements, each one of which is a complete determination consisting of 5 to 10, or more, readings when the receiver was heated electrically and when it was heated radiometrically. From the data at hand it appears that the value of the radiation constant is of the order of

$$\sigma = 5.72 \times 10^{-12} \text{ watt cm}^{-2} \text{ deg.}^{-4}$$

or

$$\sigma = 1.365 \times 10^{-12} \text{ gr} - \text{cal cm}^{-2} \text{ deg.}^{-4}.$$

Using Planck's equation and Millikan's value of the unit electric charge, this value of the constant of total radiation indicates that the constant of spectral radiation is c = 14320 cm.-deg.

WASHINGTON, July 2, 1915.[43]

---

[43] Since this paper has been written Warburg and Müller (Ann. der Phys., 48, 410, 1915) have published a value of the spectral radiation constant which is of the order: c=14300. This seems to indicate a remarkable agreement between theory and experiment; especially so, in view of the great difficulties encountered in the experimental work.

NOTE I: ON STANDARDS OF RADIATION

The foregoing researches mark what is hoped to be the beginning of real progress in the measurement of radiant energy in absolute value, especially the determination of the mechanical equivalent of light (that is, the evolution of the visible radiations, "light," in absolute measure), which research was authorized and undertaken four years[1] ago. In the meantime various subsidiary researches have been finished, including (1) the design and construction of a spectrometer[2] with triple achromatic lenses of special construction by Zeiss; (2) the investigation and construction of thermopiles and an illuminator attachment[3] to the spectroscope for measuring light stimuli; (3) the investigation of the loss of radiant energy by diffuse reflection[4] from the surface of the radiometer; (4) the investigation of light filters[5] which absorb all the infra-red rays; (5) the selective emission[6] of the mercury in quartz arc and the acetylene flame; (6) the preparation of standards of radiation[7] for calibrating the radiometer in absolute measure; and recently (7) the determination of the coefficient of total radiation[8] of a black body. This includes the development of an instrument for accurately evaluating weak sources of radiant energy in absolute measure.

In the absence of reliable apparatus of sufficient sensitivity for evaluating visible radiant energy, "light," in absolute measure, it seemed best to prepare the above-mentioned standards of radiation (consisting of seasoned incandescent lamps) by comparing their radiations with that of a black body on the assumption that the coefficient of total radiation of the latter is $\sigma = 5.7 \times 10^{-12}$ watt per cm^2 per degree^4. The foregoing research, which gave a value of $\sigma = 5.72 \times 10^{-12}$, shows that the assumed value is close to the experimentally determined value of various observers.

---

[1] This Bulletin, **7**, p. 661; 1911.

[2] This Bulletin, **7**, p. 243; 1911.

[3] This Bulletin, **11**, p. 154; 1914.

[4] This Bulletin, **9**, p. 283; 1913.

[5] This Bulletin, **7**, p. 655, 1911; **9**, p. 110, 1912.

[6] This Bulletin, **9**, p. 97; 1912. For data on the acetylene flame, see this Bulletin, **7**, p. 243, 1911; **9**, p. 98, 1912. A further investigation was made of the energy distribution of the acetylene flame, radiating flatwise and edgewise into a mirror spectrometer and also into a spectrometer with lenses having excellent optical corrections. In a forthcoming paper new data will be given which will supercede the data previously published.

[7] This Bulletin, **11**, p, 87; 1914. Copies of these radiation standards have been used since then by Nutting (Trans. Illum. Eng. Soc., **9**, p. 633; 1914) and by Ives, Coblentz and Kingsbury (Phys. Rev., **5**, p. 269; 1915) in preliminary measurements on the mechanical equivalent of light.

[8] This Bulletin, **12**, pp. 503 and 553, 1916.

The determination of the mechanical equivalent of light, primarily of monochromatic spectral lines, was undertaken some years ago when the problem was presented to provide standards of radiation for calibrating the radiometers which are used in evaluating light stimuli in absolute measure. These light stimuli are being used by psychologists, physiologists, and physicists in researches often involving photometric measurements. The simplest method of calibrating the radiometer would therefore seem to be to photometer and radiometer a standard spectral line of known mechanical equivalent. Then, knowing the candlepower, the energy value is known in absolute measure and by exposing the radiometer (say a thermopile) to this spectral line the instrument is calibrated for all light stimuli of which the energy value is to be determined. The above-mentioned incandescent lamp radiation standards were further investigated and restandardized recently, and used in an investigation of the mechanical equivalent of light.[9] In this manner a tentative value was obtained for the relation between the candlepower and the radiation of the green mercury line. Unfortunately, through a misunderstanding as to the thickness of the absorption cells used in part of this work, some of the data are rendered uncertain by the presence [10] of infra-red radiation. It is therefore not advisable to use this data for an accurate calibration of a radiometer by means of the green mercury line. However, as a check on one's work, in addition to the calibration with the incandescent lamp radiation standard, a rough test could be made with the green mercury line on the basis that 1 cp = 0.02 watt.

---

[9] Ives, Coblentz, and Kingsbury, Phys. Rev., 5, p. 269; 1915.

[10] The absorption cell was 1 cm in thickness instead of the writer's specifications of 2 cm as given in this Bulletin, 7, p. 655, 1911; 9, p. 110, 1912.

In view of the fact that the writer's investigation of the mechanical equivalent of light has been in progress for some years, and, as already stated, much of the preliminary work is already completed, the suggestion[11] that this problem and the allied one of determining the visibility of radiation should be investigated at this Bureau is somewhat belated, although welcomed as indicating the demand for such data. Most of the subsidiary problems have now been worked out, excepting perhaps the question of the method of photometry. The psychologist and the physicist differ as to photometric procedure, the former claiming that the equality of brightness method of photometry is the only one that will give a true evaluation. The physicist, however, finds that the flicker method is the more precise. The main outstanding question, which involves other investigations as well as the mechanical equivalent of light, is the photometric procedure.

WASHINGTON, November 20, 1915.

---

[11] Ives, Trans. Illum. Eng. Soc., **10**, p. 318, 1915; Phys. Rev. (2), **6**, p. 332, 1915.

ILLUMINATION FROM A RADIATING DISK

By Paul D. Foote, Assistant Physicist

Some time ago there appeared in the transactions of the Illuminating Engineering Society a discussion of the illumination produced by a diffusely and uniformly radiating circular disk at any point in space on a surface parallel to the disk.[1]

The integral there obtained is incorrect and the approximation method given in detail is not warranted, since the correct solution is very readily obtained and is much simpler and more convenient to use than the approximation formula presented in the paper above referred to. Very possibly the solution of this problem has appeared in various journals, but it seemed worth while to call attention to the question at the present time.

REFERRING TO FIG. 1

Problem: To find the illumination E due to a circular disk of uniform brightness i and radius a, at any point in space on a surface parallel to the disk.

Assumptions: (1) Diffuse radiation and hence the validity of Lambert's cosine law.

(2) Inverse square law for infinitesimal points.

Notation: OC = perpendicular to the disk at its center.

D = point at which the illumination is desired.

X = perpendicular from D to line OC, i. e. = distance of D from axis of symmetry.

K = perpendicular distance of D from plane of disk.

Other quantities are evident from Fig. 1.

---

[1] Jones, Ill. Eng. Soc. Trans., 5, p. 281; 1910.

$E = \frac{i\,dw\cos^2\theta}{r^2}$ due to element dw at B.

$$(dw = \rho d\psi d\rho)$$

$$= i\,K^2\int_0^a\int_0^{2\pi}\frac{\rho d\psi d\rho}{(x^2+\rho^2+K^2-2x\rho\sin\psi)^2}$$

Put $\cos\alpha = \sin\psi$ and integrate in respect to $d\alpha$.
Let $P = x^2+\rho^2+K^2$.
$Q = -2x\rho$

$$E = i\,K^2\int_0^a\rho d\rho\int_0^{2\pi}\frac{d\alpha}{(P+Q\cos\alpha)^2}$$

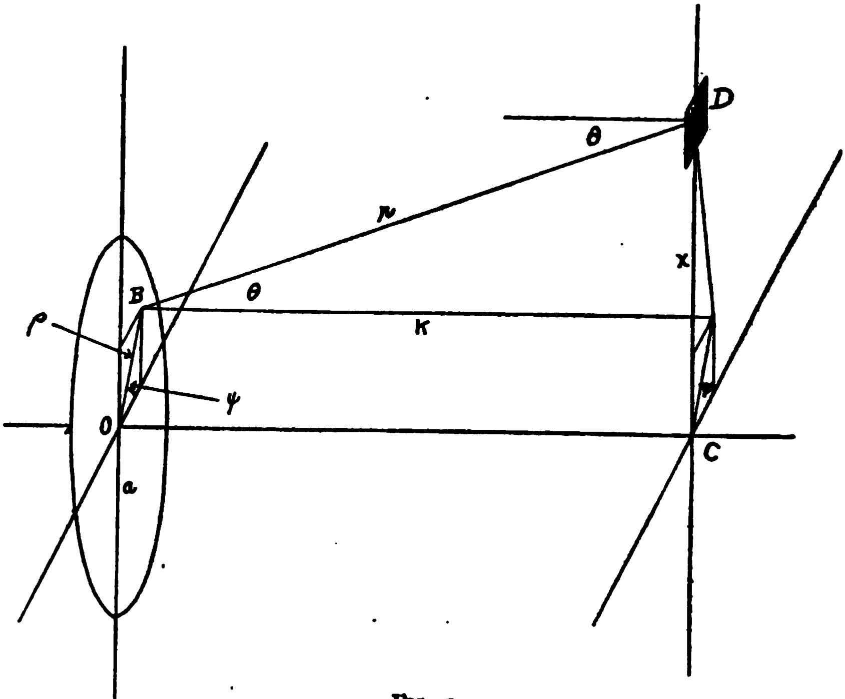


FIG. 1

P^2 must be greater than Q^2 for any possible combination of variables or constants, if $K > o$ when $x < a$. Hence:

$$E = 2\pi i\,K^2\int_0^a\frac{P\rho d\rho}{(P^2-Q^2)^{\frac{3}{2}}} \qquad K > o \text{ when } x < a$$

Put $A = x^2+K^2$
$B = -2(x^2-K^2)$
Let $y = \rho^2$

Then:

$$E = A\pi i K^2 \int_0^{a^2} \frac{dy}{(A^2+By+y^2)^{\frac{3}{2}}} + \pi i K^2 \int_0^{a^2} \frac{y\,dy}{(A^2+By+y^2)^{\frac{3}{2}}}$$

$$(1)\ E = \frac{\pi i}{2}\left[1 - \frac{x^2+K^2-a^2}{\sqrt{x^4+2(K^2-a^2)x^2+(K^2+a^2)^2}}\right] \text{—}$$

illumination at any point in space on surface parallel to the disk, except $K=o$ when $x<a$.

SPECIAL CASES

Problem: Illumination at point on a line perpendicular to disk from a point on the circumference of the disk, i. e., $x=a$.

$$(2)\ E = \frac{\pi i}{2}\left[1 - \frac{K}{\sqrt{K^2+4a^2}}\right] \qquad K \neq o$$

Problem: Illumination at point on a line perpendicular to disk at the center, i. e., $x=o$.

$$(3)\ E = \frac{\pi i}{2}\left[\frac{2a^2}{K^2+a^2}\right] \qquad K \neq o$$

which is the form derived by Hyde.[2]

As a particular example, let $\pi i/2 = 1$ unit, and $a = 1$ unit.

For equation (1) let $x = 1.1$ units and let it be desired to obtain the illumination at various distances K along a line parallel to the axis of symmetry but distant 1.1 units from it.

By equation (2) the illumination is obtained at various distances K along a line parallel to the axis of symmetry but distant from it by 1 unit.

By equation (3) the illumination along the axis of symmetry is given. These three equations take the following form for the above special conditions.

$$(1)'\ E' = \left[1 - \frac{K^2+0.21}{\sqrt{K^4+4.42\,K^2+0.0441}}\right] \qquad \text{(no restriction on } K\text{)}$$

$$(2)'\ E' = \left[1 - \frac{K}{\sqrt{K^2+4}}\right] \qquad K \neq o$$

$$(3)'\ E' = \frac{2}{K^2+1} \qquad K \neq o$$

---

[2] Hyde, this Bulletin, 8, p. 88; 1907.

These three curves are represented graphically by Fig. 2. The general equation derived above does not hold for $K=0$ when $x<a$, but is correct for any distance greater than zero, even though this distance be infinitely small. The illumination approaches, for example, when $x=1$, the value of 1, as K approaches 0, but it is conceivable that the illumination may have a different value at the mathematical point $K=0$.

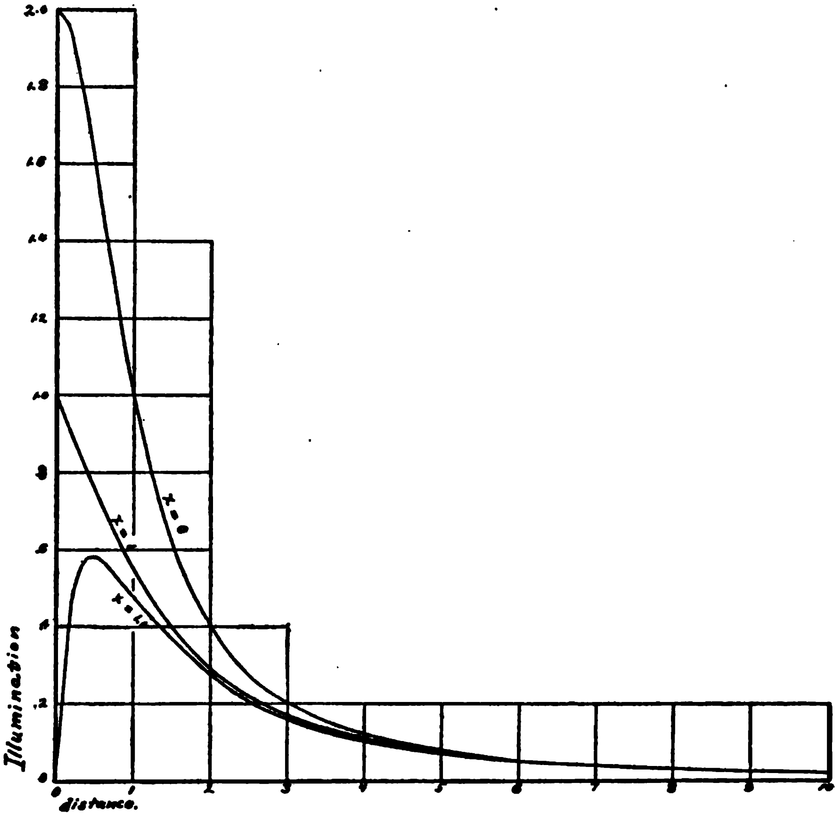


FIG. 2.—*Illumination versus distance from diffusely radiating disk*

Acknowledgment is due Dr. Olshausen, of this Bureau, for critically reading this manuscript and making suggestions.

WASHINGTON, October 30, 1915.

PHOTOMETRY OF THE GAS-FILLED LAMP [1]

By G. W. Middlekauff and J. F. Skogland

CONTENTS

Page.
I. Introduction 587
II. New variables and photometric difficulties 588
III. Experimental investigation of the variables 589
  1. Lamps, apparatus, and methods 589
  2. Preliminary measurements 590
  3. Current variation at constant speed with different voltages 590
  4. Current variation at constant voltage with different speeds 592
  5. Voltage variation at constant current with different speeds 594
  6. Candlepower variation at constant voltage with different speeds 595
IV. Law of the variations 596
V. Two photometric methods 597
  1. Rotating lamp method 597
  2. Total flux method 598
VI. Possible errors introduced by rotation 599
VII. Investigation of the cause of the variations observed 600
VIII. A probable explanation of the cause of the variations observed 601
IX. Summary 602

I. INTRODUCTION

In a recent paper [2] the authors have shown that for all vacuum tungsten lamps, within a wide range of wattage, the voltage-current-candlepower characteristics are the same regardless of the make or method of manufacture. When this investigation was extended to include gas-filled lamps, it was soon discovered that before consistant and reproducible results could be obtained on the photometer, new and unexpected variables, not present in the vacuum lamp, had to be carefully considered. It is with an experimental study of these variables that this paper is particularly concerned.

---

[1] A preliminary paper on this subject was read at a meeting of the scientific staff, Bureau of Standards, Nov. 20, 1914, and afterwards published in the Electrical World, Dec. 26, 1914, p. 1248.

[2] Middlekauff and Skogland, Characteristic Equations of Tungsten Filament Lamps and Their Application in Heterochromatic Photometry, this Bulletin, 11, p. 483 (Scientific Paper No. 238).

II. NEW VARIABLES AND PHOTOMETRIC DIFFICULTIES

In the gas-filled lamp the filament is coiled into the form of a closely wound spiral or helix, the diameter of which, though small, is large in comparison with the filament itself. Hence, as viewed from certain directions perpendicular to the axis of the lamp, one section of this helical filament may, more or less completely, cut off the light from another section. In addition to this cause of irregularity in the light distribution, there is usually a lack of symmetry in the arrangement of the filament as a whole about the vertical axis of the lamp. Consequently, the light on the photometer screen flickers excessively when the lamp is rotated at ordinary speeds. Hence, without auxiliary apparatus, accurate measurements of mean horizontal candlepower are practically impossible; the weight of the lamps, especially of the larger units, the necessity of using mercury in the rotator, etc., preventing a sufficient reduction of the flicker by the use of high speeds of rotation. Fortunately, this difficulty is readily overcome by placing back of the lamp two mirrors[3] inclined to each other so that the photometer screen is illuminated simultaneously by the direct light from the lamp and the reflected light from the mirrors. With this arrangement very low speeds of rotation may be used.

Although this expedient obviates the flicker difficulty, it does not eliminate the most serious trouble caused by rotation. It was found that at constant voltage both current and candlepower have different values when the lamp is rotating than when it is stationary, the current changing in one direction and the candlepower always in the opposite direction. In other words, there is a change in the operating efficiency of the lamp although the voltage remains the same.[4] It was found, further, that the values of current and candlepower may be either increased or decreased by rotation, depending upon the speed, and by amounts sufficiently great to affect seriously the result of candlepower and efficiency determinations.

---

[3] This Bulletin, **2**, p. 416; 1906.

[4] This general effect was observed about the same time also at the Electrical Testing Laboratories, New York, and described by Dr. C. H. Sharp, Trans. Illum. Eng. Soc., **9**, p. 1021; 1914.

The gas-filled lamp therefore presents a new problem in photometry, involving not only greater difficulties on account of greater color difference over those encountered in photometering vacuum lamps but also the difficulty of dealing with variables which, if not properly considered, may introduce considerable uncertainty in the results.

Below are given some experimental results obtained in a systematic study of the lamp and from these the law of the variations is deduced. From a consideration of this law a practical method of measuring mean horizontal candlepower free from the errors caused by rotation is discovered and an explanation of the cause of the variations observed is offered.

III. EXPERIMENTAL INVESTIGATION OF THE VARIABLES

1. LAMPS, APPARATUS, AND METHODS

The lamps used in the principal experiments described below were of the nitrogen-filled 450-watt series and 750-watt and 1000-watt multiple types, all having round bulbs with the filaments centrally mounted. The number of anchor wires in the different lamps varied from 5 to 9. Additional experiments were made with a number of 200-watt series and 750-watt multiple lamps with straight side bulbs, the former having one anchor wire, the latter five. Other experiments were made with an ordinary vacuum lamp filled with nitrogen and also with a vacuum lamp having a coiled filament such as used in the gas-filled lamp.

Measurements of voltage and current were made simultaneously by means of two potentiometers. The rotator was supplied with mercury-cup connections and two sockets so that the lamp could be mounted either tip up or tip down. The rotating mechanism was under perfect control at every speed, which was determined by a specially designed indicator, and the current to the lamp was supplied from a storage battery, thus insuring perfectly steady working conditions. In the candlepower measurements use was made of the two-mirror arrangement described above.

A few rough preliminary tests indicated that for any change in speed the percentage change in candlepower was at least several times the percentage change in current but always in the opposite direction. As current can be measured with considerably higher accuracy than candlepower, such measurements furnish a convenient method of determining approximately the variation of the candlepower. It was therefore decided that before making careful determinations of candlepower, it would be advisable to make a series of measurements of voltage, current, and speed, varying only one of these quantities at a time.

2. PRELIMINARY MEASUREMENTS

The general effect of rotation on current and power consumption with voltage constant was first determined in three series of fairly accurate measurements made on a 116-volt, 750-watt, short-necked lamp at 90, 105, and 116 volts, respectively. In each series current readings were taken at different predetermined speeds, a reading with the lamp stationary being taken as a check between the changes of speed. During these measurements the lamp was used first tip up and then tip down. With the lamp in either position and the speed above 40 revolutions per minute the current was less than with the lamp stationary and decreased with increasing speed, the effect with tip up being about twice as great as with tip down. Other lamps of the same size and of different sizes (450 watts to 1000 watts) and of different manufacture were tested in the same manner and similar results were obtained in every case.

In these preliminary measurements it was learned that very consistent and reproducible current values could be obtained provided the speed of rotation is accurately known and properly controlled; otherwise apparently inconsistent and erratic results were to be expected. Hence, in all subsequent tests the speed was as carefully measured and controlled as the voltage and current.

3. CURRENT VARIATION AT CONSTANT SPEED WITH DIFFERENT VOLTAGES

The next step taken was to determine with great care the changes in current due to rotation at a specified speed at different values of voltage through a wide range. This test was applied to two lamps, the 750-watt lamp mentioned above and a 1000-watt lamp, both rated at 116 volts. Readings of current corresponding to a chosen voltage were first made with the lamp stationary, then with it rotating at a speed of 150 rpm. In this way

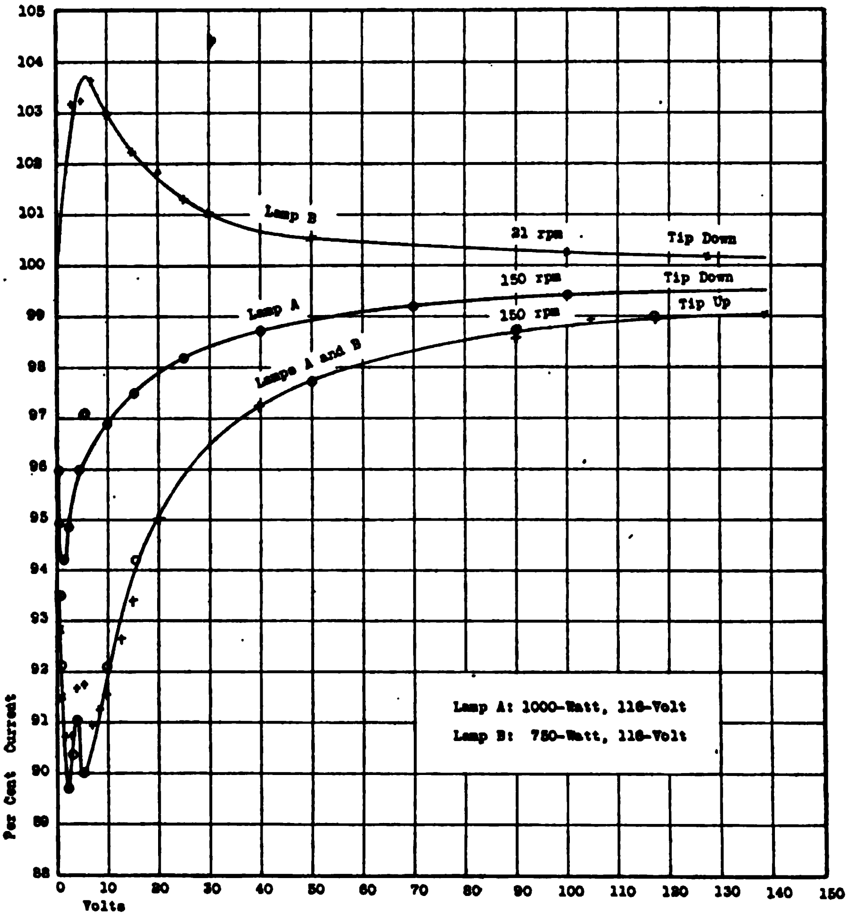


FIG. 1.—*Ratio of current with lamp rotating at constant speed to current with lamp stationary*

the changes in current for each lamp at different voltages over a range from 0.5 volt to 100 volts were obtained.

The results for both lamps rotated tip up are shown by the lowest curve of Fig. 1, values obtained for the 750-watt lamp being represented by crosses and those for the 1000-watt lamp by circles. The behavior of the current in these two lamps is thus shown to be practically identical at every value of voltage except from 3 volts to 8 volts, which is a critical region. It is probable that even within this region these curves would be found to coincide also were measurements made at a sufficiently large number of neighboring points. The upward indentation in the curve at this point is as real and reproducible as any other part of the curve.

Upon rotating these lamps tip down, it was found that for the same speed the percentage change in current for all voltages was only about half as great as when the lamps were rotating tip up. The values for the 1000-watt lamp tip down are given by the middle curve of Fig. 1. This curve is traced through all values obtained except one in the neighborhood of 5 volts. This value indicates that an indentation would probably have been found in this curve also had sufficient time been taken to pass by small voltage steps through this region. It might be stated that, in order to obtain sufficiently steady conditions for accurate measurements of current in this region, a period of at least 10 minutes was required at each point, as small variations of speed resulted in relatively large changes in current. These two curves are similar in form, both becoming asymptotic to the axis of abscissas at about 1 per cent and 0.5 per cent, respectively, below the current observed for the lamp stationary. That is, for this speed, at about 85 per cent of the normal voltage, the current reaches a value which from that point on bears a practically constant ratio to the value at zero speed.

4. CURRENT VARIATIONS AT CONSTANT VOLTAGE WITH DIFFERENT SPEEDS

In order to investigate further the behavior of the current in the critical region above mentioned, as well as at other points, measurements of current at constant voltage, with variable speed, in succession at different values of voltage were made on the 1000-watt lamp through a considerable range extending as low as 0.5 volt. The speed was varied from 5 rpm to 200 rpm, the lamp being rotated tip down. The results of this test are shown in

Fig. 2. The important fact indicated by these curves is that as the speed is increased from zero the current, at constant voltage, first increases to a maximum and then decreases to the stationary value, and continues decreasing below the stationary value as the speed is farther increased. (By stationary value is meant the value obtained with the lamp stationary.)

In order to confirm this change in sign of the current variation the 750-watt lamp was tested tip down while rotating at a speed of 21 rpm. The results of this test are shown by the uppermost curve of Fig. 1. It will be noted that this curve is of practically the same form as the other two curves of this figure, but that it is turned in the opposite direction and lies entirely above the 100 per cent or stationary values. That is, at this speed of rotation the current is higher than at zero speed for all values of voltage. Hence, correct qualitative results may be obtained at any practical voltage.

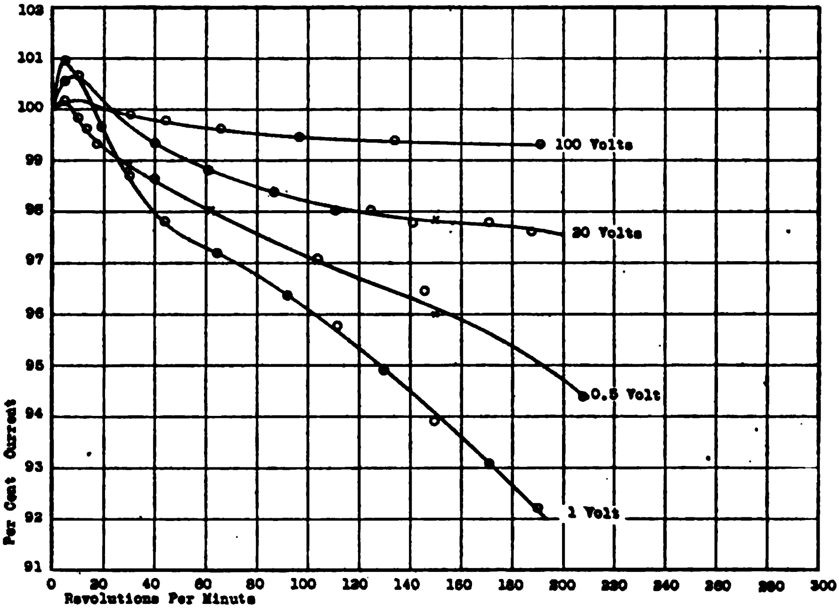


FIG. 2.—*Ratio of current with lamp rotating tip down at different speeds to current with lamp stationary; voltage constant at different values*

Upon carefully testing all available 450-watt to 1000-watt lamps, including both series and multiple types in round bulbs and one 750-watt lamp in a straight-side bulb, it was found that in every lamp while rotating with the tip down the current at constant voltage returned to the stationary value at a speed between 33 rpm and 40 rpm. This held true through a considerable range of voltage, extending above and below the normal, or rated, value. The series lamps included in this test had the same form of mount and practically the same arrangement of the filament as the multiple lamps.

For 200-watt, 6.6-ampere lamps, with V-shaped filament and a single anchor wire, it was found necessary to use a speed of from 140 to 160 rpm to obtain the stationary value of the current, the value of the current being greater than the stationary value up to the speed mentioned. It appears, therefore, that the smaller the number of loops and corresponding anchor wires in the lamp the greater the speed that is required to give the stationary value of the current.

Although there was a considerable difference in the speeds required for lamps having different numbers of loops in the filament, it was possible to find for every lamp tested a particular speed at which the current returned to the stationary value, the speed being in every case such as can be used in practice.

5. VOLTAGE VARIATION AT CONSTANT CURRENT WITH DIFFERENT SPEEDS

The effect of rotation on voltage was determined by testing a few of the 450-watt lamps, tip down, with the current held constant at 6.6 amperes and the speed varied over a wide range, beginning at 5 rpm as the lower limit. For every lamp it was found that as the speed was increased from zero the voltage first decreased to a minimum and then increased to the stationary value and continued increasing above the stationary value as the speed was further increased. In other words, the effect of rotation upon the voltage at constant current was opposite in sign to the effect upon the current in the same lamps at constant voltage.

Since the variable quantity (i. e., voltage or current) in either case returns to its stationary value, it is evident that the speed at which this occurs, and therefore at which the power consumption is the same as when the lamp is stationary, may be determined by holding either quantity (voltage or current) constant and observing the variation of the other as the speed is gradually increased from zero. Principally as a matter of convenience, therefore, all subsequent tests were made at constant voltage.

6. CANDLEPOWER VARIATION AT CONSTANT VOLTAGE WITH DIFFERENT SPEEDS

The effect of rotation on candlepower was very strikingly shown by the following experiment: The voltage was so adjusted that the filament when stationary was just visible in a darkened room. Upon rotating the lamp at a speed of about 180 rpm it was found that, although the current decreased in value, the intensity of the light largely increased, the effect being greater with the tip up than with the tip down, thus corroborating the preliminary measurements of current and candlepower at higher voltages. With the lamp in either position and either rotating or stationary the upper loops of the filament glowed more brightly than the lower ones, thereby indicating a considerable difference in temperature between closely neighboring regions of the gas.

Since there was a speed at which the power consumption was the same as when the lamp was stationary, it was considered probable that at this same speed the candlepower also would return to the stationary value. This led to a series of simultaneous measurements of current and relative values of the candlepower, with voltage constant and speed varied over a wide range.

Results obtained in this way with the 750-watt lamp operated at 90 volts are shown by the curves of Fig. 3. All values of current and candlepower, respectively, are expressed in percentages of the value of each obtained with the lamp stationary and the tip down. In determining the candlepower with the lamp stationary, readings were taken at intervals of 7.5° about the vertical axis, there being a total of 48 determinations.

The curves for the lamp with the tip down are the result of two independent series of measurements made on different days, values obtained in the two series being represented by crosses and circles, respectively. The agreement of these two series shows that current and candlepower values are reproducible to a high degree of precision.

The ratio of the ordinates to which these curves were drawn was purposely so chosen as to clearly represent to the eye the constant relation of candlepower to current throughout the whole range of speed employed.

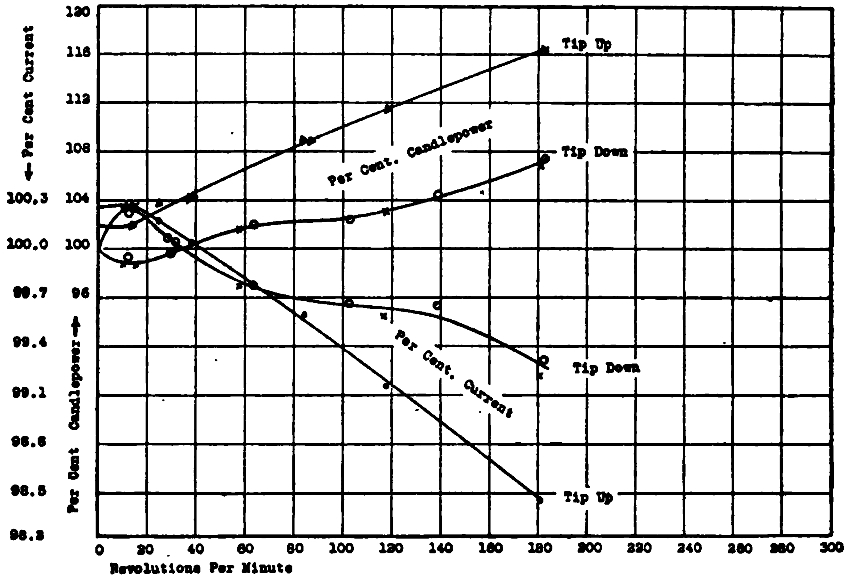


FIG. 3.—*Ratio of current and candlepower with lamp rotating tip up and also tip down to current and candlepower, respectively, with lamp stationary tip down*

IV. LAW OF THE VARIATIONS

The results of these experiments may be summarized as follows, in which the voltage is to be regarded as constant:

1. With the lamp stationary, both current and candlepower have higher values when the lamp is mounted tip up than when it is mounted tip down.

2. Rotation of the lamp causes both current and candlepower to change in value but in opposite directions, the change in both being about twice as great when the tip is up as when it is down.

3. As the speed is increased from zero the current first increases to a maximum and then returns to the value it has when the lamp is stationary and continues decreasing below this value as the speed is further increased. The candlepower varies in the opposite direction.

4. For each position of the lamp there is a particular speed at which both current and candlepower have each the same value as when the lamp is stationary, the speed at which this occurs being practically the same for all lamps having the same number of loops in the filament. For lamps having different forms of filament mounting the speed for the above condition varies from lamp to lamp, being highest for those having the smallest number of loops.

V. TWO PHOTOMETRIC METHODS

1. ROTATING LAMP METHOD

From a consideration of the above results it is evident that if the voltage be held constant and the lamp be rotated at the speed at which the current has the same value as when the lamp is stationary an accurate determination of mean horizontal candlepower may be made and the same efficiency rating given to the lamp as it would have if operated stationary at the same voltage. It is fortunate that in the position in which gas-filled lamps are most generally used in practice—that is, tip down—the speed for the above condition is somewhat higher and therefore more practical for most lamps than the speed for the same condition with the tip up.

Although the above method has its limitations which are stated below, it fulfills all the requirements for determining the voltage-current-candlepower characteristics of gas-filled lamps by the use of an ordinary bar photometer outfit. With a knowledge of the reduction factor, the watts per candle of the lamp may be determined for any given voltage, and then with the lamp stationary and with this voltage as a starting point the variations of current, candlepower, and watts per candle with voltage are as readily determined for a gas-filled lamp as for one of the vacuum type. In watts per candle determinations the factor expressing the relation of mean spherical to horizontal candlepower in the given direction must, of course, be applied to the candlepower readings.

For groups of lamps having the same form of mounting, and therefore the same reduction factor, the method of rotation is perfectly satisfactory in determining relative candlepower and efficiency ratings, and practically the same speed may be used for all lamps of the group. However, as stated above, this method presupposes a knowledge of the reduction factor, which for gas-filled lamps in general, as now made in the present more or less experimental stage of manufacture, varies considerably from lamp to lamp, principally on account of the great variety of methods of mounting the filament.

2. TOTAL FLUX METHOD

During life tests of vacuum lamps the blackening of the bulb in all directions is about proportional to the distribution of the light, and hence a measure of the reduction in light during life may be obtained by observing the reduction in the mean horizontal candlepower. In the gas-filled lamp, however, the greater proportion of the blackening occurs at the top of the bulb, the volatilized material thrown off from the filament being carried upward by the gas, and hence a true measure of the reduction in light during the life of the lamp can not be obtained by mean horizontal candlepower measurements alone but by determinations of the total flux or mean spherical candlepower. This can be most conveniently done by means of an integrating photometer, such as the Ulbricht sphere. The advantage of the sphere over other forms of integrating photometers, such as the Matthews instrument, in measuring gas-filled lamps is that the lamps are measured while stationary and thus the complications arising from rotation are avoided.

For the determination of the voltage-current-candlepower characteristics the sphere has no particular advantage over the method of rotation and has the possible disadvantage in that if the coating of the sphere is not perfectly nonselective the ratio of the candlepower at different voltages may be more or less affected by a change in the absorption of the light at the different colors.

VI. POSSIBLE ERRORS INTRODUCED BY ROTATION

The accompanying table shows the errors possible in efficiency determinations and the consequent errors in life values introduced by measuring candlepower and watts while the lamp is rotating. The values given are the result of photometric measurements made at different speeds on a 750-watt, 115-volt, nitrogen-filled lamp having nine anchor wires supporting the filament.

Data on Errors in Efficiency and Life Values

Position of lamp	Speed in rpm	Rated volts for 0.70 watt per candle, lamp rotating	Watts per candle at rated volts; lamp stationary, tip down	Per cent error in rated volts	Per cent error in rated watts per candle	Per cent life at rated volts; lamp stationary, tip down
Tip down	0	115.0	0.700	0.0	0.0	100
	35	115.0	0.700	0.0	0.0	100
	70	113.5	0.718	1.3	2.6	121
	120	112.8	0.726	1.9	3.7	131
	180	109.9	0.760	4.4	8.6	184
Tip up	0	113.8	0.713	1.0	1.9	115
	14	113.8	0.713	1.0	1.9	115
	70	110.1	0.757	4.3	8.1	178
	120	107.4	0.793	6.6	13.3	252
	180	104.4	0.837	9.2	19.6	375

On account of the increased efficiency of this lamp caused by rotation, for example, at 70 rpm tip down, it was found to have a specific consumption of 0.7 watt per candle at 113.5 volts instead of at 115 volts as found at zero speed with the tip down. If, therefore, the lamp had been photometered at the factory while rotating at this speed, it would have been rated at 113.5 volts and would have been operated on the life rack at 0.718 watt per candle instead of 0.7 watt per candle and would have had a life 21 per cent greater than would be expected from the rating. If, however, it had been photometered with the tip up at 180 rpm (as is sometimes done in practice), it would have been rated as having a specific consumation of 0.7 watt per candle at 104.4 volts. If put upon the life rack at this voltage, it would operate at 0.837 watt per candle instead of 0.7 watt per candle and give a life 275 per cent in excess of normal. In other words, a long-life performance is fictitious at the given specific consumption rating, or the consumption rating is fictitious for the given life. This holds true if the specific consumption is determined at any speed above that at which the current and candlepower have the same value as when the lamp is stationary with the tip down. For this lamp the particular speed is 35 rpm. For speeds below this value with the tip down, the specific consumption rating would be too high and the life value would be lower than expected from the rating.

VII. INVESTIGATION OF THE CAUSE OF THE VARIATIONS OBSERVED

The following experiments were made with a view to finding the cause of the variations observed in this type of lamp:

1. To determine the effect of increased external ventilation of the bulb caused by rotation, a draft of air was driven against a 1000-watt lamp by means of an electric fan and a series of measurements of current at constant voltage were made at different speeds with "air on" and "air off." It was found that at every speed the cooling effect of the air caused a small but measurable increase in the current but that this increase was very small in comparison with the decrease in current caused by the rotation of the lamp. Hence, the decrease in current can not be attributed to an increase in external ventilation of the bulb by rotation.

2. Another test was made to determine whether or not the earth's magnetic field had a noticeable effect in producing a counter electromotive force, thus changing the effective voltage on the lamp. This was done by placing the lamp in the middle of a solenoid having a field strength of about 500 gausses, and the current in the lamp was observed when the current in the solenoid was thrown on and off, both with the lamp stationary and rotating. As no change was detected it was concluded that the much weaker field of the earth certainly did not produce a counter emf of sufficient magnitude to account for any part of the changes noted.

3. To determine the effect of possible distortion of the filament upon the candlepower in the horizontal plane, candlepower measurements at various angles with the vertical were made upon the

1000-watt lamp (lamp A, figs. 1 and 2) rotating with the tip down, first at a speed of 45 rpm and then at a speed of 200 rpm. There was practically the same percentage increase in candlepower at every angle when the lamp was rotating at the higher speed—that is, the spherical reduction factor, which was about 0.86, was not appreciably changed by rotation. Hence distortion of the filament was not the cause of the changes observed.

4. A 100-watt vacuum tungsten lamp having a coiled filament similar to that of the gas-filled lamp, and similarly mounted in the bulb, when rotated exhibited none of the changes observed in the case of the gas-filled lamp. The same was found to be true for a 500-watt vacuum tungsten lamp having an ordinary (not coiled) filament mounted in the usual manner. On the other hand an ordinary 250-watt lamp when filled with nitrogen exhibited all the variations of the coiled filament gas-filled lamp, the current at constant voltage returning to the stationary value at a speed of about 100 rpm. It was therefore concluded that the gas alone must be the disturbing element which causes the changes observed in the gas-filled lamp.

VIII. A PROBABLE EXPLANATION OF THE CAUSE OF THE VARIATIONS OBSERVED

The results of the above tests show quite conclusively that practically the whole of the effect observed in the change in current, candlepower, and specific consumption arises from a change in the convection currents of the gas within the bulb. At low speeds, as seen above, there is an increase in the current over the stationary value for all voltages, the greater percentage increase being at the low voltages. At high speeds there is a decrease in current as compared with the stationary value for all voltages, the greatest percentage decrease being also at low voltages.

When the lamp is stationary there is a considerable difference in temperature between the gas at the top and bottom of the bulb. Very low speeds of rotation disturb the steady current of hot gas which is rising in the center and flowing down at a distance from the axis. The result is an increase in heat convection by the gas, and consequently a lowering of the temperature of the filament.

As the filament has a positive temperature-resistance coefficient, the resistance decreases and consequently the current increases at constant voltage. At high values of voltage the mean temperature of the gas is higher than at low voltage, consequently the temperature of the filament is reduced in a lesser degree by rotation and the change in current, though positive, is less at high than at low voltage.

On the other hand, when the lamp is rotated at high speeds the cooler gas at the bottom of the bulb is thrown out by centrifugal force, which varies as the square of the speed, and the hot gas tends to remain near the center, thus considerably retarding the convection currents which cool the filament. The result is an increase in the resistance of the filament as a whole and a decrease in the value of the current. At high voltages the mean temperature of the gas is higher than at low voltage and the retarding effect due to centrifugal force is decreased; hence the change in current due to rotation is less than at low voltages.

At low speeds, as seen above, the temperature of the filament is lowered and consequently the candlepower is decreased. At high speeds the temperature of the filament is raised, and hence there is an increase in the candlepower.

IX. SUMMARY

The new high efficiency gas-filled lamp introduces variables not hitherto encountered in the photometry of incandescent electric lamps. On account of the comparative broadness of the filament spiral and the dissymmetry of the filament mounting, there is considerable irregularity in the distribution of the light about the vertical axis. Consequently, when the lamp is rotated, as is commonly done in rating lamps at the factory, the light as seen in the photometer flickers so excessively as to render accurate measurements of candlepower practically impossible without the use of auxiliary apparatus. However, as is sometimes done, if two mirrors inclined to each other be placed back of the lamp, the flickering is so much reduced as to permit accurate candlepower measurements even at very low speeds of rotation.

But this expedient does not eliminate the most serious trouble caused by rotation. It was found that at constant voltage both the current consumed and the candlepower are different when the lamp is rotating than when it is stationary, the current changing in one direction and the candlepower always in the opposite direction; that is, there is a change in the operating efficiency of the lamp. Furthermore, this change in efficiency may be either positive or negative, depending upon the speed, and it is about twice as great when the lamp is rotating tip up as when it is rotating tip down.

Fortunately, from the standpoint of photometry, there is for each lamp in either position a particular speed at which the current and the candlepower have the same values, respectively, as when the lamp is stationary. Hence, with the lamp rotating at this speed its candlepower can be measured with accuracy in spite of its rotation. The speed for the above condition is practically the same for all lamps having the same number of loops in the filament; but for lamps having different forms of filament mounting it varies from lamp to lamp, being greatest for those having the smallest number of loops in the filament.

If the above precaution as to speed adjustment is not observed and lamps are rated while rotating at speeds ordinarily used in photometering vacuum lamps, the errors which enter may amount to as much as 1 to 2 per cent in current, or watts, in one direction, and as much as 15 to 20 per cent in candlepower in the opposite direction. Hence the voltage found for a desired operating efficiency may be so much in error as to give a lamp on test at this rated voltage a fictitious life value three or four times as large as the lamp would give if it were operated stationary at a voltage corresponding to that efficiency which during the rating was only apparent. That is, the lamp may be given credit for a much longer life than it really deserves. On the other hand, the speed may be such as to cause errors in the opposite direction resulting in a lamp life much shorter than would be expected from the apparent efficiency rating.

Another peculiarity of the gas-filled lamp is that while it burns the blackening occurs, not all over the bulb in approximate proportion to the light distribution as in the vacuum lamp, but principally at the top of the bulb because the volatilized material is carried upward by the gas. Hence in making a life test a true measure of the reduction in total light during the life of the lamp can not be obtained, in the usual manner, by mean horizontal candlepower measurements, but by determinations of the total flux or mean spherical candlepower. This is accomplished most rapidly and conveniently by means of an integrating photometer, such as the Ulbricht sphere, in which the lamp is measured stationary, and thus all the complications arising from rotation are entirely avoided.

As to the cause of the variations observed in candlepower and efficiency when the lamp is rotated, it is concluded from the results of a number of special tests that the whole effect is produced by a change in the convection currents of the gas, a consequent variation in the temperature distribution in the bulb, resulting in a change in the resistance, and therefore a variation in the current and candlepower of the lamp.

The authors acknowledge their indebtedness to Dr. E. B. Rosa for his kindly interest and valuable suggestions, and to H. B. Sinelnick for efficient assistance in the laboratory work.

WASHINGTON, January 25, 1915.

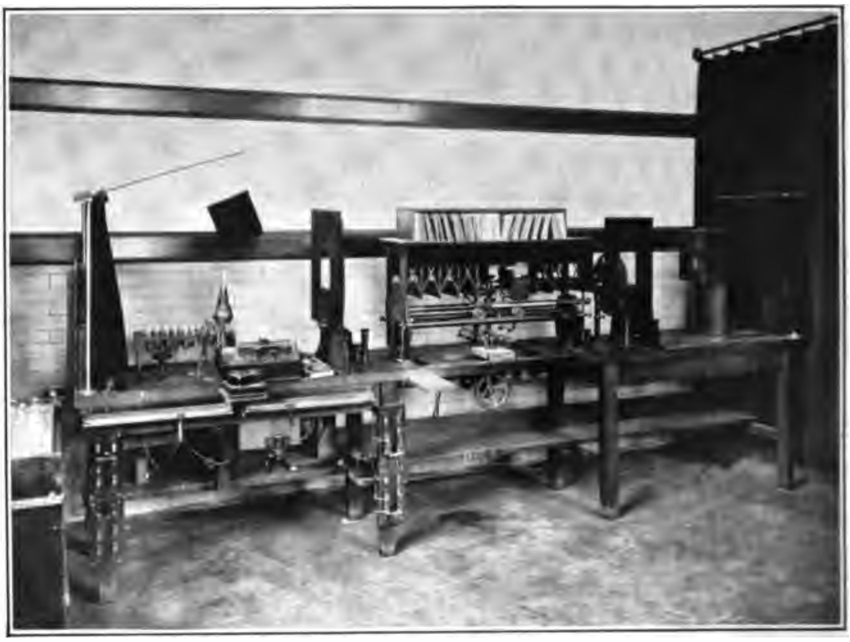


FIG. 1.—*The life-test photometer*

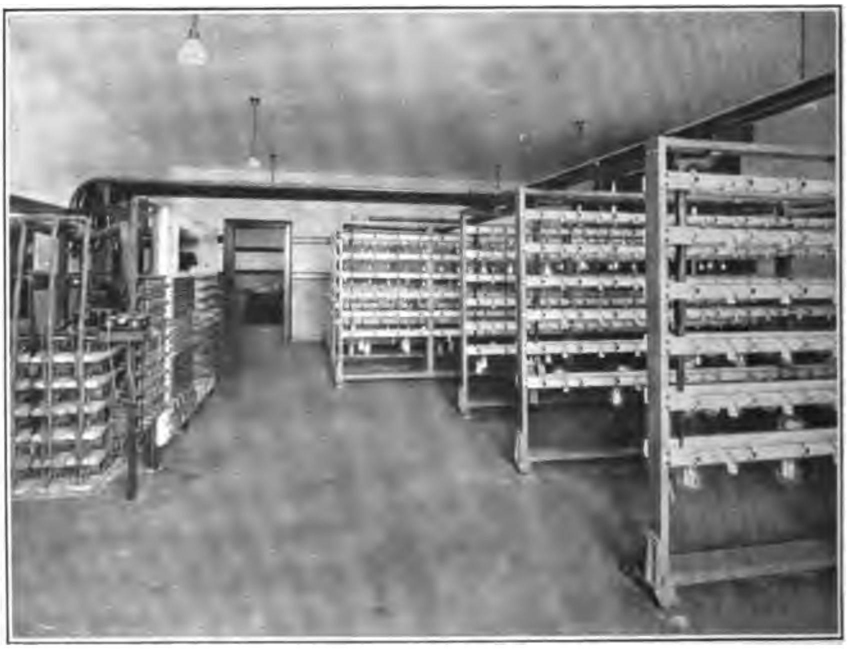


FIG. 2.—*Transformers, switchboard, and life racks*

LIFE TESTING OF INCANDESCENT LAMPS AT THE BUREAU OF STANDARDS

By G. W. Middlekauff, B. Mulligan, and J. F. Skogland

CONTENTS

Page
I. INTRODUCTION 605
II. PURPOSES OF A LIFE TEST 607
  1. General 607
  2. Special purposes of Bureau of Standards tests 608
III. SELECTION OF LIFE-TEST LAMPS 609
IV. MEASUREMENT OF LIFE-TEST LAMPS 610
  1. The life-test photometer 610
    (a) General construction 610
    (b) Instruments and candlepower scales 611
    (c) Wiring and special resistances 612
    (d) The watts-per-candle computer 614
    (e) The recording device 615
    (f) Features of the record 620
      α. Detection and compensation of errors 620
      β. Increased accuracy in life values 621
  2. Methods of measuring and recording observed values 621
    (a) Rating of lamps for life test 621
    (b) Details of a photometric run 622
V. THE LIFE TEST 625
  1. Design of the installation 625
    (a) Wiring and voltage adjustment 625
    (b) Voltage regulation 627
    (c) Current generator and voltage transformers 627
    (d) The life-test racks 628
    (e) Measurement of life-test periods 628
  2. Records taken during life test 629
VI. SUMMARIES OF LIFE VALUES 629
VII. SUMMARY 630

I. INTRODUCTION

The first edition of "Standard Specifications for the Purchase of Incandescent Electric Lamps,"[1] issued in 1907, was the result of concerted action on the part of the Federal Government departments, representative lamp manufacturers, the Electrical Testing Laboratories, and the Bureau of Standards. The purpose of these specifications was to establish such standard methods of initial inspection and life testing as would permit their adoption by the Government and make them available to the general public; so that all purchasers of incandescent lamps, by including these specifications in contracts, might realize the benefits of their use.

---

[1] These specifications are issued by the Bureau of Standards as Circular No. 13, which has been revised from time to time and is now in the seventh edition.

Application of these specifications necessitates careful initial inspection and reliable life tests. The specified life-test procedure is so exacting and the quantity of lamps to be tested on any considerable contract so large that the purchaser, unless his facilities for testing are complete, must of necessity refer at least the life-test work to some reputable testing laboratory. It was, therefore, the natural outcome that the Bureau of Standards should be sought and recognized by departments of the Government as the authority on life tests. Initial inspection is so closely related to life-test procedure and its efficiency so pronounced in the effect on the results of life test that the Bureau, almost of necessity, undertook this part of the work as well.

The design of a life-test installation was therefore begun early in 1908. This was developed by Messrs. E. P. Hyde, F. E. Cady, C. F. Sponsler, and H. B. Brooks, under the direction of Dr. E. B. Rosa, chief of the electrical division, which included the photometric section. A lamp inspector was appointed in July and the plant was put into operation in October of the same year (1908). About this time Dr. Hyde and Mr. Cady left the service of the Bureau and the work has since been carried on and developed mainly by the authors of the present paper, under the direction of the chief of the electrical division.

The whole life-test equipment was originally installed in the mechanical building which houses the power plant of the Bureau. In 1913 the life racks, transformers, and photometric apparatus were removed to two adjoining rooms on the third floor of the new electrical building which was then nearing completion. Although some parts of the equipment are differently arranged in the new building, the general plan has remained the same as originally designed.

The introduction of new classes of lamps, however, rendered it advisable to make considerable changes in the original photometric equipment and in the details of the method of testing. These changes have been made from time to time by those who have been most intimately associated with the work. The equipment as it now stands and the present method of the Bureau's life-testing procedure in all its details are, therefore, the result of a gradual development in which various persons have been of assistance.

From the beginning the magnitude of the work of inspection and life testing has been constantly increasing year by year in consequence of the natural growth of the Government's purchases of incandescent lamps. Fortunately, however, the quality of the lamps supplied has, in most cases, been fairly uniform and also above the requirements of the specifications, so that full and reliable data on the lamps supplied by each manufacturer have been obtained by submitting to life test a yearly total of not over 5,000 lamps which represent about one and a quarter millions of inspected lamps.

Since inspections and tests are made primarily for departments of the Government, outside tests are accepted only "when special circumstances make the test of more than usual importance." A specified fee is charged for work of this kind.[2]

In the following description of apparatus and methods of life test an attempt is made to indicate the essential features of this work and the manner in which the testing is at present actually conducted.

II. PURPOSES OF A LIFE TEST

1. GENERAL

A life test may be run for any one of several reasons. For example, a manufacturer who desires quick results in order to test the effect of some modified construction or change in material may choose to burn the lamps selected at a voltage greatly in excess of that employed in normal operation, thus causing the lamps to fail in a few hours. Unwarranted confidence is sometimes placed in tests of this kind for other purposes, and attempts are made to evaluate life at normal voltage from the test results, whereas no known constants for these life corrections will apply in all cases. Although relative results may be of some value, they often point to conclusions not at all in agreement with those which might be drawn from a test at a voltage corresponding more nearly to rated efficiency.

---

[2] Fees for Electric, Magnetic, and Photometric Testing; Bureau of Standards Circular No. 6, p. 26; 1914.

Comparative tests of greater value may be run at or near normal operating efficiency, even on a line of uncertain voltage regulation, by placing both tests side by side on the same circuit. However, the voltage applied to the lamps of each test must be such that the average efficiency of the two groups is the same, or, if differently rated and burned at one voltage, correction factors must be applied to reduce the test results of one group to their equivalent life at the efficiency of the other group. In all cases the initial (test) efficiency must be known, if test results are to be correctly interpreted. It should be emphasized that relative results only are obtained by such a test, unless the voltage regulation is that indicated in the specifications under which the lamps are tested.

In contradistinction to these rough tests are those in which actual values of life at normal efficiency are obtained for any group of lamps. This necessitates great care in initial rating and constancy of voltage at which the lamps are operated on the life test. By choosing test efficiencies within a range through which factors for life correction have been fully established, the time necessary to complete the tests may be materially shortened. Life tests at the Bureau of Standards are of this kind.

2. SPECIAL PURPOSES OF BUREAU OF STANDARDS TESTS

Although the chief concern of departments of the Government in connection with tests under Standard Specifications is to secure reasonably prompt delivery of lamps which meet the specified requirements, a consideration almost equal in importance is the determination from the life tests of the relative standing of the various manufactures as regards quality of output. The relative quality thus determined is referred to and given due weight in deciding upon future awards of contract.

The evaluation of a lamp life to as high a degree of accuracy as is possible in testing a large quantity of lamps has no doubt guided the manufacturers to some extent in their improvements of efficiency ratings, notably in the tungsten lamp. Consequently manufacturers and purchasers receive all available service and assistance not only from the actual test results but also from conclusions drawn therefrom.

III. SELECTION OF LIFE-TEST LAMPS

The Standard Specifications, in accordance with which all Bureau tests of lamps for Government are made, recognize the importance of a proper selection of samples for life test. It is assumed that no lamp can accurately represent the life of a group unless it accurately represents the group in other respects. Hence, great care is exercised in the selection of the samples for life test, and no sample is taken unless the lamps have first passed the prescribed initial tests.

These initial tests are made by Bureau inspectors[3] at the factory of the manufacturer, and regular factory apparatus is used. Such testing equipment as is required in the work of inspection is usually assembled in an inspection department, so that factory work is not interfered with. In the larger factories, where initial tests under specifications are made for a number of purchasers, certain operators are employed most or all of the time in the inspection department. It is their duty to render the inspectors such assistance as may be required in making initial tests. Besides one or more photometers, this department contains vacuum test equipment, special sockets supplied with current for lighting up the test lamps, and, in factories manufacturing tungsten lamps, racks for seasoning or "aging" the lamps selected. This last-named equipment has been introduced as required by Standard Specifications, because of the new process of exhaust, which produces a ductile filament, not, however, stable in its electrical characteristics; so that a certain amount of burning is necessary before the current and candlepower reach values sufficiently steady for accurate measurement.

---

[3] One inspector is employed continuously and another is sent out to assist him when necessary.

The quantity of lamps selected for initial tests is specified as 5 per cent of the total of a lot including only lamps of the same size, class, and voltage range, and not less than 10 lamps from any one lot. The number of lamps to be included in a lot is left to the judgment of the inspector.

The lamps must conform to certain specified requirements as regards bulbs, bases, filaments, and vacuum. Lamps which pass these requirements are then run on the photometer, and in determining their acceptability tables of allowable limits of watts and candlepower, or of watts per candle, as given in the specifications are applied. In calibrating the photometer for these tests the inspector uses standards which have been certified by the Bureau for candlepower and current. A lot of lamps is accepted if the number of defective lamps on either test is below the specified percentage of the total.

The next step is to compute from the records of the photometric test the mean values of individual groups of test lamps representing not more than 250 lamps from any one lot. The lamp nearest the mean value of each group is selected, labeled, and sent to the Bureau to represent the group on life test.

IV. MEASUREMENT OF LIFE-TEST LAMPS

In order to facilitate the photometric measurements of the life-test lamps and still secure a permanent, accurate, and as nearly as possible automatic card record of each lamp tested, certain modifications and additions have been made to the photometer used in this work at the Bureau. As these features are decidedly special and not found elsewhere, their construction and use are fully explained in what follows, not only that the method of measurement described later may be better understood but also that the equipment may be duplicated by anyone desiring to use it.

1. THE LIFE-TEST PHOTOMETER

(*a*) General Construction.—The general construction of the life-test photometer is shown in Fig. 1 (frontispiece). A Lummer-Brodhun contrast photometer head is mounted upon a movable carriage between the test lamp and comparison source, the distance between the last two mentioned being 250 cm. The comparison lamp, a 100-watt tungsten, is placed in a mirror-backed box fronted by a ground-glass window. This window presents an approximately uniform illuminated surface to the photometer, so that the glass plate acts as the effective light source and is so considered. The mirrors within the box are employed to increase the illumination of the window to a practical working value, the effective area of the window being adjusted by means of a variable diaphragm or shutter,[4] this adjustment being used in calibrating the photometer. By the screening system used stray light is so effectively excluded from the photometer screen that measurements are made in a curtained booth about 8 feet high under conditions which might be defined as approximately "semidaylight."

The standard lamp socket may be rotated in a direction depending upon the position of a knee switch which reverses the current in the armature of the motor, so that lamps may be rapidly turned in or out of the socket and may be rotated during measurements.

Current and voltage leads are joined to the lamp rotator by means of mercury cup connectors. Storage battery current is used in all measurements and available line voltage is adjusted by means of end-cell switches.

(*b*) INSTRUMENTS AND CANDLEPOWER SCALES.—Current through the standard or test lamp is read on a millivoltmeter connected across a separate shunt. Standard or test lamp voltage is read on a Brooks deflection potentiometer.[5] On this instrument the balanced portion of the emf is read from the dial which is arranged in steps of 2 volts. The unbalanced portion produces current in the galvanometer circuit, with consequent motion of a pointer over a scale calibrated in 0.1 volt divisions through a range of 1.5 volts above and below the dial setting, so that 0.01 to 0.02 volt is the smallest readable deflection, and the precision of any setting is within these limits. In practice a null method is used and voltages corresponding to dial settings are chosen in the measurement of all test lamps. Certain modifications described later have been made in the connections of this instrument to facilitate the convenient handling of large quantities of lamps.

---

[4] This arrangement of the comparison lamp and of a special resistance, described later, were introduced by Ives and Woodhull, who, for a short time, were associated with this work. See this Bulletin, **5**, p. 555.

[5] Brooks, H. B., A New Potentiometer for the Measurement of Electromotive Force and Current, this Bulletin, **2**, p. 225; 1906.

Several candlepower scales are mounted on a brass drum which fits within the front tube of the track. The normal scale is used when the photometer receives unmodified light from both test and comparison lamps. The choice of other scales depends upon the opening of the sectored disk[6] or the transmission of the glass screen used and upon whether these auxiliaries are used on the test or on the comparison side of the photometer. In routine work these scales are used only in calibrating the photometer, because the equipment installed eliminates all reference to actual values on the scales.

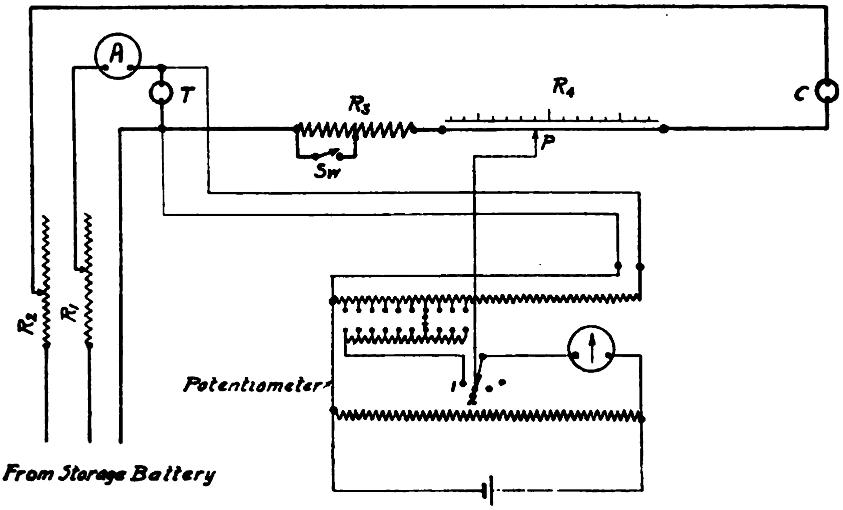


FIG. 3.—*Wiring diagram of the life-test photometer*

(*c*) WIRING AND SPECIAL RESISTANCES.—As shown in Fig. 3, the test and the comparison lamps are wired in separate circuits in order to permit a wide voltage range on the former without affecting the voltage on the latter. In the comparison lamp circuit, besides the adjustable rheostat R_2, there are two special resistances designated by R_3 and R_4, respectively. The purpose of these special resistances is to maintain the comparison lamp at certain definite colors and still permit a precise calibration of the photometer in terms of the group of standards used without making tedious experimental adjustments of resistance.

---

[6] For all work on this photometer an adjustable sectored disk is used.

With the resistance R_3 all in circuit the comparison lamp operates at the color of carbon test lamps. With a fixed amount of R_3 short-circuited by the switch *SW*, a color used in the measurement of tungsten lamps is obtained. When the standards are operated at the same color as the test lamps, a color match with the comparison lamp is obtained by placing a blue glass screen (the transmission of which need not be known) on the comparison side of the photometer. This is done in order that the comparison lamp may be operated at a comparatively low efficiency, and thus prolong its useful life. In case it is desired to run test lamps at an efficiency higher than that which would be safe for the standards, a glass screen of known transmission must be used with the comparison lamp while measuring the test lamps, but in calibrating the photometer the screen is replaced by the sectored disk so set that the percentage opening is equal to the transmission of the screen. In this way the standards are operated at the unmodified color of the comparison lamp and the test lamps at any desired color for which a color screen of the proper density for color match with the comparison lamp is selected.

The potentiometer button *2*, to which the galvanometer is switched in setting the comparison lamp, is connected to contact *P* on the slide-wire resistance R_4, which will be described presently. In the position shown it is evident that the drop from *P* across the portion of R_3 in circuit is measured. This drop is proportional to the current in the comparison-lamp circuit, and hence by a proper choice of resistance R_3 (which is large in comparison with R_4) the exact current in the comparison lamp for carbon color is obtained. As the voltage on the standards or test lamps is set with the switch lever on button *1*, a check can be kept on the current in the comparison lamp without disturbing the potentiometer setting by simply switching the lever to button *2*. Any necessary adjustment in the current is made by means of resistance R_2 to bring the galvanometer pointer back to zero.

In calibrating the photometer the adjustment of the comparison source is easily made to within 1 or 2 per cent in candlepower by means of the adjustable shutter on the ground-glass window. The final adjustment is made by moving contact *P* along the slide-wire resistance R_4 a distance corresponding to the desired small change in candlepower as read from a scale of candlepower differentials placed under the wire. The changes of current produced by moving *P* are small, so that the changes in color of the comparison lamp thus produced are entirely negligible. Ives and Woodhull[7] made use of an adjustable resistance, but the null method made possible by the modified potentiometer connections and the calibrated slide-wire resistance just described was introduced later.

(*d*) THE WATTS-PER-CANDLE COMPUTER.—Two sets of special scales are used in connection with this photometer. One set is used in computing watts-per-candle from the observations while the other set is used in connection with a recording device. The wpc computer, which operates on the principle of an ordinary slide rule, consists of an ampere scale and a wpc scale both logarithmic and calculated on the same base.[8] These are placed parallel to the photometric axis between the photometer head and the carriage, the wpc scale (showing white in FIG. 1) being attached to the carriage so as to move with it.

The design of the computer is based upon the fact that a logarithmic scale may be constructed which practically coincides with the candlepower scale over a range extending from one-half to double the candlepower reading at the middle of the scale. The base of such a logarithmic scale for a 250 cm photometer is 71.25 cm, and the maximum differences of a scale so constructed from the true candlepower scale, the middle division of which is 20 candlepower, are only 0.08 cm, corresponding to about 0.25 per cent in candlepower and occurring at approximately the 14 and 28 candlepower divisions. These differences, even at the points of maximum value, are entirely negligible for the purposes of this photometer and the advantages gained by employing the logarithmic scale fully offset the small errors introduced.

---

[7] See note 4, p. 611.

[8] The base of a common logarithmic scale is the distance from 1 to 10, 10 to 100, etc., on the scale.

The two parts of the computer are logarithmic scales constructed in this manner, but the divisions are labeled amperes and wpc, respectively, instead of candlepower.

Now, it is evident that, with the photometer set to a given candlepower, the ampere scale may be moved horizontally to a point where for a given voltage the corresponding wpc will appear under any chosen value of current (which then corresponds to the wattage), and that after this setting the correct wpc value will appear under the corresponding current at all points of the scale. Now, if a lamp is run at this same voltage and the photometer is moved to the point of balance the correct wpc will still appear under the observed current, because the wpc scale attached to the photometer carriage has been moved in its relation to the ampere scale by a distance corresponding to the change in candlepower. The ampere scale must be reset for every change of voltage, but by proper grouping of lamps a large number may be run in succession at one voltage, so that these changes are infrequent during any single run.

(*e*) THE RECORDING DEVICE.—The recording device consists of a stamping magnet, a cylinder carrying a number of scales, and a car for holding the record cards. The magnet and cylinder are attached to the photometer carriage, and therefore move with it. The cylinder is mounted normal to the photometric axis and carries three logarithmic scales running parallel to its length, one being an hour scale, the other two being wpc scales for use in measuring tungsten and carbon lamps, respectively. The magnet is supported by a rod placed parallel to the cylinder, so that the pointer carried by the magnet may be set at any division on any one of the three scales, the desired scale being presented by turning the cylinder.

The car may be moved on a track parallel to the photometric axis but is held at any one of a number of nearly equally spaced points by means of a pin placed in a corresponding hole in the track. The distance between any two adjacent holes corresponds to half the distance from 100 per cent to 80 per cent candlepower as read from the true candlepower scale. These holes are labeled with two series of the same letters, one series being printed in red, the other in black, the letters of the red series being placed two spaces nearer the comparison lamp than the corresponding letters of the series in black. The use of these letters will be described presently.

The observations are recorded as points stamped on plain white cards approximately 12.5 cm by 20 cm, there being one card for each lamp. (See Fig. 4.) These cards are placed in the car with their long dimension normal to the photometric axis and therefore parallel to the scales on the cylinder. Now, it is evident that the short dimension of the card may be looked upon as a candlepower scale and the long dimension as an hour scale or a wpc scale, depending upon which of these two quantities is to be measured and recorded. The position for the card on the photometer is so chosen at the initial measurement that the candlepower record will be made sufficiently high to permit all values during the life of the lamp to fall on the card. This is regarded as the normal position of the card and is designated

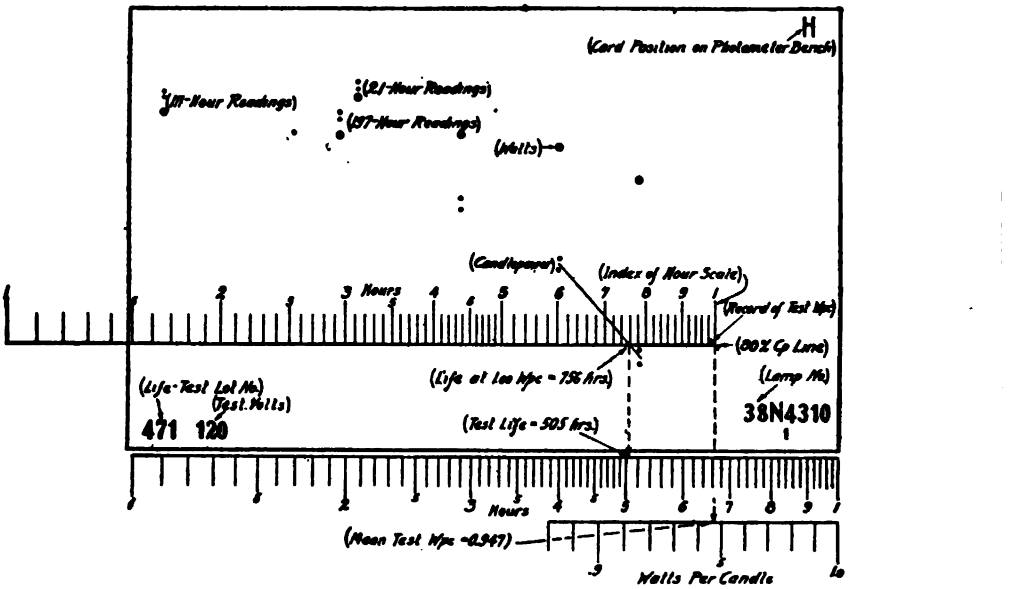


FIG. 4.—*Completed test record on a lamp card showing the scales used in placing the record points and in evaluating corrected life*

This lamp, a 40-watt, 110-volt, tungsten, was tested at 0.947 wpc and had a life of 505 hours which is equivalent to 756 hours at 1.00 wpc. It was rated at 1.05 wpc by the manufacturer and the life specified was 1,000 hours which is equivalent to 697 hours at 1.00 wpc. Hence, the life of the lamp was 756÷697= 108.5 per cent of the life required by the specifications. (For further explanation, see p. 618) by the corresponding *black* letter which is then written on the card. The card is placed in this position during all but the initial measurements, the reason for this exception being given in the following section.

The two most important quantities to be recorded are the initial (test) wpc and the life. The latter is defined as the number of hours required for a lamp to reduce to 80 per cent of its initial candlepower, or to burn out, if within that period. Now, it is evident that, so far as making the record is concerned, motion of the card toward higher candlepower on the photometer is equivalent to moving the photometer in the opposite direction. If, therefore, during the initial measurement of candlepower the card be set, not at its normal (black letter) position, but at that designated by the corresponding red letter, the record point of the observed candlepower will fall at a position corresponding to 80 per cent of the value observed. This point therefore establishes on the record the limiting line of life as defined by the specifications.

As the record of the initial measurement does not include the element of time, it may be made at any point along the 80 per cent line. Hence, if the stamping magnet be set at the point on the wpc scale (which scale for tungsten lamps is reproduced under the card in the figure) corresponding to observed initial wpc, not only 80 per cent of the initial candlepower but also the initial wpc as well may be recorded by the same point. To distinguish these initial points from the rest of the records, they are stamped in red (indicated by + in the figure) while all the others are stamped in black. For all but the initial measurements the card is set at the black letter position and the magnet is set at a point on the hour scale (which scale is also reproduced under the card in the figure) corresponding to the total number of hours the lamp has burned. The candlepower-hour record points are stamped in succession across the card as many times as necessary, depending upon the life of the lamp, the hour scale reading for each succeeding series being ten times the value it had in the series next preceding. The complete record thus obtained on any card graphically represents the performance of the corresponding lamp and the actual test life is indicated by the point of intersection of the curve of candlepower performance with the 80 per cent candlepower line.

So far as obtaining a record is concerned, any scale might have been adopted for use on the cylinder in recording test life and initial wpc, but the scales here employed have been so chosen in respect to their relative lengths and relative position on the cylinder as to permit the evaluation of corrected life from test wpc to rated wpc directly from the card record without computation or reference to tables of factors. This arrangement was based upon the following considerations.

It has been shown that within certain limits the relation between life and wpc may be expressed by the formula,

$$\text{Life ratio} = (\text{wpc ratio})^m \qquad (1)$$

in which m has been found to have a value of about 7.4 for tungsten lamps and 5.83 for carbon lamps. From equation (1) is derived

$$\log \text{life ratio} = m \log \text{wpc ratio} \qquad (2)$$

analogous to the equation

$$y = mx \qquad (3)$$

which is the equation of a straight line. Hence a logarithmic hour scale and a similarly constructed wpc scale with a base equal to m times the base of the hour scale may be used together as a slide rule for making life corrections from one efficiency to another. Life in hours on the one scale is set opposite the corresponding wpc on the other, and life at any other wpc, not exceeding the limits through which m has a constant value, is read by referring to the corresponding wpc division.

The hour scale on the cylinder of the recording device was plotted to a base of 20 cm (equal to the approximate length of the record cards) with divisions from 1 to 10, as in all slide-rule scales, and hence the base taken for the wpc scale for tungsten lamps was $7.4 \times 20 = 148.0$, and for carbon lamps, $5.83 \times 20 = 116.6$.

Life requirements in the specifications are expressed in hours at rated wpc, being at present 1,000 hours for tungsten lamps and from 120 to 450 hours for carbon lamps, depending upon the size. In order to facilitate the test of tungsten lamps, they have been burned at from 0.90 to 0.95 wpc but carbon lamps have been tested at or near their normal rating. To avoid complications in the recording device and occasional changes to correspond to changes which, from time to time, are made in the rating of lamps as the art of manufacture improves, it was considered best to arrange for the correction of test life values to their equivalent at a certain chosen wpc for each class of lamp. Accordingly 1.00 wpc was chosen for tungsten lamps and 3.05 wpc for carbon lamps and by means of equation (1) the life requirements of all sizes of each class at rated wpc were reduced to their equivalent at the corresponding chosen wpc. These values then are referred to as the required life instead of those given in the specifications and the life of any lamp, or of any group of lamps of the same size, is expressed in per cent of the required life. (See explanation under Fig. 4.)

The logarithmic hour and wpc scales constructed as above described were then so placed on the cylinder that the 1.00 wpc division of the tungsten scale was in line with the 1000-hour division of the hour scale, as shown in Fig. 4, and the 3.05 wpc division of the carbon scale in line with the 450-hour division. The wpc points on the card are thus recorded on a logarithmic scale and in a definite relation to the hour scale. Now, if the 1000-hour division of the scale in the case of tungsten lamps or the 450-hour division in the case of carbon lamps be taken as an index and a duplicate of the hour scale be placed, as shown in Fig. 4, with the proper index on the mean of the wpc points of the record and with its reading edge on the 80 per cent line, the test life, corrected to the chosen wpc, may be read at the intersection of the scale and the candlepower performance curve.

In case a lamp burns out above 80 per cent of its initial candlepower value, a vertical line is drawn across the 80 per cent line at the proper point as determined by the life test log and the hour scale, but the procedure in obtaining corrected life is the same as in the case of lamps which have burned to 80 per cent.

For lamps having other than the specified mean spherical reduction factors, the index may be so chosen that the corresponding difference is made in the corrected life. Certain special lamps, for example, lamps in tubular and round bulbs, are thus evaluated.

(*f*) FEATURES OF THE RECORD.—(α) *Detection and Compensation of Errors.*—One characteristic of these record points of the initial readings of wpc and candlepower (Fig. 4) is of interest and importance in that it serves as a visual check upon the correctness of the records. Rarely do two observers on the photometer check each other exactly, but the precision of electrical instruments and the constancy of electric lamps during the relatively short time they are in circuit on the photometer are such that the ampere reading is usually repeated to within 0.001. Suppose, now, that at the same current the second observer reads a candlepower value higher than that recorded by the first. The wpc computer will, consequently, indicate a lower value, since the candlepower is higher for the same watts. Referring to Fig. 4, it will be seen that the second point will be placed above and to the left of the first. For a candlepower reading lower than the first, the current remaining the same, the point will be placed below and farther to the right. Suppose, now, that one or other of the ampere readings is in error, the second being appreciably higher than the first. The apparent wpc of the second observation is then higher than it should be, regarding the first as correct, and the effect is to change the slope of the line connecting the two observations. Displacements may occur also in case of errors in transfer to the record card or as combination of errors.

Now it is evident that the equation

$$\text{Watts} = \text{cp} \times \text{wpc} = \text{constant} \quad \ldots\ldots\ldots\ldots (4)$$

expresses, for a steady lamp, the condition for correct reading. This is the equation of an equilateral hyperbola. Although somewhat modified by the logarithmic scale of the recording equipment, it is closely approximated in form by correctly recorded points under conditions of constant watts; so that the slope of the line connecting the initial wpc points may be used as an indication of their precision, and any considerable deviation from the correct slope indicates that some error has been made. Any lamps, the records of which, show such deviations are, therefore, rephotometered.

Another interesting feature of the card record of a normal lamp is that the slope of the candlepower-life curve between its last two points is often very nearly the same as that of the line joining the two initial wpc points; consequently in these cases comparatively large differences in distance between initial points effect no considerable change in corrected life, which may be evaluated with small error from any point in the line connecting the wpc points. Observational errors in initial readings are therefore always compensated for to some extent by the fact that the candlepower-life and initial wpc curves always slope in the same general direction. It is doubtful if any other than this system of photometry and recording possesses these advantages.

(β) *Increased Accuracy in Life Values.*—It has already been stated that in evaluating lamps which have burned to 80 per cent a straight line is drawn between the last two points on the record cards, one of which is above and other below the 80 per cent candlepower line. (Fig. 4.) If this line be transferred to rectangular coordinates it will be found that it is slightly curved, being convex downward toward the life axis. As this is characteristic of a true candlepower-life curve, this method gives, on an average, a closer approximation to the actual time of crossing the 80 per cent line than that obtained by direct interpolation.

2. METHODS OF MEASURING AND RECORDING OBSERVED VALUES

(*a*) RATING OF LAMPS FOR LIFE TEST.—Two methods are in common use in rating lamps for life test. The first distinguishes two voltages, namely, "photometer" voltage, which usually corresponds to rated voltage, and "rack" voltage. Rack voltage is computed from photometer voltage and the corresponding wpc by the characteristic equation expressing the relation of volts to wpc. By this method the lamps are always run on the photometer, both initially and during life test, at photometer voltage. They are operated on life test at rack voltage, which of course corresponds to test wpc within the desired limits. By the second method the lamps are photometered and operated on life test at rack voltage. In the case of vacuum tungsten lamps, the characteristics of which are well known, either method may be used. Advocates of the first method claim advantages for it in the greater certainty of candlepower observations made at or near a color match with the standards. These are no doubt real advantages, as there is now practically no uncertainty introduced by computations based on well-established values within certain limits of wpc for normal lamps.

The Bureau, however, employs the second method. Although this method was adopted before the characteristics of tungsten lamps were as well known as they now are, it is still used because it introduces no uncertainties due to possible failure of any lamps to conform to the characteristic relations. Although an extra scale for reading rack voltage could easily be added to those above described, thus permitting measurements at photometer voltage, a careful investigation of the possible added advantages thus secured as weighed against a somewhat greater complexity of apparatus and consequent added liability of error would first have to be made if a change to the first method should ever be contemplated.

(*b*) DETAILS OF A PHOTOMETRIC RUN.—As a Lummer-Brodhun photometer is used, all measurements are made at as nearly a color match as possible. By the method at present in use, the photometer is always calibrated by six tungsten standards selected at random from a much larger group. The values of candlepower and current for the individual lamps of this group, over a wide range of voltage (and color), are tabulated on a card within view of the electrical operator and in what follows these are designated as "certified" values. The comparison lamp is adjusted in current so as to give the proper color to match the lamps to be tested, this being done by simply balancing the potentiometer against the voltage drop across resistance R_3 (Fig. 3), the small adjustment necessary being made by means of resistance R_2. Switch *SW* is open or closed, depending upon whether carbon or tungsten lamps are to be measured. The first standard is then placed in the socket and adjusted in voltage to match the modified or unmodified color of the comparison lamp depending upon the efficiency at which the test lamps are to be measured. (See p. 613.)

After the color adjustment, the certified candlepower value of the standard, at the voltage to which adjustment was made, is called off by the electrical operator, and the photometer operator so adjusts the shutter on the ground glass window which fronts the comparison lamp box that a balance is secured at approximately the certified value as read on the candlepower scale. After this approximate calibration, a stamped record of about ten individual settings is made for each of the six standards. After the observed values of a standard are recorded, the certified value is called off by the electrical operator and, with the photometer set at this point on the scale, this value also is stamped on the card. A copy of a short section of the candlepower scale is used to read off the algebraic differences between the certified and the observed candlepower values. In this manner the difference between observed and certified values of all the standards are determined and the mean difference in candlepower is computed. Correction for this mean difference is then made by moving the sliding contact P of the resistance R_4 (p. 614) the proper number of scale divisions. This necessitates a small adjustment of the comparison lamp current which is now made by means of resistance R_2. The electrical operator has, in the meantime, compared the observed current with the certified current and determined a mean correction for ammeter readings; or, in case lamps whose ampere readings are considerably different from that of the standards are to be run, the proper ampere standard is selected from a group of seasoned lamps used only for this purpose, and the mean ampere correction thus established is applied throughout the run. The standard check is the last direct reference made to actual values on the candlepower scale.

Having determined by trial the even voltage (e. g., 118, 120, etc.) corresponding to a dial setting on the potentiometer at which the first test lamp falls within the desired range of test wpc, the ampere scale is set to a point corresponding to this voltage. (See p. 615.) Opposite the ampere value called off by the electrical operator is read the test wpc. With the corresponding lamp card so placed that the value to be recorded will be at least two-thirds of the way down the card, the index carried by the stamping magnet is set at the observed wpc, the circuit through the magnet is closed by pressing a button, thus making the record of the wpc and also 80 per cent of the candlepower as a single point in red. The red letter indicating the card position is noted and a card bearing this letter is selected from the file within reach and placed face down on the photometer bench, the first record card being turned over and placed upon it. As the different lamps are photometered the corresponding lamp cards and position cards are added in regular order. The same voltage is applied to each lamp in succession until one is reached which requires a change of voltage, when the ampere scale is reset to correspond to this voltage. Readings are continued at this new voltage to a point where another change of voltage is required, etc. "Information cards" designating voltage, disk opening, card position, etc., are introduced in the proper place to indicate the changes to be made in succeeding measurements.

The photometer calibration is checked by two or three standards at intervals during the run and the indicated changes of comparison lamp current are made when required (p. 613). After the first run, cards for lamps of the same voltage, disk opening, card position, etc., are grouped together to the best advantage, the extra information cards being removed and filed for future use. The life-test lot number, the voltage, and position letter are then printed or written on each card, and the lamps rearranged for a second run in the order determined by the card positions, thus facilitating the work. After the second run, for which the two operators exchange places, such additional check measurements as are found necessary (p. 620) are made. The lamps are then ready for the life-test racks, where they are burned at the respective voltages found.

After the first period of burning on the life test, the lamps are removed, placed in the proper order and again run on the photometer at the test (rack) voltage. The cards are now set to the *black* letter position (p. 617) indicated on the information cards and on each lamp card, and the stamping-magnet index is placed at the point on the hour scale corresponding to the number of hours the lamps of the lot have burned.

The ampere scale is set as in the initial run, and, after the observed candlepower value is recorded, the photometer is set so that an index on the movable part of the wpc computing device is opposite the observed current value and a record of the position is stamped. As the voltage at every measurement of a given lamp is the same, this record shows the variations in the watts during the life of the lamp. (These points are surrounded by circles in Fig. 4.)

Measurements are made in this manner after each test life period until all lamps of the lot have crossed the 80 per cent candlepower line or burned out above it.

V. THE LIFE TEST

1. DESIGN OF THE INSTALLATION

At the time when the design of the life test equipment was under discussion, the common method in use elsewhere of setting individual lamps or racks of lamps to a desired test voltage was by means of a resistance in series with each lamp or rack. The disadvantages of this method were apparent, and search was therefore made for an arrangement of equipment which would be free from these disadvantages, but which would still conform to the requirements to be met. An arrangement of auto-transformers proposed by Mr. Brooks was adopted because of its simplicity, convenience, and general conformity to the requirements of life-test operation. Other laboratories have since adopted the essential features of this arrangement which are fully described below.

(*a*) Wiring and Voltage Adjustment.—Referring to the wiring diagram, Fig. 5, which exhibits the essential features of the system, it is seen that alternating current is supplied by the generator to the center of distribution. Auto-transformers T_1 to T_4 supply current to the bus bars at the voltage indicated.[9] These bus-bars are mounted on the back of the switchboard panel farthest to the right. (Fig. 2, frontispiece.) One terminal of each rack (horizontal row) is connected to the common bus through the secondary of a corresponding regulating transformer *B*; the other terminal is connected to a plug hole in this same panel. Hence, to energize a given rack *R* a connecting cable is plugged from the corresponding plug hole to the bus maintaining the voltage nearest to that desired. The conductors from the switchboard to the racks are of No. 4 wire carried through 10 lines of 2-inch conduit running over the tops of the switchboard and racks to junction boxes from which connection is made to the terminals of the copper rod conductors of the racks.

---

[9] No provision has yet been made for low voltage or series burning lamps.

The special autotransformer ST maintains voltages of +5, +10, +15 to +50, and corresponding negative voltages on bus bars located on the front of the middle panel. One primary

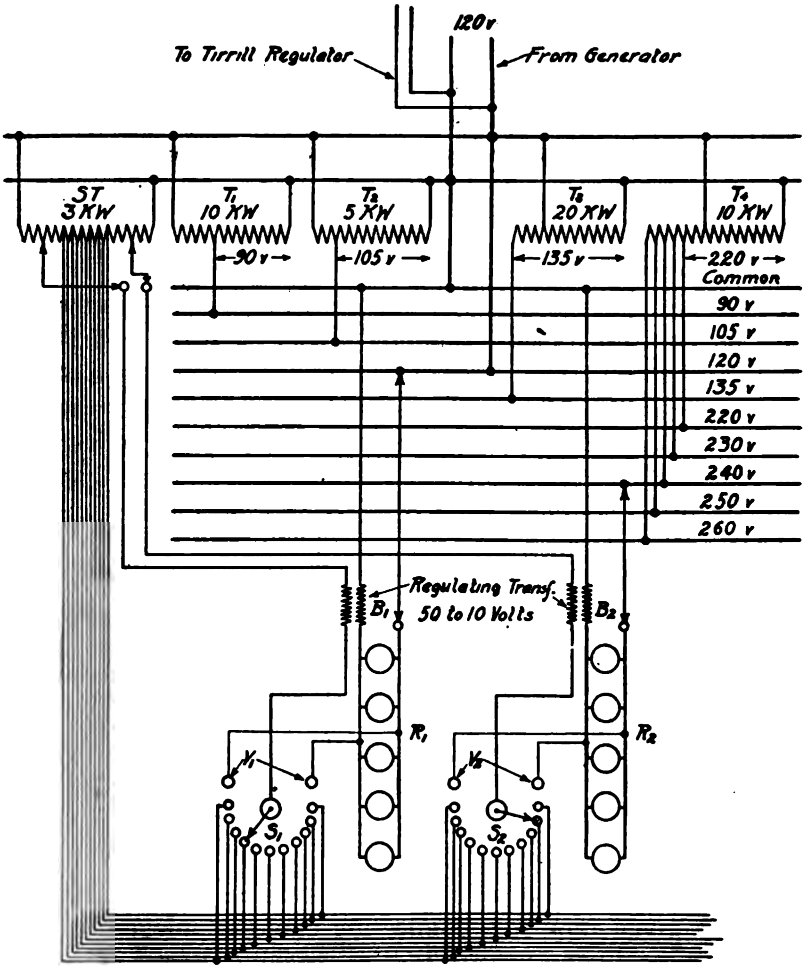


FIG. 5.—*Wiring diagram of the switchboard and life racks.*

terminal of each of the regulating transformers B ends in a corresponding plug hole also on this panel. As the ratio of transformation of each regulating transformer is 5 to 1, it follows that + or — changes of 1, 2, 3 to 10 volts may be made effective on the rack. Hence, by plugging from the transformer terminal to the proper bus bar on this panel, a second approximation to the exact voltage desired on rack *R* is obtained.

The other primary terminal of each of the regulating transformers ends in the lever of a corresponding dial switch *S* located on the left panel of the switchboard. The buttons of each switch are maintained 1 volt apart over a range of 10 volts, by leads from adjacent 1-volt subdivisions of transformer *ST*, but because of the 5 to 1 transformation in *B* each volt at the switch is effectively 0.2 volt on the rack. Hence, by properly setting the switch 1, the exact voltage desired is approximated to within 0.1 volt.

The voltage of each rack is adjusted at the switchboard by reference to a portable voltmeter which may be connected to the corresponding pair of binding posts forming the terminals of the potential leads *V* from the center of the rack. The voltage of a rack is thus adjusted by actual measurement in every case. Each pair of binding posts appears on the corresponding dial switch. As these switches are grouped on a single panel, any number of racks may be quickly set without inconvenience with the voltmeter kept in a fixed position on its stand.

(*b*) VOLTAGE REGULATION.—A Tirrill regulator, which operates by periodically short-circuiting a resistance in series with the exciter field, maintains the voltage at the center of distribution in the life test room constant to within the limits of plus or minus one-quarter of 1 per cent as required by the specifications. A continuous record of this voltage is obtained on an accurate recording voltmeter located in the dynamo room.

(*c*) CURRENT GENERATOR AND VOLTAGE TRANSFORMERS.—The generator which supplies current for the life test is a 40-kw, 125-volt, 360 rpm, single-phase, rotating-field alternator, directly connected to the driving engine, the exciter being mounted upon the same shaft. Transformers *B* (shown back of the switchboard in Fig. 2) are one-half-kw, air-cooled, shell-type; while *ST* and T_1 to T_4 are oil-immersed, auto-transformers of the capacities indicated in Fig. 5, the relative capacities of T_1 to T_4 being roughly in proportion to the number of lamps usually run at their respective voltages.

(*d*) THE LIFE-TEST RACKS.—The supporting frames of the racks are built up of steel members consisting of vertical end posts of channel section and equally spaced intermediate posts of I-beam section connected by heavy angles to horizontal top and bottom pieces of channel section, the whole being supported by cast-iron feet bolted to the composition (Tileine) floor. Bolted to each side of the vertical members are six equally spaced horizontal strips of asbestos board which support porcelain cleat sockets, spaced on 12-inch (30.5 cm) centers, with soldered electrical connections to copper rods 5 mm in diameter. Midway between each pair of these sockets, which are arranged for burning the lamps in a horizontal position, conducting straps are soldered at one end to the 5 mm copper rods and at the other end to the terminals of porcelain cleat sockets arranged for burning lamps in a vertical position. The long racks (17 feet (5.18 m)) have 31 sockets on each side; the short racks (13 feet (3.96 m)), 23. On a few of the lower racks the sockets for vertical burning are spaced on 18-inch (45.7 cm) centers. The large lamps burned on these racks are thus kept well separated during life test. The total number of vertical sockets is 1200 and of horizontal sockets, 1296.

The eight stacks of racks are spaced 4 feet 10.5 inches (1.49 m) apart, which gives a symmetrical arrangement in the life-test room with sufficient space to permit safe and convenient handling of lamps.

(*e*) MEASUREMENT OF LIFE-TEST PERIODS.—An important detail in conducting a life test is the accurate measurement of the time the lamps have burned. For this purpose an electric clock which measures time in hours from one to one thousand is used. This clock is connected in the master clock circuit of the Bureau and is short-circuited by a relay when the power is cut off. The log of any life test contains the clock time to the nearest 0.1 hour, corresponding to the time of placing the lamps on and removing them from the rack circuit. The time of burnouts during the night is either recorded by the watchman who visits the room every two hours, or the lamps are considered as having burned until 9 o'clock the following morning.

2. RECORDS TAKEN DURING LIFE TEST

Summarizing the records which are taken during life test, as described above, it will be found that the following have been mentioned:

(*a*) Test voltage; initial candlepower and initial wpc at test voltage.

(*b*) Candlepower and watts at certain periods during test life. For carbon and metallized filament lamps the specifications require measurement after approximately 50 hours of burning and "at least every hundred hours thereafter" throughout useful life. Five measurements, the first approximately one-twentieth of the test life period, after the initial are specified for tungsten lamps.

(*c*) Recording voltmeter records of main life test voltage.

(*d*) Test log showing clock reading from which test life periods are computed.

In addition to the above there are, of course, required such other records as will permit orderly clerical procedure. A card record system is used throughout, but the details, which have been worked out to take care of features in some cases peculiar to the Bureau tests only, would hardly be of general interest.

VII. SUMMARIES OF LIFE VALUES

After the completion of a sufficient number of test lamps to warrant quality comparisons, life values of lamps of the same type, size, and manufacture, and of a voltage range through which a given life value is specified, are averaged. A summary giving the date, type, size, manufacture, voltage range, number of lamps, corrected life and percentage of required life is prepared from these data, so that a manufacturer may, at his request, refer to the summary for information regarding the quality of his lamps and those of other manufacturers supplying lamps under the annual contract. In case lamps are rejected as the result of life test the manufacturer and purchaser are promptly notified, each being given the life value on which rejection is based.

Additions of other lamps are made to this summary from time to time, so that average quality values to the corresponding date are indicated; except that in case of a drop in quality of certain items so decidedly below the required life that rejection of the defective lamps is necessary, the figures for accepted and rejected lamps are kept separate until the end of the tests, when the average life of accepted and rejected lamps combined is reported as a final value.

VII. SUMMARY

The method employed by the Bureau of Standards in the inspection and life testing of incandescent lamps for the Federal Government is outlined and a description of the power plant, the life racks, and the photometer is given. Particular attention is directed to the special equipment of the photometer. This includes a watts-per-candle computer and a recording device by which observed values of candlepower, watts, watts per candle, and actual life are recorded on a separate card for each lamp. These records are made in such a way that life at forced efficiency is corrected to life at normal without computation or reference to tables of factors. The procedure in actual measurement and testing is described with considerable detail.

WASHINGTON, July 31, 1915.

INDEX TO VOLUME 12.

A.

Absolute Thermopile, 503.
Aneroid Calorimeter, 23.
Antennas, Radiotelegraphic, Resistance of, 365.
Austin, L. W., Note on the resistance of radiotelegraphic antennas, 365.

B.

Bimetallic Wires, resistance and inductance of, 207
Black body, Present status of determination of constant of total radiation from a, 553.
Briggs, C. A., and M. P. Shoemaker, F. A. Wolff, Construction of primary mercurial resistance standards, 375.
Burgess, G. K., and P. D. Foote, Characteristics of radiation pyrometers, 91.
——, ——, Emissivity of metals and oxides, IV, 83.
——, *and P. D. Sale*, Study of the quality of platinum ware, 289.
Burns, K., Interference measurements of wave lengths in the iron spectrum, 179.

C.

Calculation of the maximum force between two coaxial circular currents, 317.
Calorimeter, Aneroid, 23.
Center of gravity and effective wave length of transmission of pyrometer color screens, and the extrapolation of the high temperature scale, 483.
Characteristics of radiation pyrometers, 91.
Characteristics of vacuum tungsten lamps, device for computing, 269.
Circular Currents, Maximum force between two coaxial, 317.
Coblentz, W. W., and W. B. Emerson, Studies of instruments for measuring radiant energy in absolute value: An absolute thermopile, 503.
Coblentz, W. W., Present status of the determination of the constant of total radiation from a black body, 553.
Coefficient, Temperature, of magnetic permeability, 1.
Color screens, Pyrometer, 483.
Comparisons of lengths of light waves, 179.
Constant of total radiation from a black body, 553.
Construction of primary mercurial resistance standards, 375.

D.

Device for computing characteristics of vacuum tungsten lamps, 269.
Dickinson, H. C., and N. S. Osborne, Aeroid calorimeter, 23.
——, ——, Specific heat and heat of fusion of ice, 49.
Direct-reading device for use in computing characteristics of vacuum tungsten lamps, 269.

E.

Earth resistivity, 469.
Effective resistance and inductance of iron and bimetallic wires, 207.
Emerson, W. B., W. W. Coblentz and, Studies of instruments for measuring radiant energy in absolute value: An absolute thermopile, 503.
Emissivity of metals and oxides. IV. Iron oxide, 83.
Extrapolation of the high temperature scale, 483.

F.

Foote, P. D., Center of gravity and effective wave length of transmission of pyrometer color screens, and the extrapolation of the high temperature scale, 483.
——, Illumination from a radiating disk, 583.
——, New relation derived from Planck's law, 479.
——, *G. K. Burgess and*, Characteristics of radiation pyrometers, 91.
——, Emissivity of metals and oxides, IV, 83.

G.

Gas-filled lamp, Photometry of, 587.
Grover, F. W., Calculation of the maximum force between two coaxial circular currents, 317.

H.

Heat of fusion of ice, 49.
High temperature scale, Extrapolation of, 483.

I.

Ice, Specific heat and heat of fusion of, 49.
Illumination from a radiating disk, 583.
Inductance of iron and bimetallic wires, 207.
Incandescent lamps, Life testing of, 605.
Inductances of low-resistance standards, 11.
Instruments for measuring radiant energy in absolute value, 503.
Interference measurements of wave lengths in iron spectrum, 179.
Iron and bimetallic wires, Resistance and inductance of, 207.
Iron oxide, 83.
Iron spectrum, Measurements of wave lengths in, 179.

L.

Lamps, Life testing of, 605.
Lengths of light waves, 179.
Life testing of incandescent lamps at the Bureau of Standards, 605.
Low-resistance standards, Measuring Inductances of, 11.

M.

Magnetic permeability, Temperature coefficient of, 1.
Maximum force between two coaxial circular currents, 317.
Measurements of wave lengths in iron spectrum, 179.
Measuring earth resistivity, Method of, 469.
Measuring inductances of low-resistance standards, 11.
Measuring radiant energy in absolute value, 503.
Meggers, W. F., Notes on comparisons of lengths of light waves by interference methods, and some wave lengths in the spectrum of neon gas, 179.
Mercurial resistance standards, Primary, Construction of, 375.
Metals and oxides, Emissivity of, IV, 83.
Method of measuring earth resistivity, 469.
Methods of measuring inductances of low-resistance standards, 11.
Middlekauff, G. W., and J. F. Skogland, Photometry of the gas-filled lamp, 587.
——, *B. Mulligan, and J. F. Skogland*, Life testing of incandescent lamps at the Bureau of Standards, 605.
Miller, J. M., Effective resistance and inductance of iron and bimetallic wires, 207.
Mulligan, B., G. W. Middlekauff, and J. F. Skogland, Life testing of incandescent lamps at the Bureau of Standards, 605.

N.

Neon gas, Wave lengths in spectrum of, 179.
New relation derived from Planck's law, 479.
Note on the resistance of radiotelegraphic antennas, 465.

O.

Osborne, N. S., H. C. Dickinson and, Aneroid calorimeter, 23.
Osborne, N. S., H. C. Dickinson and, Specific heat and heat of fusion of ice, 49.
Oxides, Metals and, Emissivity of, IV, 83.

P.

Permeability, Magnetic, Temperature coefficient of, 1.
Photometry of the gas-filled lamp, 587.
Planck's law, New relation derived from, 479.
Platinum ware, Quality of, 289.
Present status of the determination of the constant of total radiation from a black body, 553.
Primary mercurial resistance standards, Construction of, 375.
Pyrometer color screens, Transmission of, 483.
Pyrometers, Radiation, Characteristics of, 91.

Q.

Quality of platinum ware, 289.

R.

Radiant energy in absolute value, 503.
Radiating disk, Illumination from, 583.
Radiation from a black body, 553.
Radiation pyrometers, 91.
Radiotelegraphic antennas, Resistance of, 365.
Resistance and inductance of iron and bimetallic wires, 207.
Resistance of radiotelegraphic antennas, 365.
Resistance standards, Measuring the inductances of, 11.
Resistance standards, Primary mercurial, Construction of, 375.

S.

Sale, P. D., G. K. Burgess and, Study of the quality of platinum ware, 289.
Sanford, R. L., Temperature coefficient of magnetic permeability within the working range, 1.
Shoemaker, M. P., F. A. Wolff, and C. A. Briggs, Construction of primary mercurial resistance standards, 375.
Silsbee, F. B., E. Weibel, F. Wenner, and, Methods of measuring the inductances of low-resistance standards, 11.
Skogland, J. F., Direct-reading device for use in computing characteristics of vacuum tungsten lamps, 269.
Skogland, J. F., G. W. Middlekauff and, Photometry of the gas-filled lamp, 587.
Skogland, J. F., B. Mulligan, G. W. Middlekauff, and, Life testing of incandescent lamps at the Bureau of Standards, 605.
Specific heat and heat of fusion of ice, 49.
Spectrum, Iron, Measurements of wave lengths in, 179.
Studies of instruments for measuring radiant energy in absolute value: An absolute thermopile, 503.
Study of the quality of platinum ware, 289.

T.

Temperature coefficient of magnetic permeability, 1.
Temperature scale, Extrapolation of, 483.
Testing of incandescent lamps, 605.
Thermopile, Absolute, 503.
Total radiation from a black body, 553.
Transmission of pyrometer color screens, 483.
Tungsten lamps, device for computing characteristics of, 269.

V.

Vacuum Tungsten lamps, Device for computing characteristics of, 269.

W.

Wave length, Center of gravity and, of Transmission of pyrometer color screens, 483.
Wave lengths in the iron spectrum, 179.
Weibel, E., F. Wenner, and F. B. Silsbee, Methods of measuring the inductances of low-resistance standards, 11.
Wenner, F., Method of measuring earth resistivity, 469.
Wenner, F., E. Weibel, and F. B. Silsbee, Methods of measuring the inductances of low-resistance standards, 11.
Wolff, F. A., M. P. Shoemaker, and C. A. Briggs, Construction of primary mercurial resistance standards, 375.

PUBLICATIONS ISSUED BY THE BUREAU OF STANDARDS

SCIENTIFIC PAPERS

1.	Recomparison of the United States Prototype Meter	*L. A. Fischer*
2.	A Study of the Silver Voltameter	*K. E. Guthe*
3.	The So-called International Electrical Units	*Frank A. Wolff*
4.	The Spectra of Mixed Gases	*P. G. Nutting*
5.	On Secondary Spectra and the Conditions under which They May Be Produced	*P. G. Nutting*
6.	Some New Rectifying Effects in Conducting Gases	*P. G. Nutting*
7.	On Fibers Resembling Quartz in Their Elastic Properties	*K. E. Guthe*
8.	On the Temperature of the Arc	*C. W. Waidner and G. K. Burgess*
9.	The Absolute Measurement of Inductance	*E. B. Rosa and F. W. Grover*
10.	The Absolute Measurement of Capacity	*E. B. Rosa and F. W. Grover*
11.	Optical Pyrometry	*C. W. Waidner and G. K. Burgess*
12.	On the Theory of the Matthews and the Russell-Leonard Photometers for the Measurement of Mean Spherical and Mean Hemispherical Intensities	*E. P. Hyde*
13.	The Testing of Clinical Thermometers	*C. W. Waidner and L. A. Fischer*
14.	Measurement of Inductance by Anderson's Method, Using Alternating Currents and a Vibration Galvanometer	*E. B. Rosa and F. W. Grover*
15.	Use of Serpentine in Standards of Inductance	*E. B. Rosa and F. W. Grover*
16.	The Silver Coulometer	*K. E. Guthe*
17.	History of Standard Weights and Measures of the United States	*L. A. Fischer*
18.	Wattmeter Methods of Measuring Power Expended upon Condensers and Circuits of Low Power Factor	*E. B. Rosa*
19.	The Relative Intensities of Metal and Gas Spectra from Electrically Conducting Gases	*P. G. Nutting*
20.	The Use of White Walls in a Photometric Laboratory	*E. P. Hyde*
21.	Influence of Wave Form on the Rate of Integrating Induction Wattmeters	*E. B. Rosa, M. G. Lloyd, and C. E. Reid*
22.	Detector for Small Alternating Currents and Electrical Waves	*L. W. Austin*
23.	The Positive Charges Carried by the Canal Rays	*L. W. Austin*
24.	Radiation from Platinum at High Temperatures	*G. K. Burgess*
25.	A Five-Thousand-Volt Generator Set	*P. G. Nutting*
26.	Talbot's Law as Applied to the Rotating Sectored Disk	*E. P. Hyde*
27.	A New Determination of the Electromotive Force of Weston and Clark Standard Cells by an Absolute Electrodynamometer	*K. E. Guthe*
28.	The Gray Absolute Electrodynamometer	*Edward B. Rosa*
29.	Construction and Calculation of Absolute Standards of Inductance	*J. G. Coffin*
30.	An Efficiency Meter for Electric Incandescent Lamps	*E. P. Hyde and H. B. Brooks*
31.	Calculation of the Self-Inductance of Single-Layer Coils	*Edward B. Rosa*
32.	Heat Treatment of High-Temperature Mercurial Thermometers	*Hobert C. Dickinson*
33.	A New Potentiometer for the Measurement of Electromotive Force and Current	*H. B. Brooks*
34.	Spectrum Lines as Light Sources in Polariscopic Measurements	*Frederick Bates*
35.	Polarimetric Sensibility and Accuracy	*P. G. Nutting*
36.	On the Platinum-Point Electrolytic Detector for Electrical Waves	*L. W. Austin*
37.	The Influence of Frequency upon the Self-Inductance of Coils	*J. G. Coffin*
38.	Experiments on the Heusler Magnetic Alloys	*K. E. Guthe and L. W. Austin*
39.	A Pocket Spectrophotometer	*P. G. Nutting*

40. Preliminary Measurements on Temperature and Selective Radiation of Incandescent Lamps . . . . . . *C. W. Waidner and G. K. Burgess*
41. Revision of the Formulæ of Weinstein and Stefan for the Mutual Inductance of Coaxial Coils . . . . *Edward B. Rosa*
42. The Mutual Inductance of Two Circular Coaxial Coils of Rectangular Section . . . . *Edward B. Rosa and L. Cohen*
43. On the Determination of the Mean Horizontal Intensity of Incandescent Lamps by the Rotating Lamp Method . . . *E. P. Hyde and F. E. Cady*
44. Purity and Intensity of Monochromatic Light Sources . . . . *P. G. Nutting*
45. Radiometric Investigations of Infra-Red Absorption and Reflection Spectra . . . . *W. W. Coblentz*
46. A Vacuum Radiomicrometer . . . . *W. W. Coblentz*
47. On the Geometrical Mean Distances of Rectangular Areas and the Calculation of Self-Inductance . . . . *E. B. Rosa*
48. The Compensated Two-Circuit Electrodynamometer . . . . *E. B. Rosa*
49. Complete Form of Fechner's Law . . . . *P. G. Nutting*
50. A Comparison of the Unit of Luminous Intensity of the United States with Those of Germany, England, and France . . . . . . *E. P. Hyde*
51. Geometrical Theory of Radiating Surfaces, with Discussion of Light Tubes . . . . *E. P. Hyde*
52. The Influence of Basic Lead Acetate on the Optical Rotation of Sucrose in Water Solution . . . . *F. J. Bates and J. C. Blake*
53. On the Calorimetric Determination of Iron, with Special Reference to Chemical Reagents . . . . *H. N. Stokes and J. R. Cain*
54. On Sulphocyanic Acid . . . . *H. N. Stokes and J. R. Cain*
55. Radiation from and Melting Points of Palladium and Platinum . . . . *C. W. Waidner and G. K. Burgess*
56. The Mutual Inductance of a Circle and a Coaxial Single-Layer Coil—The Lorenz Apparatus and the Ayrton-Jones Absolute Electrodynamometer . . . . *E. B. Rosa*
57. On the Establishment of the Thermodynamic Scale of Temperature by Means of the Constant-Pressure Thermometer . . . *Edgar Buckingham*
58. An Exact Formula for the Mutual Inductance of Coaxial Solenoids . . . . *Louis Cohen*
59. The Mutual Inductance of Coaxial Solenoids . . . *E. B. Rosa and L. Cohen*
60. The Production of High-Frequency Oscillations from the Electric Arc . . . . *L. W. Austin*
61. An Explanation of the Short Life of Frosted Lamps . . . . *E. P. Hyde*
62. Melting Points of the Iron Group Elements by a New Radiation Method . . . . *G. K. Burgess*
63. On the Determination of the Mean Horizontal Intensity of Incandescent Lamps . . . . *Edward P. Hyde and F. E. Cady*
64. Simultaneous Measurement of the Capacity and Power Factor of Condensers . . . . *F. W. Grover*
65. A New Determination of the Ratio of the Electromagnetic to the Electrostatic Unit of Electricity . . . . . *E. B. Rosa and N. E. Dorsey*
66. A Comparison of the Various Methods of Determining the Ratio of the Electromagnetic to the Electrostatic Unit of Electricity . . . . *E. B. Rosa and N. E. Dorsey*
67. Preliminary Specifications for Clark and Weston Standard Cells . . . . *F. A. Wolff and C. E. Waters*
68. Calorimetric Resistance Thermometers and the Transition Temperature of Sodium Sulphate . . . . . *H. C. Dickinson and E. F. Mueller*
69. On the Standard Scale of Temperature in the Interval 0° to 100° C. . . . . *C. W. Waidner and H. C. Dickinson*
70. Clark and Weston Standard Cells . . . . . *F. A. Wolff and C. E. Waters*
71. The Electrode Equilibrium of the Standard Cell . *F. A. Wolff and C. E. Waters*
72. A Comparative Study of Plain and Frosted Lamps . *E. P. Hyde and F. E. Cady*
73. The Variation of Resistances with Atmospheric Humidity . . . . *E. B. Rosa and H. D. Babcock*
74. On the Self-Inductance of a Toroidal Coil of Rectangular Section . . . . *Edward B. Rosa*

75. On the Self-Inductance of Circles . . . . *Edward B. Rosa and Louis Cohen*
76. The Influence of Frequency on the Resistance and Inductance of Solenoidal Coils . . . . *Louis Cohen*
77. The Atomic Weight of Hydrogen . . . . *W. A. Noyes*
78. The Best Method of Demagnetizing Iron in Magnetic Testing . *C. W. Burrows*
79. A Deflection Potentiometer for Voltmeter Testing . . . . *H. B. Brooks*
80. The Self and Mutual Inductance of Linear Conductors . . . *Edward B. Rosa*
81. The Atomic Weight of Chlorine . . . . *W. A. Noyes and H. C. P. Weber*
82. The Preparation of Chloroplatinic Acid by Electrolysis of Platinum Black . . . . *H. C. P. Weber*
83. The Self-Inductance of a Coil of any Length and any Number of Layers of Wire . . . . *Edward B. Rosa*
84. Self-Inductance of a Solenoid of any Number of Layers . . . *Louis Cohen*
85. Instruments and Methods Used in Radiometry . . . . *W. W. Coblentz*
86. A Quartz Compensating Polariscope with Adjustable Sensibility . . *F. J. Bates*
87. An Apparatus for Determining the Form of a Wave of Magnetic Flux . . . . *M. G. Lloyd and J. V. S. Fisher*
88. Effect of Wave Form upon the Iron Losses in Transformers . *Morton G. Lloyd*
89. The Luminous Properties of Electrically Conducting Helium Gas . . . . *P. G. Nutting*
90. Function of a Periodic Variable Given by the Steady Reading of an Instrument, with a Note on the Use of the Capillary Electrometer with Alternating Voltages . . . . *Morton G. Lloyd*
91. Selective Radiation from the Nernst Glower . . . . *W. W. Coblentz*
92. The Testing of Glass Volumetric Apparatus . . *N. S. Osborne and B. H. Veazey*
93. Formulæ and Tables for the Calculation of Mutual and Self-Inductance . . . . *Edward B. Rosa and Louis Cohen*
94. Some Contact Rectifiers of Electric Currents . . . . *L. W. Austin*
95. A Method for Producing Feebly Damped High-Frequency Electrical Oscillations for Laboratory Measurements . . . . *L. W. Austin*
96. On the Advantages of a High Spark Frequency in Radio-Telegraphy . . . . *L. W. Austin*
97. Selective Radiation from Various Solids . . . . *W. W. Coblentz*
98. Remarks on the Quartz Compensating Polariscope with Adjustable Sensibility . . . . *Frederick Bates*
99. Methods of Obtaining Cooling Curves . . . . *George K. Burgess*
100. Note on the Approximate Values of Bessel's Functions for Large Arguments . . . . *Louis Cohen*
101. The Influence of Terminal Apparatus on Telephonic Transmission . *Louis Cohen*
102. The Principles Involved in the Selection and Definition of the Fundamental Electrical Units to be Proposed for International Adoption . . . . *F. A. Wolff*
103. The Luminous Equivalent of Radiation . . . . *P. G. Nutting*
104. The Temperature Formula of the Weston Standard Cell . . . *F. A. Wolff*
105. Radiation, Constants of Metals . . . . *W. W. Coblentz*
106. Dependence of Magnetic Hysteresis upon Wave Form . . . *Morton G. Lloyd*
107. A New Form of Standard Resistance . . . . *Edward B. Rosa*
108. Errors in Magnetic Testing with Ring Specimens . . . . *Morton G. Lloyd*
109. The Testing of Transformer Steel . . . . *M. G. Lloyd and J. V. S. Fisher*
110. A New Method of Determining the Focal Length of a Converging Lens . . . . *Irwin G. Priest*
111. A New Method for the Absolute Measurement of Resistance . *Edward B. Rosa*
112. The Theory of Coupled Circuits . . . . *Louis Cohen*
113. A Volt Scale for a Watts-per-candle Meter . . . . *Herbert E. Ives*
114. The Coefficient of Reflection of Electrical Waves at a Transition Point . . . . *Louis Cohen*
115. A Tungsten Comparison Lamp in the Photometry of Carbon Lamps . . . . *Herbert E. Ives and L. R. Woodhull*
116. The Determination of the Ratio of Transformation and of the Phase Relations in Transformers . . . . *E. B. Rosa and M. G. Lloyd*
117. The Determination of the Magnetic Induction in Straight Bars . . . . *Charles W. Burrows*

118. A Method for Constructing the Natural Scale of Pure Color . . *P. G. Nutting*
119. An Approximate Method for the Analysis of EMF. Waves . . *P. G. Agnew*
120. The Thermoelectric Properties of Tantalum and Tungsten . *W. W. Coblentz*
121. The Estimation of the Temperature of Copper by Means of Optical Pyrometers . . . . . . *George K. Burgess, assisted by J. F. Crowe*
122. The Resolving Power of Objectives . . . . *P. G. Nutting*
123. The Theory of the Hampson Liquefier . . . . . . *Edgar Buckingham*
124. Platinum Resistance Thermometry at High Temperatures . . . . *C. W. Waidner and G. K. Burgess*
125. The Daylight Efficiency of Artificial Illuminants . . . . . *H. E. Ives*
126. Coupled Currents in which the Secondary has Distributed Inductance and Capacity . . . . *Louis Cohen*
127. Effect of Phase of Harmonics upon Acoustic Quality . . . . *M. G. Lloyd and P. G. Agnew*
128. White Light from the Mercury Arc and its Complementary . *Herbert E. Ives*
129. The Regulation of Potential Transformers and the Magnetizing Current . . . . *M. G. Lloyd and P. G. Agnew*
130. The Determination of the Constants of Instrument Transformers . . . . *P. G. Agnew and T. T. Fitch*
131. Selective Radiation from Various Solids, II . . . . . . *W. W. Coblentz*
132. Luminous Efficiency of the Firefly . . *Herbert E. Ives and W. W. Coblentz*
133. Luminosity and Temperature . . . . *P. G. Nutting*
134. A Theoretical and Experimental Study of the Vibration Galvanometer . . . . *F. Wenner*
135. Specific Heat of Some Calcium Chloride Solutions between −35° C. and +20° C . . . . *H. C. Dickinson, E. F. Mueller, and E. B. George*
136. On the Definition of the Ideal Gas . . . . *Edgar Buckingham*
137. Mica Condensers as Standards of Capacity . . . . . . *Harvey L. Curtis*
138. The Mutual Induction of Two Parallel Coaxial Circles in Terms of Hypergeometrical Series . . . . *Frederick W. Grover*
139. A New Method for the Absolute Measurement of Electric Quantity . . . . *Burton McCollum*
140. The Comparative Sensitiveness of Some Common Detectors of Electrical Oscillations . . . . *Louis W. Austin*
141. Photometric Units and Nomenclature . . . . *E. B. Rosa*
142. A Modified Method for the Determination of Relative Wave Lengths, Especially Adapted to the Establishment of Secondary Standards . . . . *Irwin G. Priest*
143. Note on the Temperature Scale between 100 and 500° C. . . . . *C. W. Waidner and G. K. Burgess*
144. A New Form of Direct-Reading Candlepower Scale and Recording Device for Precision Photometers . . . . . . *George W. Middlekauff*
145. A Device for Measuring the Torque of Electrical Instruments . . . . *F. G. Agnew*
146. The Intensities of Some Hydrogen, Argon, and Helium Lines in Relation to Current and Pressure . . . *P. G. Nutting and Orin Tugman*
147. The Temperature Coefficient of Resistance of Copper . . . *J. H. Dellinger*
148. The Electrical Conductivity of Commercial Copper . . . . *F. A. Wolff and J. H. Dellinger*
149. On the Constancy of the Sulphur Boiling Point . . . . *C. W. Waidner and G. K. Burgess*
150. Note on Oscillatory Interference Bands and Some of their Practical Applications . . . . *G. O. Squier and A. C. Crehore*
151. The Effect of Preliminary Heat Treatment upon the Drying of Clays . . . . *A. V. Bleininger*
152. The Reflecting Power of Various Metals . . . . *W. W. Coblentz*
153. The Action of Sunlight and Air upon Some Lubricating Oils . *C. E. Waters*
154. The Visibility of Radiation. A Recalculation of Konig's Data . *P. G. Nutting*
155. A Photometric Attachment for Spectroscopes . . . . . . *P. G. Nutting*
156. Selective Radiation from Various Substances, III . . . . *W. W. Coblentz*
157. The Measurement of Electrical Oscillations in the Receiving Antenna . . . . *L. W. Austin*

158. Some Experiments with Coupled High-Frequency Circuits . . . *L. W. Austin*
159. Some Quantitative Experiments in Long Distance Radiotelegraphy . . . *L. W. Austin*
160. The Behavior of High-Boiling Oils on Heating in the Air . . . *C. E. Waters*
161. The Determination of Vanadium in Vanadium and Chrome-Vanadium Steels . . . *J. R. Cain*
162. On the Computation of the Constant C_2 of Planck's Equation by an Extension of Paschen's Method of Equal Ordinates . . . *Edgar Buckingham and J. H. Dellinger*
163. A Comparison of American Direct Current Switchboard Voltmeters and Ammeters . . . *T. T. Fitch and C. J. Huber*
164. Study of the Current Transformer with Particular Reference to Iron Loss . . . *P. G. Agnew*
165. Thermodynamics of Concentration Cells . . . *Henry S. Carhart*
166. The Capacity and Phase Difference of Paraffined Paper Condensers as Functions of Temperature and Frequency . . . *Frederick W. Grover*
167. The Steam Expansion Line on the Mollier Diagram and a Short Method of Finding the Reheat Factor . . . *Edgar Buckingham*
168. Radiometric Investigation of Water of Crystallization, Light Filters, and Standard Absorption Bands . . . *W. W. Coblentz*
169. Formulas and Tables for the Calculation of Mutual and Self Induction (2d edition, revised and enlarged) . *E. B. Rosa and Frederick W. Grover*
170. The Correction for Emergent Stem of a Mercurial Thermometer . . . *Edgar Buckingham*
171. Thermometric Lag . . . *D. R. Harper*
172. Deflection Potentiometers for Current and Voltage Measurements . . . *H. B. Brooks*
173. Outline of Design of Deflection Potentiometers with Notes on the Design of Moving-Coil Galvanometers . . . *H. B. Brooks*
174. The Determination of Total Sulphur in India Rubber . . . *C. E. Waters and J. B. Tuttle*
175. The Measurement of the Inductances of Resistance Coils . . . *Frederick W. Grover and Harvey L. Curtis*
176. Luminous Properties of Electrically Conducting Helium Gas. II. Reproducibility . . . *P. G. Nutting*
177. Resistance Coils for Alternating Current Work . *H. L. Curtis and F. W. Grover*
178. The Hydrolysis of Sodium Oxalate and Its Influence Upon the Tests for Neutrality . . . *William Blum*
179. Wave-Lengths of Neon . . . *Irwin G. Priest*
180. On the Deduction of Wien's Displacement Law . . . *Edgar Buckingham*
181. The Four-Terminal Conductor and the Thomson Bridge . . . *Frank Wenner*
182. Standardization of Potassium Permanganate Solution by Sodium Oxalate . . . *R. S. McBride*
183. Benzoic Acid as an Acidimetric Standard . . . *G. W. Morey*
184. A Tubular Electrodynamometer for Heavy Currents . . . *P. G. Agnew*
185. Thermometric Lag. . . . *D. R. Harper*
186. Determination of Manganese as Sulphate and by the Sodium Bismuthate Method . . . *William Blum*
187. A New Precision Colorimeter . . . *P. G. Nutting*
188. Instruments and Methods Used in Radiometry, II . . . *W. W. Coblentz*
189. Antenna Resistance . . . *L. W. Austin*
190. Energy Losses in Some Condensers Used in High-Frequency Circuits . . . *L. W. Austin*
191. Selective Radiation from Various Substances, IV . . . *W. W. Coblentz*
192. On a Modified Form of Stability Test for Smokeless Powder and Similar Materials . . . *H. C. P. Weber*
193. Atomic Weight of Bromine . . . *H. C. P. Weber*
194. The Silver Voltameter.—Part I. First Series of Quantitative Experiments . . . *E. B. Rosa and G. W. Vinal*
195. The Silver Voltameter.—Part II. The Chemistry of the Filter Paper Voltameter and the Explanation of Striations . *E. B. Rosa and G. W. Vinal*
196. The Diffuse Reflecting Power of Various Substances . . . *W. W. Coblentz*

197. Density and Thermal Expansion of Ethyl Alcohol and of Its Mixtures with Water . . . *N. S. Osborne, E. C. McKelvy, and H. W. Bearce*
198. A Micropyrometer . . . . *G. K. Burgess*
199. A Simplified Formula for the Change in Order of Interference Due to Changes in Temperature and Pressure of Air . . . . . . *I. G. Priest*
200. New Calorimetric Resistance Thermometers . *H. C. Dickinson and E. F. Mueller*
201. The Silver Voltameter.—Part III. Second Series of Quantitative Experiments and the Preparation and Testing of Silver Nitrate . . . . *E. B. Rosa, G. W. Vinal, and A. S. McDaniel*
202. Note on Cold-Junction Corrections for Thermocouples . . . . *P. D. Foote*
203. The Analysis of Alternating Current Waves by the Method of Fourier, with Special Reference to Methods of Facilitating the Computations . . . . *F. W. Grover*
204. The Constants of Spectral Radiation of a Uniformly Heated Inclosure or So-Called Black Body, Part I . . . . . . *W. W. Coblentz*
205. Melting Points of the Refractory Elements.—I. Elements of Atomic Weight from 48 to 59 . . . . . . *G. K. Burgess and R. G. Waltenberg*
206. High-Frequency Ammeters . . . . *J. H. Dellinger*
207. A Comparative Study of American Direct-Current Watthour Meters . . . . *T. T. Fitch and C. J. Huber*
208. Windage Resistance of Steam Turbine Wheels . . . . *E. Buckingham*
209. Latent Heat of Fusion of Ice . *H. C. Dickinson, D. R. Harper 3d, and N. S. Osborne*
210. Observations on Ocean Temperatures in the Vicinity of Icebergs and in other Parts of the Ocean . *C. W. Waidner, H. C. Dickinson, and J. J. Crowe*
211. Accuracy of the Formulas for the Ratio, Regulation, and Phase Angle of Transformers . . . . *P. G. Agnew and F. B. Silsbee*
212. Melting Points of Some Refractory Oxides . . . . *C. W. Kanolt*
213. The Critical Ranges A2 and A3 of Pure Iron . . *G. K. Burgess and J. J. Crowe*
214. Note on the Setting of a Mercury Surface to a Required Height . *M. H. Stillman*
215. Micrometer Microscopes . . . . *A. W. Gray*
216. The Pentane Lamp as a Working Standard . . *E. C. Crittenden and A. H. Taylor*
217. Testing Potential Transformers . . . . *H. B. Brooks*
218. Comparison of the Silver and Iodine Voltameters and the Determination of the Faraday . . . . *G. W. Vinal and S. J. Bates*
219. Production of Temperature Uniformity in an Electric Furnace . *A. W. Gray*
220. The Silver Voltameter.—Part IV. Third Series of Quantitative Experiments and Special Investigations . *E. B. Rosa, G. W. Vinal, and A. S. McDaniel*
221. Influence of Atmospheric Conditions in the Testing of Sugars . . . . *F. J. Bates and F. P. Phelps*
222. Flame Standards in Photometry . . . . . *E. B. Rosa and E. C. Crittenden*
223. The Testing of Potentiometers . . . . . . *Frank Wenner and Ernest Weibel*
224. The Emissivity of Metals and Oxides.—I. Nickel Oxide (NiO) in the Range 600 to 1300° C . . . . *G. K. Burgess and P. D. Foote*
225. Adjustments of the Thomson Bridge in the Measurement of Very Low Resistances . . . . *Frank Wenner and Ernest Weibel*
226. Quantitative Experiments in Radiotelegraphic Transmission . *L. W. Austin*
227. Measurements on Standards of Radiation in Absolute Value . *W. W. Coblentz*
228. An Experimental Study on the Koepsel Permeameter . . *Charles W. Burrows*
229. Various Modifications of Bismuth-Silver Thermopiles Having a Continuous Absorbing Surface . . . . *W. W. Coblentz*
230. Combustion Calorimetry and the Heats of Combustion of Cane Sugar, Benzoic Acid, and Naphthalene . . . . *H. C. Dickinson*
231. Specific Heat of Copper in the Interval 0° to 50° C.; with a Note on Vacuum-Jacketed Calorimeters . . . . *D. R. Harper 3d*
232. Equilibrium in the System: Lead Acetate, Lead Oxide, and Water, at 25° . . . . *Richard F. Jackson*
233. A Watthour Meter Method of Testing Instrument Transformers . *P. G. Agnew*
234. Insulating Properties of Solid Dielectrics . . . . *Harvey L. Curtis*
235. A Direct-Reading Instrument for Measuring the Logarithmic Decrement and Wave Length of Electromagnetic Waves . . *Frederick A. Kolster*
236. Electrical Resistance and Critical Ranges of Pure Iron . . . . *G. K. Burgess and I. N. Kellberg*
237. Absorption, Reflection, and Dispersion Constants of Quartz . *W. W. Coblentz*

238. Characteristic Equations of Tungsten Filament Lamps and Their Application in Heterochromatic Photometry . *G. W. Middlekauff and J. F. Skogland*
239. A Vibration Electrometer . . . . *W. W. Coblentz*
240. Studies on the Silver Voltameter . . . . . *G. A. Hulett and G. W. Vinal*
241. A Wheatstone Bridge for Resistance Thermometry . . . . *C. W. Waidner, H. C. Dickinson, E. F. Mueller, and D. R. Harper 3d*
242. The Emissivity of Metals and Oxides.—II. Measurements with the Micropyrometer . . . . *G. K. Burgess and R. G. Waltenberg*
243. The Emissivity of Metals and Oxides.—III. The Total Emissivity of Platinum and the Relation Between Total Emissivity and Resistivity . . . . *Paul D. Foote*
244. A Comparison of Stellar Radiometers and Radiometric Measurements on 110 Stars . . . . *W. W. Coblentz*
245. Temperature Coefficient of Magnetic Permeability Within the Working Range . . . . *R. L. Sanford*
246. Methods of Measuring the Inductances of Low Resistance Standards . . . . *Frank Wenner, Ernest Weibel, and F. B. Silsbee*
247. An Aneroid Calorimeter . . . . *H. C. Dickinson and N. S. Osborne*
248. Specific Heat and Heat of Fusion of Ice . *H. C. Dickinson and N. S. Osborne*
249. The Emissivity of Metals and Oxides.—IV. Iron Oxide . . . . *George K. Burgess and Paul D. Foote*
250. Characteristics of Radiation Pyrometers . *George K. Burgess and Paul D. Foote*
251. Interference Measurements of Wave Lengths in the Iron Spectrum (2851–3701), with Notes on Comparisons of Lengths of Light Waves by Interference Methods, and Some Wave Lengths in the Spectrum of Neon Gas . . . . *Keivin Burns and W. F. Meggers*
252. Effective Resistance and Inductance of Iron and Bimetallic Wires . . . . *John M. Miller*
253. A Direct-Reading Device for use in Computing Characteristics of Vacuum Tungsten Lamps . . . . *J. F. Skogland*
254. A Study of the Quality of Platinum Ware . *George K. Burgess and P. D. Sale*
255. Calculation of the Maximum Force Between Two Coaxial Circular Currents . . . . *Frederick W. Grover*
256. Construction of Primary Mercurial Resistance Standards . . . . *F. A. Wolff, M. P. Shoemaker, and C. A. Briggs*
257. Note on the Resistance of Radiotelegraphic Antennas . . . . *L. W. Austin*
258. A Method of Measuring Earth Resistivity . . . . *Frank Wenner*
259. A New Relation Derived from Planck's Law . . . . *Paul D. Foote*
260. "Center of Gravity" and "Effective Wave Length" of Transmission of Pyrometer Color Screens, and the Extrapolation of the High Temperature Scale . . . . *Paul D. Foote*
261. Studies of Instruments for Measuring Radiant Energy in Absolute Value: An Absolute Thermopile . . . *W. W. Coblentz and W. B. Emerson*
262. Present Status of the Determination of the Constant of Total Radiation from a Black Body . . . . *W. W. Coblentz*
263. Illumination from a Radiating Disk . . . . *Paul D. Foote*
264. Photometry of the Gas-Filled Lamp . *G. W. Middlekauff and J. F. Skogland*
265. Life Testing of Incandescent Lamps at the Bureau of Standards . . . . *G. W. Middlekauff, B. Mulligan, and J. F. Skogland*

CIRCULARS

1. Verification of Standards and Measuring Instruments.
2. Measurements of Length and Area, Including Thermal Expansion.
3. Verification of Standards of Mass.
4. Verification of Standards of Capacity.
5. Testing of Clinical Thermometers.
6. Fees for Electric, Magnetic, and Photometric Testing.
7. Pyrometer Testing and Heat Measurements.
8. Testing of Thermometers.
9. Testing of Glass Volumetric Apparatus.
10. Legal Weights (in pounds) per Bushel of Various Commodities.
11. The Standardization of Bomb Calorimeters.

12. Verification of Polariscopic Apparatus.
13. United States Government Standard Specifications for Incandescent Electric Lamps.
14. Analyzed Irons and Steels—Methods of Analysis.
15. The International Unit of Light.
16. The Testing of Hydrometers.
17. Magnetic Testing.
18. Standard Gauge for Sheet and Plate Iron and Steel.
19. Standard Density and Volumetric Tables.
20. Testing of Electrical Measuring Instruments.
21. Precision Measurements of Resistance and Electromotive Force.
22. Standard Specifications for Transformers, Oil-immersed, Self-cooled, 60-cycle, 2200 Volts.
23. Standardization of Electrical Practice in Mines.
24. Publications of the Bureau of Standards.
25. Standard Analyzed Samples—General Information.
26. Analyzed Iron and Manganese Ores—Methods of Analysis.
27. The Testing and Properties of Optical Instruments.
28. The Determination of the Optical Properties of Materials.
29. Announcement of a Change in the Value of the International Volt.
30. Lime: Its Properties and Uses.
31. Copper Wire Tables.
32. Standards of Gas Service.
33. United States Government Specification for Portland Cement.
34. The Relation of the Horsepower to the Kilowatt.
35. Melting Points of Chemical Elements.
36. The Testing and Properties of Electrical Condensers.
37. Electric Wire and Cable Terminology.
38. The Testing of Mechanical Rubber Goods.
39. Specifications for and Measurement of Standard Sieves.
40. Sodium Oxalate as a Standard in Volumetric Analysis.
41. Testing and Properties of Textile Materials.
42. Metallographic Testing.
43. The Metric Carat.
44. Polarimetry.
45. The Testing of Materials.
46. The Testing of Barometers.
47. Units of Weight and Measure; Definitions and Tables of Equivalents.
48. Standard Methods of Gas Testing.
49. Safety Rules to be Observed in the Operation of Electrical Equipment and Lines.
50. National Standard Hose Couplings and Fittings for Public Fire Service.
51. Measurement of Time and Tests of Timepieces.
52. Regulation of Electrotyping Solutions.
53. The Composition, Properties, and Testing of Printing Inks.
54. Proposed National Electrical Safety Code.
55. Measurements for the Household.
56. Standards for Electric Service.
57. United States Standard Tables for Petroleum Oils.
58. Invar and Related Nickel Steels.
59. United States Standard Baumé Hydrometer Scales.
60. Electric Units and Standards.

TECHNOLOGIC PAPERS

1. The Effect of Preliminary Heat Treatment upon the Drying of Clays (53 pp.) . . . . *A. V. Bleininger*
2. The Strength of Reinforced Concrete Beams. Results of Tests of 333 Beams (first series) (200 pp.) . . . . . *R. L. Humphrey and L. H. Losse*
3. Tests of Absorptive and Permeable Properties of Portland Cement Mortars and Concretes, Together with Tests of Damp Proofing and Waterproofing Compounds and Materials (127 pp.) . . . . *Rudolph J. Wig*
4. The Effect of Added Fatty and Other Oils upon the Carbonization of Mineral Lubricating Oils (14 pp.) . . . . *C. E. Waters*

5. The Effect of High-Pressure Steam on the Crushing Strength of Portland Cement Mortar and Concrete (25 pp.) . . . . *R. J. Wig*
6. The Determination of Chromium and Its Separation from Vanadium in Steels (6 pp.) . . . . *J. R. Cain*
7. The Testing of Clay Refractories, With Special Reference to Their Load Carrying Capacity at Furnace Temperatures (78 pp.) . . . *A. V. Bleininger and G. H. Brown*
8. A Rapid Method for the Determination of Vanadium in Steels, Ores, etc., Based on Its Quantitative Inclusion by the Phosphomolybdate Precipitate (20 pp.) . . . . . . *J. R. Cain and J. C. Hostetter*
9. The Density and Thermal Expansion of Linseed Oil and Turpentine (27 pp.) . . . . *H. W. Bearce*
10. The Melting Points of Fire Bricks (17 pp.) . . . . *C. W. Kanolt*
11. Comparison of Five Methods Used to Measure Hardness (27 pp.) . *Ralph P. Devries*
12. Action of the Salts in Alkali Water and Sea Water on Cements (157 pp.) . . . . *P. H. Bates, A. J. Phillips, and R. J. Wig*
13. The Evaporation Test for Mineral Lubricating and Transformer Oils (13 pp.) . . . . *C. E. Waters*
14. Legal Specifications for Illuminating Gas (31 pp.) . *E. B. Rosa and R. S. McBride*
15. Surface Insulation of Pipes (44 pp.) . . . *Burton McCollum and O. S. Peters*
16. The Manufacture of Lime (130 pp.) . . . . *W. E. Emley*
17. The Function of Time in the Vitrification of Clays (26 pp.) . . . . *G. H. Brown and G. A. Murray*
18. Electrolysis in Concrete (137 pp.) . *E. B. Rosa, Burton McCollum, and O. S. Peters*
19. Physical Testing of Cotton Yarns (31 pp.) . . . . *W. S. Lewis*
20. Determination of Sulphur in Illuminating Gas (46 pp.) . . . . . *R. S. McBride and E. R. Weaver*
21. The Dehydration of Clays (23 pp.) . . . *G. H. Brown and E. T. Montgomery*
22. The Effect of Overfiring Upon the Structure of Clays (23 pp.) . . . *A. V. Bleininger and E. T. Montgomery*
23. The Technical Control of the Colloidal Matter of Clays (118 pp.) . *H. E. Ashley*
24. The Determination of Phosphorus in Steels Containing Vanadium (11 pp.) . . . . *J. R. Cain and F. H. Tucker*
25. Electrolytic Corrosion of Iron in Soils (69 pp.) . *Burton McCollum and K. H. Logan*
26. Earth Resistance and Its Relation to Electrolysis . . . . *Burton McCollum and K. H. Logan*
27. Special Studies in Electrolysis Mitigation.—I. A Preliminary Study of Conditions in Springfield, Ohio, with Recommendations for Mitigation and Control (55 pp.) . . . . *E. B. Rosa and Burton McCollum*
28. Methods of Making Electrolysis Surveys . *Burton McCollum and A. H. Ahlborn*
29. The Variation in Results of Sieving with Standard Cement Sieves (16 pp.) . . . . *R. J. Wig and J. C. Pearson*
30. The Viscosity of Porcelain Bodies (11 pp.) . *A. V. Bleininger and Paul Teetor*
31. Some Leadless Boro-Silicate Glazes Maturing at about 1100° C. (22 pp.) . . . . *E. T. Montgomery*
32. Special Studies in Electrolysis Mitigation.—II. Electrolysis from Electric Railway Currents and Its Prevention—Experimental Test on a System of Insulated Negative Feeders in St. Louis (34 pp.) . . . . *E. B. Rosa, Burton McCollum, and K. H. Logan*
33. Determination of Carbon in Steel and Iron by the Barium Carbonate Titration Method (12 pp.) . . . . *J. R. Cain*
34. Determination of Ammonia in Illuminating Gas (23 pp.) . . . . *J. D. Edwards*
35. Combustion Method for the Direct Determination of Rubber (11 pp.) . *L. G. Wesson*
36. Industrial Gas Calorimetry (150 pp.) . . . *C. W. Waidner and E. F. Mueller*
37. Iodine Number of Linseed and Petroleum Oils (17 pp.) . . . . . *W. H. Smith and J. B. Tuttle*
38. Observations on Finishing Temperatures and Properties of Rails (63 pp.) . . *G. K. Burgess, J. J. Crowe, H. S. Rawdon, and R. G. Waltenberg*
39. Analysis of Printing Inks (20 pp.) . . . . . *J. B. Tuttle and W. H. Smith*
40. The Veritas Firing Rings (10 pp.) . . . . *A. V. Bleininger and G. H. Brown*
41. Lead Acetate Test for Hydrogen Sulphide in Gas (46 pp.) . . . . *R. S. McBride and J. D. Edwards*
42. Standardization of No. 200 Cement Sieves (51 pp.) *R. J. Wig and J. C. Pearson*

43. Hydration of Portland Cement (71 pp.) . . . *A. A. Klein and A. J. Phillips*
44. Investigation of the Durability of Cement Drain Tile and Alkali Soils (56 pp.) . . *R. J. Wig and G. M. Williams, in cooperation with S. H. McCrory, E. C. Bebb, and L. R. Ferguson*
45. A Study of Some Recent Methods for the Determination of Total Sulphur in Rubber (16 pp.) . . . . *J. B. Tuttle and A. Isaacs*
46. A Study of the Atterberg Plasticity Method (18 pp.) . . . *Charles S. Kinnison*
47. Value of the High-Pressure Steam Test of Portland Cements (34 pp.) . . . . *R. J. Wig and H. A. Davis*
48. An Air-Analyzer for Determining the Fineness of Portland Cement (74 pp.) . . . . *H. C. Pearson and W. H. Sligh*
49. Emergent Stem Correction for Thermometers in Creosote Oil Distillation Flasks (21 pp.) . . . . *R. M. Wilhelm*
50. Viscosity of Porcelain Bodies High in Feldspar (7 pp.) . . . . *A. V. Bleininger and C. S. Kinnison*
51. Use of Sodium Salts in the Purification of Clays and in the Casting Process (40 pp.) . . . . *A. V. Bleininger*
52. Electrolysis and Its Mitigation (37 pp.) . . *E. B. Rosa and Burton McCollum*
53. An Investigation of Fusible Tin Boiler Plugs (143 pp.) . . . . *George K. Burgess and P. D. Merica*
54. Special Studies in Electrolysis Mitigation.—III. A Report on Conditions in Springfield, Ohio, with Insulated Feeder System Installed (64 pp.) . . . . *Burton McCollum and G. H. Ahlborn*
55. Special Studies in Electrolysis Mitigation.—IV. A Preliminary Study of Electrolysis Conditions in Elyria, Ohio, with Recommendations for Mitigation (49 pp.) . . . . *Burton McCollum and K. H. Logan*
56. Protection of Life and Property Against Lightning (127 pp.) . . . *O. S. Peters*
57. Difference in Weight Between Raw and Clean Wools (5 pp.) . . *Walter S. Lewis*
58. Compressive Strength of Portland Cement Mortars and Concretes . . . . *R. J. Wig, G. M. Williams, and E. R. Gates*
59. Standard Test Specimens of Zinc Bronzes, .88 cu, .10 Sn, .02 Zn.—Part I. Preparation of Specifications. Part II. Microstructure (67 pp.) . . . . *C. P. Karr and Henry S. Rawdon*
60. Microstructural Changes Accompanying the Annealing of Bronze (Cu .88, Sn .10, Zn .02) (17 pp.) . . . . *Henry S. Rawdon*
61. Some Foreign Specifications for Railway Materials (132 pp.) . . . . *George K. Burgess and P. D. Merica*
62. Modern Practice in the Construction and Maintenance of Rail Joints and Bonds in Electric Railways (123 pp.) . . . . *E. R. Shepard*
63. Leakage of Current from Electric Railways (31 pp.) . . . . *Burton McCollum and K. H. Logan*
64. Determination of Barium Carbonate and Barium Sulphate in Vulcanized Rubber Goods (5 pp.) . . . . *J. B. Tuttle*
65. Determination of Oil and Resin in Varnish (32 pp.) . . . . *E. W. Boughton*
66. Detection of Resin in Drier (9 pp.) . . . . *E. W. Boughton*
67. Some Qualitative Tests for Gum Arabic and Its Quantitative Determination (15 pp.) . . . . *C. E. Waters and J. B. Tuttle*
68. Standardization of Automobile Tire Fabric Testing (18 pp.) . . . . *W. S. Lewis and C. J. Cleary*
69. Determination of Carbon in Steels and Irons by Direct Combustion in Oxygen at High Temperatures (10 pp.) . . . *J. R. Cain and H. E. Cleaves*
70. Durability of Stucco and Plaster Construction . . . . *R. J. Wig*
71. Effect of Certain Pigments on Linseed Oil (16 pp.) . . . . *E. W. Boughton*
72. Influence of Frequency of Alternating or Infrequently Reversed Current on Electrolytic Corrosion . . . . *Burton McCollum and G. H. Ahlborn*
73. Data on the Oxidation of Automobile Cylinder Oils . . . . . *C. E. Waters*

MISCELLANEOUS

International Metric System. (Chart.)
The International Metric System of Weights and Measures. (Pamphlet.)
First Conference on the Weights and Measures of the United States.
Second Annual Conference on the Weights and Measures of the United States.
Third Annual Conference on the Weights and Measures of the United States.
Fourth Annual Conference on the Weights and Measures of the United States.
Fifth Annual Conference on the Weights and Measures of the United States.
Sixth Annual Conference on the Weights and Measures of the United States.
Seventh Annual Conference on the Weights and Measures of the United States.
Eighth Annual Conference on the Weights and Measures of the United States.
Ninth Annual Conference on the Weights and Measures of the United States.
Tenth Annual Conference on the Weights and Measures of the United States.
State and National Laws Concerning the Weights and Measures of the United States (second edition).
The National Bureau of Standards. (Descriptive pamphlet.)
Report of the International Committee on Electrical Units and Standards.

◆

This book should be returned to the Library on or before the last date stamped below.

A fine of five cents a day is incurred by retaining it beyond the specified time.

Please return promptly.

JAN 21 1932

[illegible] 1932

CANCELLED

CANCELLED

MAR 1 1965 H

537-[illegible]